Collider Phenomenology

— From basic knowledge to new physics searches

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Outline:

Lecture I: Colliders and Detectors

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Lecture II: Basics Techniques and Tools

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Lecture III: (a). An e^+e^- Linear Collider

(b). Perturbative QCD at Hadron Colliders

(c). Hadron Colliders Physics

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Main reference: TASI 04 Lecture notes
hep-ph/0508097,
plus the other related lectures in this school.

III(a). An e^+e^- Linear Collider (ILC)

(A.) Simple Formalism

Event rate of a reaction:

$$\begin{aligned} R(s) &= \sigma(s)\mathcal{L}, \quad \text{for constant } \mathcal{L} \\ &= \mathcal{L}\int d\tau \frac{dL}{d\tau}\,\sigma(\hat{s}), \quad \tau = \frac{\hat{s}}{s}. \end{aligned}$$

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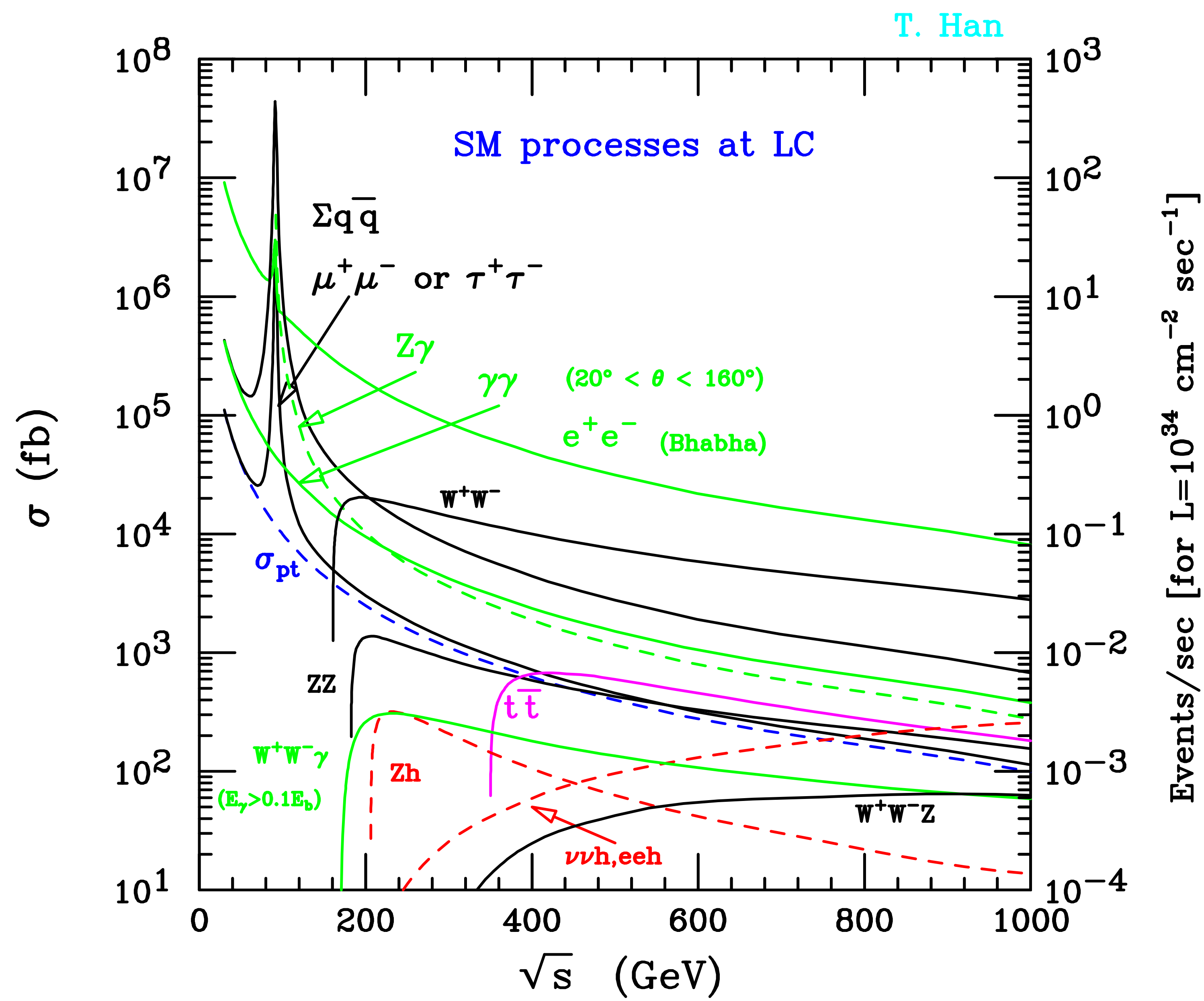
As for the differential production cross section of two-particle a, b,

$$\frac{d\sigma(e^+e^- \to ab)}{d\cos\theta} = \frac{\beta}{32\pi s}\overline{\sum}|\mathcal{M}|^2$$

where

- $\beta = \lambda^{1/2}(1, m_a^2/s, m_b^2/s)$, is the speed factor for the out-going particles in the c.m. frame, and $p_{cm} = \beta\sqrt{s}/2$,
- $\overline{\sum}|\mathcal{M}|^2$ the squared matrix element, summed and averaged over quantum numbers (like color and spins etc.)
- unpolarized beams so that the azimuthal angle trivially integrated out,

Total cross sections and event rates for SM processes:



(B). Resonant production: Breit-Wigner formula

$$\frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}$$

If the energy spread $\delta\sqrt{s} \ll \Gamma_V$, the line-shape mapped out:

$$\sigma(e^+e^- \to V^* \to X) = \frac{4\pi(2j+1)\Gamma(V \to e^+e^-)\Gamma(V \to X)}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2} \; \frac{s}{M_V^2},$$

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If $\delta\sqrt{s} \gg \Gamma_V$, the narrow-width approximation:

$$\begin{aligned}\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2} \quad &\to \quad \frac{\pi}{M_V\Gamma_V}\ \delta(s-M_V^2),\\ \sigma(e^+e^- \to V^* \to X) \quad &= \quad \frac{2\pi^2(2j+1)\Gamma(V\to e^+e^-)BF(V\to X)}{M_V^2}\frac{dL(\hat{s}=M_V^2)}{d\sqrt{\hat{s}}}\end{aligned}$$

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`Exercise 3.1: sketch the derivation of these two formulas, assuming a Gaussian distribution for`

$$\frac{dL}{d\sqrt{\hat{s}}} = \frac{1}{\sqrt{2\pi}\ \Delta}\exp[\frac{-(\sqrt{\hat{s}}-\sqrt{s})^2}{2\Delta^2}].$$

Note: Away from resonance

For an s-channel or a finite-angle scattering:

$$\sigma \sim \frac{1}{s}.$$

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For an s-channel or a finite-angle scattering:

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For forward (co-linear) scattering:

$$\sigma \sim \frac{1}{M_V^2} \ln^2 \frac{s}{M_V^2}.$$

(C). Fermion production:

Common processes: $e^-e^+ \to f\bar{f}$.
For most of the situations, the scattering matrix element can be casted into a $V \pm A$ chiral structure of the form (sometimes with the help of Fierz transformations)

$$\mathcal{M} = \frac{e^2}{s} Q_{\alpha\beta}\ [\bar{v}_{e^+}(p_2)\gamma^\mu P_\alpha u_{e^-}(p_1)]\ [\bar{\psi}_f(q_1)\gamma_\mu P_\beta \psi'_{\bar{f}}(q_2)],$$

where $P_\mp = (1 \mp \gamma_5)/2$ are the L, R chirality projection operators, and $Q_{\alpha\beta}$ are the bilinear couplings governed by the underlying physics of the interactions with the intermediate propagating fields.
With this structure, the scattering matrix element squared:

$$\begin{aligned} \overline{\sum} |\mathcal{M}|^2 &= \frac{e^4}{s^2} \left[(|Q_{LL}|^2 + |Q_{RR}|^2)\ u_i u_j + (|Q_{LL}|^2 + |Q_{RL}|^2)\ t_i t_j \right. \\ &+ \left. 2Re(Q^*_{LL} Q_{LR} + Q^*_{RR} Q_{RL}) m_f m_{\bar{f}} s \right], \end{aligned}$$

where $t_i = t - m_i^2 = (p_1 - q_1)^2 - m_i^2$ and $u_i = u - m_i^2 = (p_1 - q_2)^2 - m_i^2$.

`Exercise 3.2: Verify this formula.`

(D). Typical size of the cross sections:

- The simplest reaction

$$\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) \equiv \sigma_{pt} = \frac{4\pi\alpha^2}{3s}.$$

In fact, $\sigma_{pt} \approx 100$ fb$/(\sqrt{s}/\text{TeV})^2$ has become standard units to measure the size of cross sections.

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- At the ILC $\sqrt{s} = 500$ GeV,

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$$\sigma_{pt} \sim \sigma(ZZ) \sim \sigma(t\bar{t}) \sim 400 \text{ fb};$$
$$\sigma(u,d,s) \sim 9\sigma_{pt} \sim 3.6 \text{ pb};$$
$$\sigma(WW) \sim 20\sigma_{pt} \sim 8 \text{ pb}.$$

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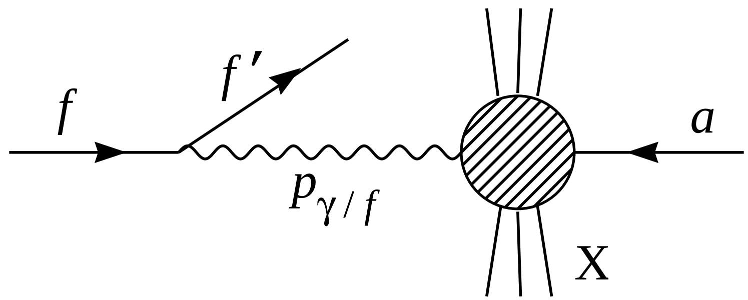
and

$$\sigma(ZH) \sim \sigma(WW \to H) \sim \sigma_{pt}/4 \sim 100\ \text{fb};$$
$$\sigma(WWZ) \sim 0.1\sigma_{pt} \sim 40\ \text{fb}.$$

(E). Gauge boson radiation:

A qualitatively different process is initiated from gauge boson radiation, typically off fermions:



The simplest case is the photon radiation off an electron, like:

$$e^+e^- \to e^+,\ \gamma^* e^- \to e^+e^-.$$

The dominant features are due to the result of a t-channel singularity, induced by the collinear photon splitting:

$$\sigma(e^- a \to e^- X) \approx \int dx\ P_{\gamma/e}(x)\ \sigma(\gamma a \to X).$$

The so called the effective photon approximation.

For an electron of energy E, the probability of finding a collinear photon of energy xE is given by

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{E^2}{m_e^2},$$

known as the Weizsäcker-Williams spectrum.

`Exercise 3.3: Try to derive this splitting function.`

We see that:

• m_e enters the log to regularize the collinear singularity;

• $1/x$ leads to the infrared behavior of the photon;

• This picture of the photon probability distribution is also valid for other photon spectrum:

Based on the back-scattering laser technique, it has been proposed to produce much harder photon spectrum, to construct a "photon collider"...

(massive) Gauge boson radiation:

A similar picture may be envisioned for the electroweak massive gauge bosons, $V = W^{\pm}, Z$.

Consider a fermion f of energy E, the probability of finding a (nearly) collinear gauge boson V of energy xE and transverse momentum p_T (with respect to $\vec{p}_f$) is approximated by

$$
\begin{aligned}
P^{T}_{V/f}(x, p_T^2) &= \frac{g_V^2 + g_A^2}{8\pi^2} \, \frac{1 + (1-x)^2}{x} \, \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2}, \\
P^{L}_{V/f}(x, p_T^2) &= \frac{g_V^2 + g_A^2}{4\pi^2} \, \frac{1 - x}{x} \, \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.
\end{aligned}
$$

Although the collinear scattering would not be a good approximation until reaching very high energies $\sqrt{s} \gg M_V$, it is instructive to consider the qualitative features.

(F). Beam polarization:

One of the merits for an e^+e^- linear collider is the possible high polarization for both beams.
Consider first the longitudinal polarization along the beam line direction. Denote the average $e^\pm$ beam polarization by $P^L_\pm$, with $P^L_\pm = -1$ purely left-handed and +1 purely right-handed.

The polarized squared matrix element, based on the helicity amplitudes $\mathcal{M}_{\sigma_{e-}\sigma_{e+}}$:

$$\overline{\sum}|\mathcal{M}|^2 = \frac{1}{4}[(1-P^L_-)(1-P^L_+)|\mathcal{M}_{--}|^2 + (1-P^L_-)(1+P^L_+)|\mathcal{M}_{-+}|^2$$
$$+(1+P^L_-(1-P^L_+)|\mathcal{M}_{+-}|^2 + (1+P^L_-)(1+P^L_+)|\mathcal{M}_{++}|^2].$$

Since the electroweak interactions of the SM and beyond are chiral:
Certain helicity amplitudes can be suppressed or enhanced by properly choosing the beam polarizations: *e.g.*, $W^\pm$ exchange ...

Furthermore, it is possible to produce transversely polarized beams with the help of a spin-rotator.

If the beams present average polarizations with respect to a specific direction perpendicular to the beam line direction, $-1 < P_{\pm}^{T} < 1$, then there will be one additional term in the limit $m_e \to 0$,

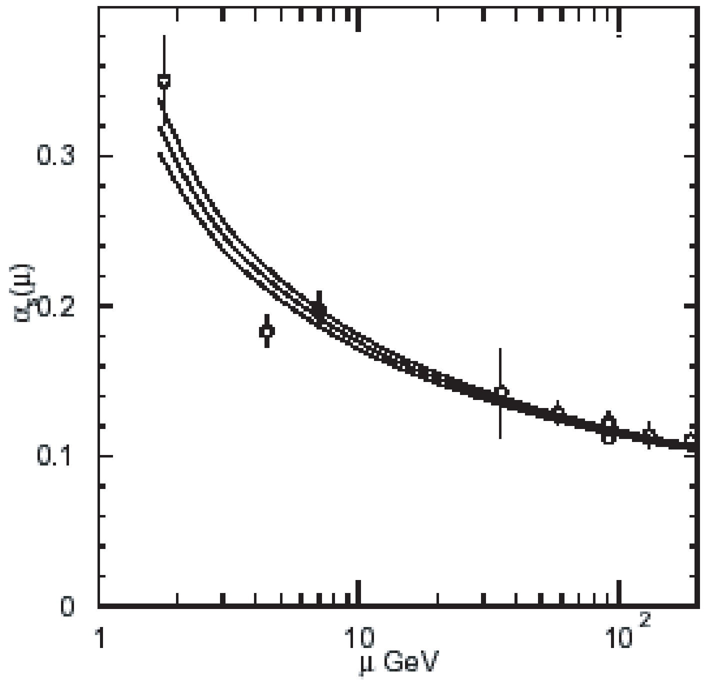
$$\frac{1}{4}\, 2\ P_{-}^{T} P_{+}^{T}\ \mathrm{Re}(\mathcal{M}_{-+}\mathcal{M}_{+-}^{*}).$$

The transverse polarization is particularly important when the interactions produce an asymmetry in azimuthal angle, such as the effect of CP violation.

III(b). Perturbative QCD

(A). Running of the strong coupling:

$$\alpha_s(Q_R^2) = \frac{12\pi}{(33 - 2n_f)\ln\frac{Q_R^2}{\Lambda_{QCD}^2}}\,, \qquad 11n_c - 2n_f > 0.$$



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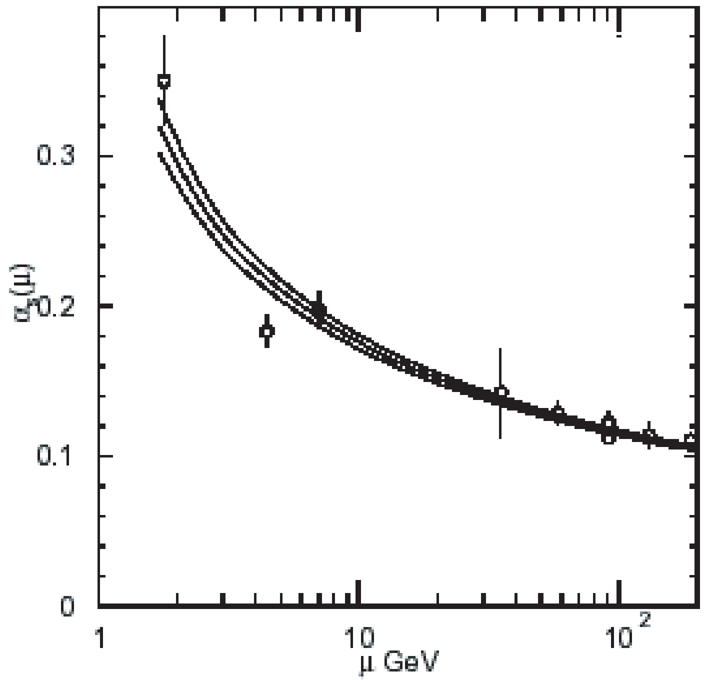

Significant implications (D. Gross, D. Politzer, F. Wilczek, Nobel Prize 2004):

† Confinement at low energies (hadrons: the observable world);

† Asymptotic freedom at high energies (quarks, gluons and perturbation techniques);

† Possibility of Grand Unification; Description of the early universe.

(B). Parton Distribution Functions (PDF)

• Factorization theorem:

In high energy collisions involving a hadron, the total cross sections can be factorized into two factors:

(1). hard subprocess of parton scattering with a large scale $\mu^2 \gg \Lambda^2_{QCD}$;

(2). "parton distribution functions" (hadronic structure with $Q^2 < \mu^2$.)

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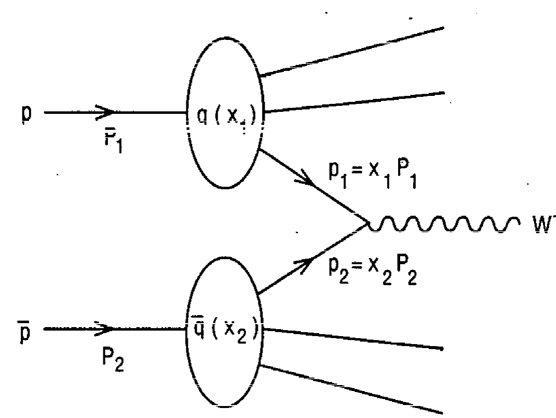
Observable cross sections at hadron level:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2)\ \hat{\sigma}_{parton}(s).$$



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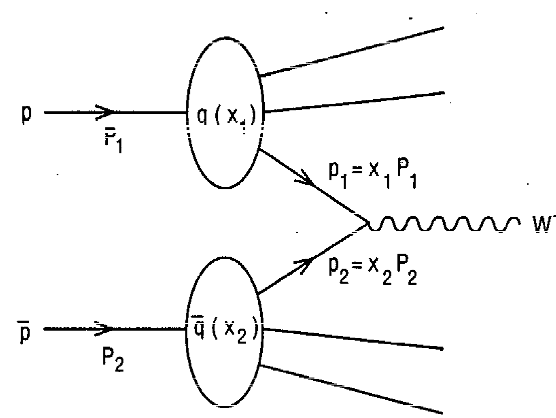

† $\hat{\sigma}_{parton}(s)$ is theoretically calculated by perturbation theory (in the SM or models beyond the SM).

Ultra violet (UV) divergence (beyond leading order) is renormalized;
Infra-red (IR) divergence is cancelled by soft gluon emissions;
Co-linear divergence (massless) is factorized into PDF
– The essence of "factorization theorem".

† $P(x, Q^2)$ is the "Parton Distribution Functions" (PDF): The probability of finding a parton P with a momentum fraction x inside a proton.

$P(x, Q^2)$ cannot be calculated from first principles, only extracted by fitting data, assuming a boundary condition at $Q_0^2 \sim$(2 GeV)^2.

The PDF's should match the parton-level cross section $\hat{\sigma}_{parton}(s)$ at a given order in α_s.

† Q^2 is the "factorization scale", below which it is collinear physics. It is NOT uniquely determined, leading to intrinsic uncertainty in QCD perturbation predictions. But its uncertainty is reduced with higher order calculations.

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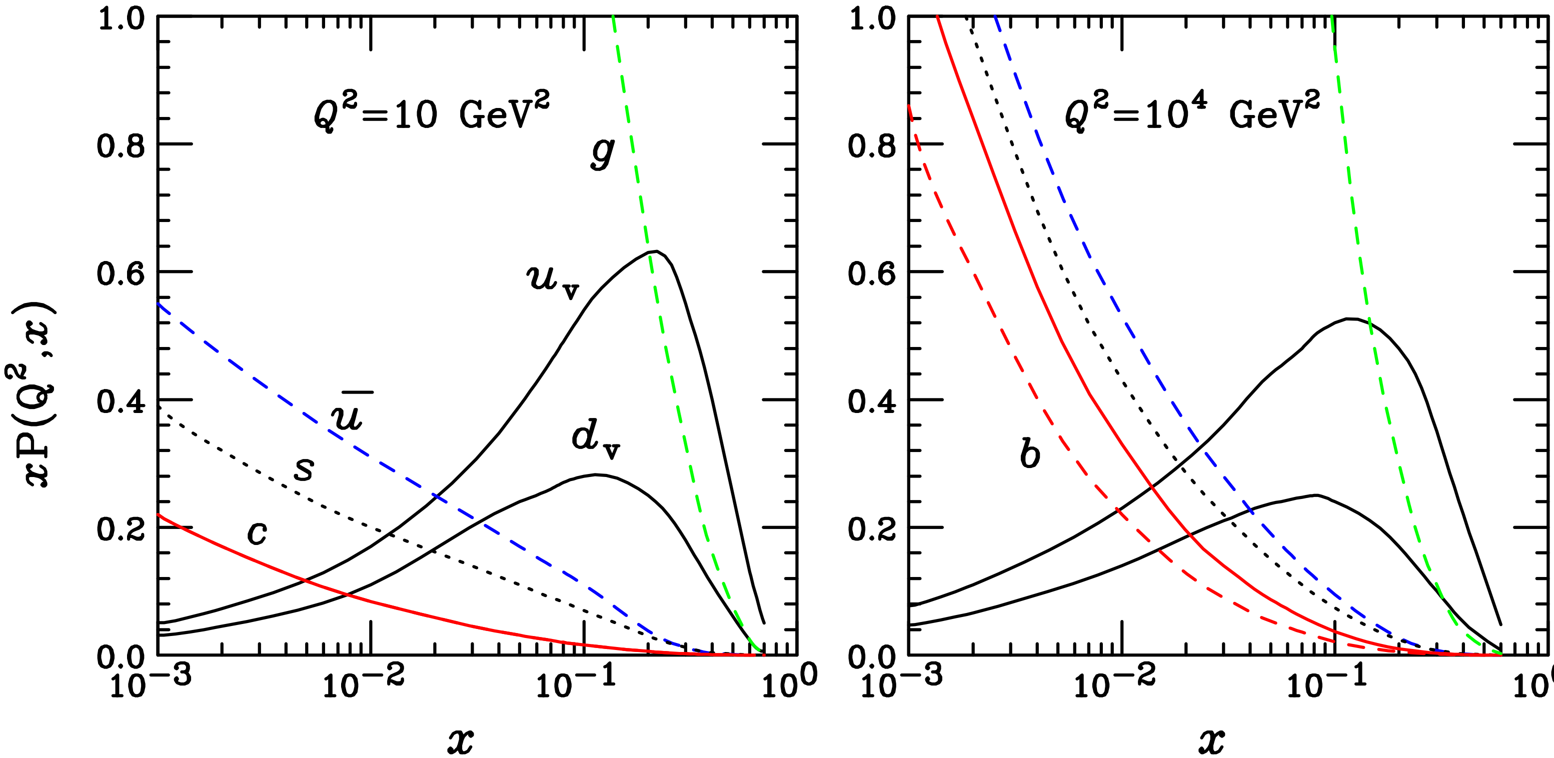
Several dedicated groups are developing PDF's:
CTEQ (Michigan State U.); MRSxxx (Durham U.) ... ...

Typical quark/gluon parton distribution functions:



(CTEQ-5)

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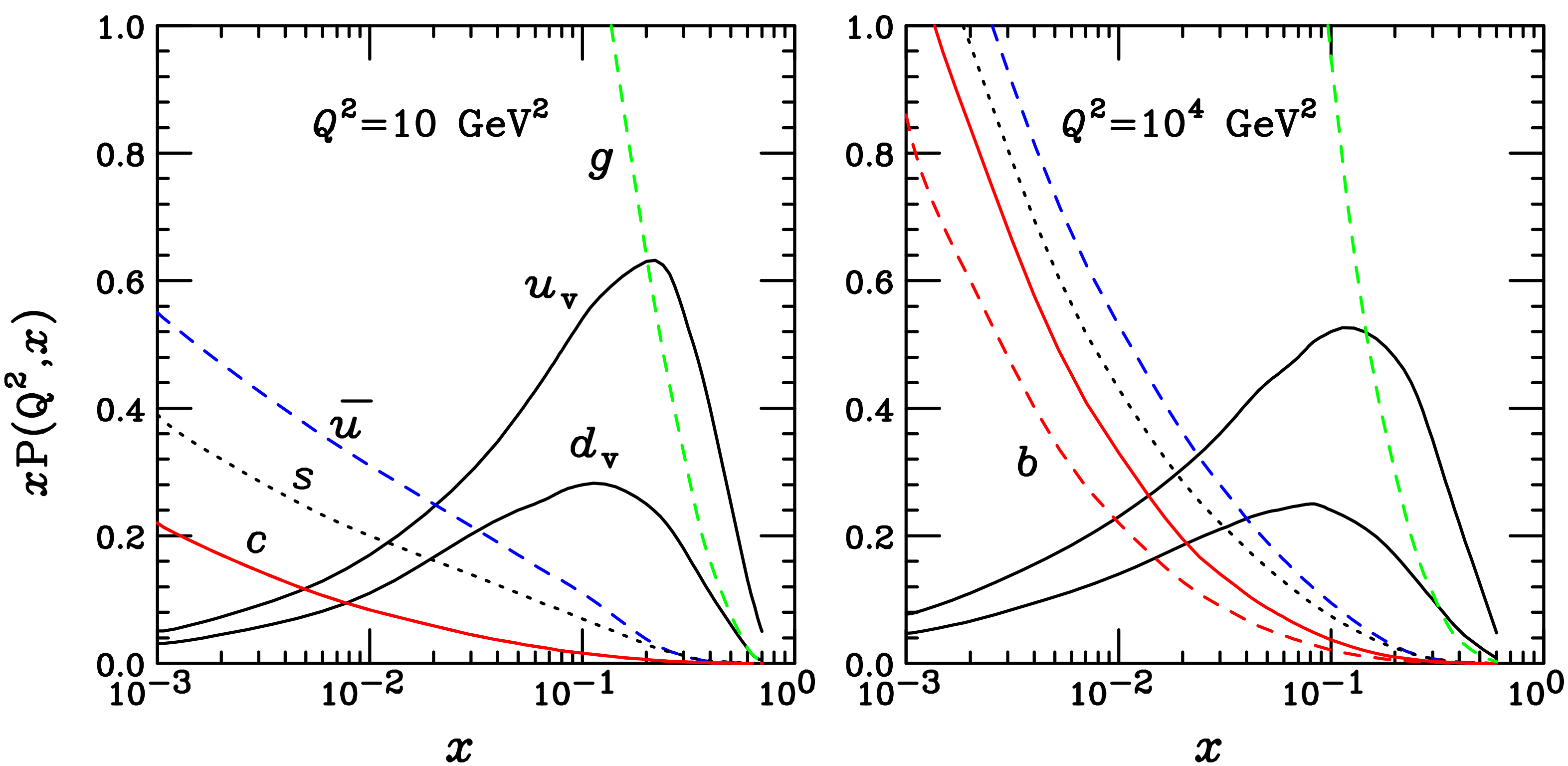

(CTEQ-5)

Better understanding of the SM cross section, in particular in QCD are crucial for observing new physics as deviations from the SM.

(C). Jets and fragmentation functions

Upon production of a colored parton (quark/gluon):

† At the scale $\Lambda_{QCD} \sim 10^{-24}$s or 1 fm, the parton "hadronizes (fragments)" into massive, color-neutral, hadrons $\pi,\ n,\ p,\ K\ \ldots$

The "fragmentation function" is like the reverse of the PDF:

$$\frac{d\sigma(pp \to hX)}{dE_h} = \sum_q \int \frac{d\sigma(pp \to qX)}{dE_q} \frac{dE_q}{E_q} f_q^h(z, Q^2)$$

where $z = E_h/E_q$.

Non-perturbative and cann't be calculated from first principles.

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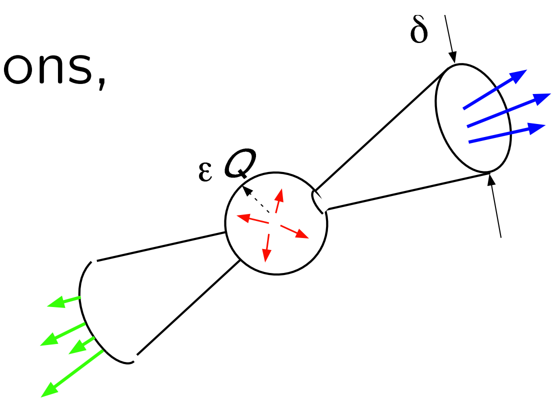
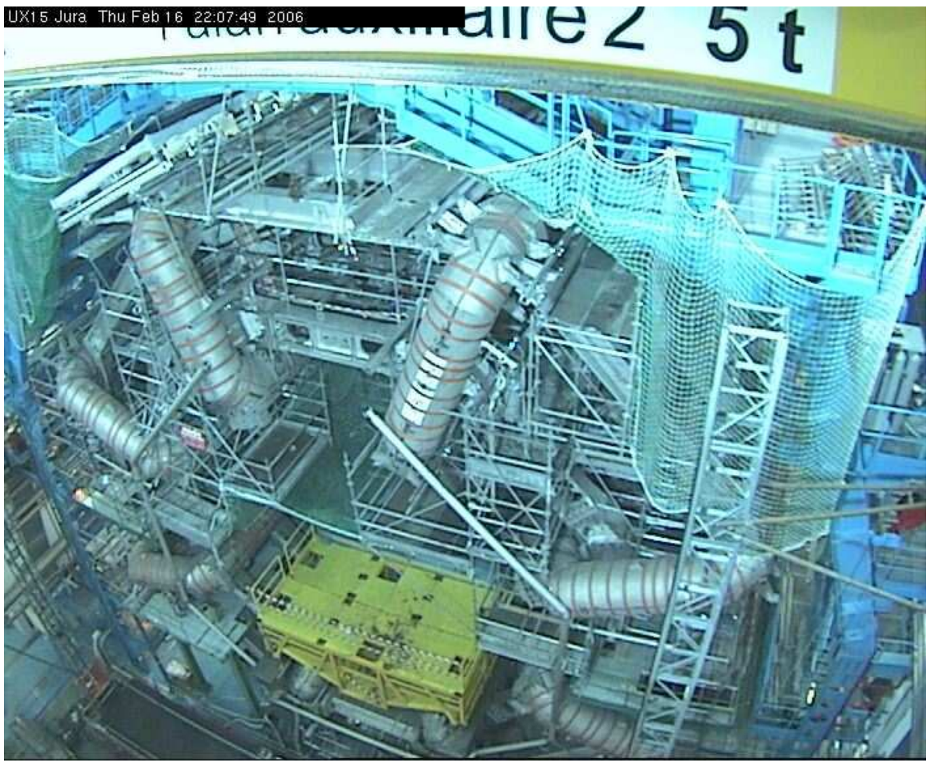
† For most of the purposes in high energy collisions, we do not need to keep track of the individual hadrons, and thus the collective and collimated hadrons form a "jet".



III(c). Hadron Collider Physics

(A). New HEP frontier: the LHC
Major discoveries and excitement ahead ...

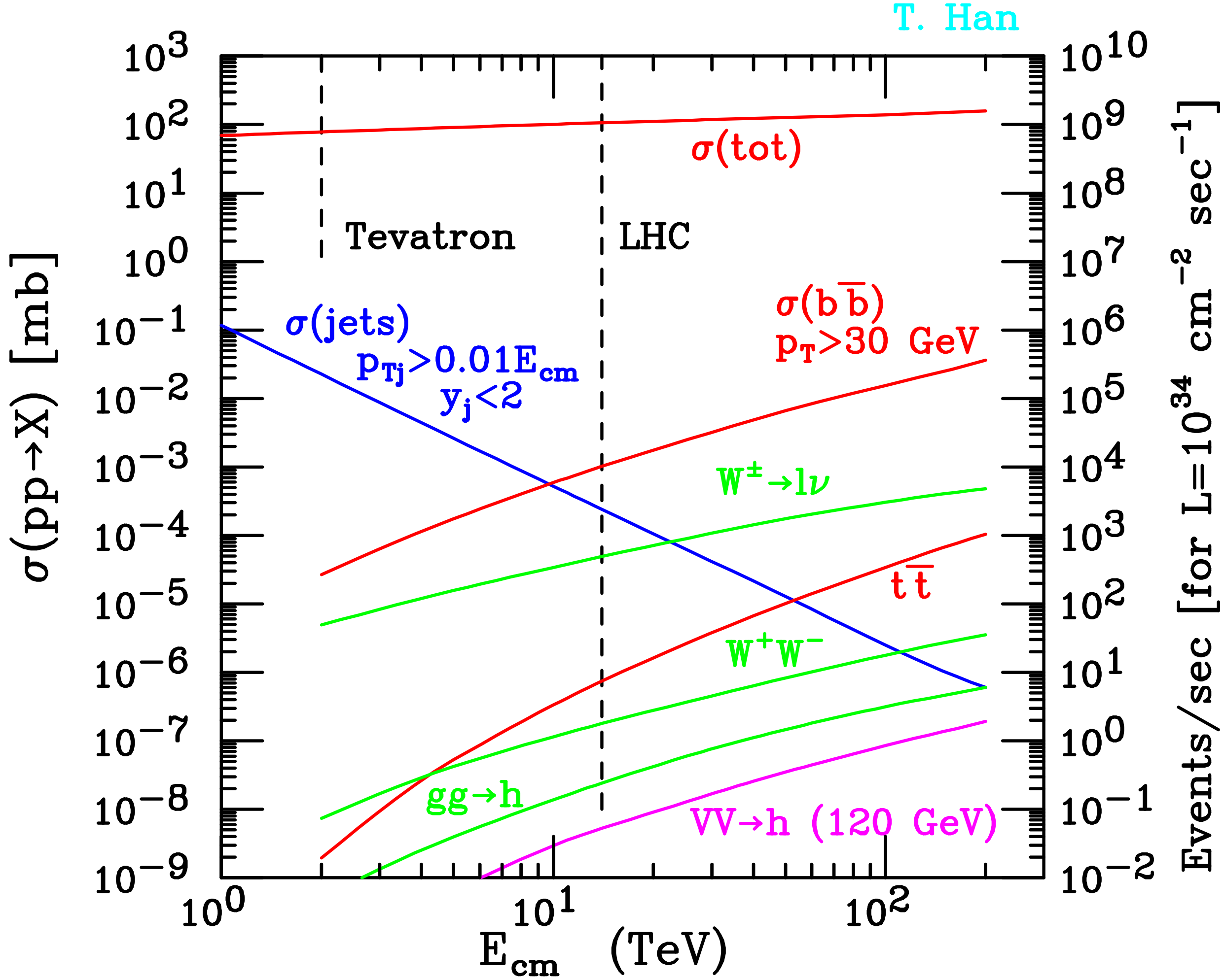


ATLAS (90m underground)

CMS

(New mission started in March 2010.)

LHC Event rates for various SM processes:



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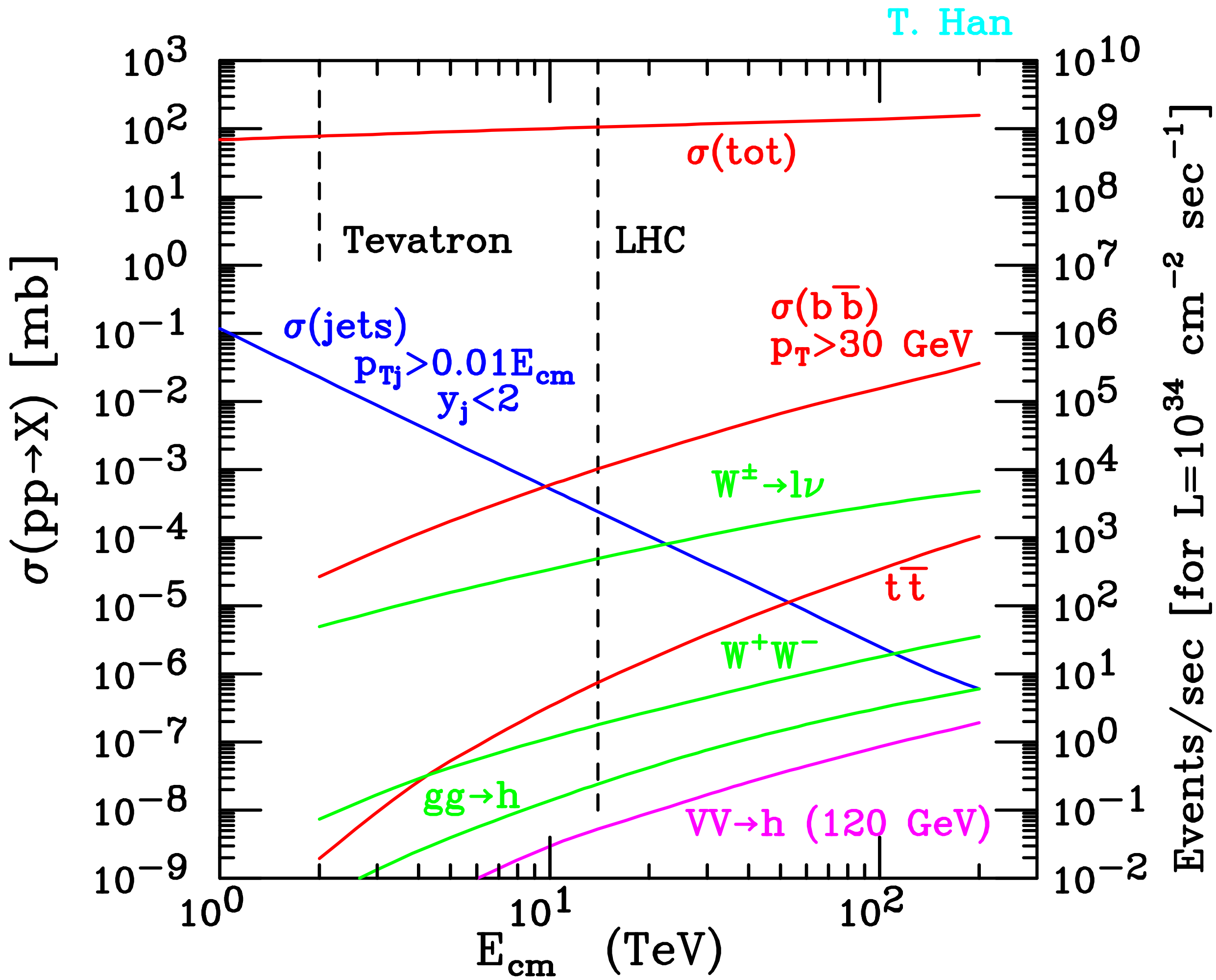

$10^{34}/\mathrm{cm}^2/\mathrm{s} \Rightarrow 100\ \mathrm{fb}^{-1}/\mathrm{yr}.$

Annual yield # of events $= \sigma \times L_{int}$:

10B $W^{\pm}$; 100M $t\bar{t}$; 10M W^+W^-; 1M H^0...

Great potential to open a new chapter of HEP!

Theoretical challenges:

Unprecedented energy frontier

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(a) Total hadronic cross section: Non-perturbative.
The order of magnitude estimate:

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$$\sigma(pp) \begin{cases} \approx 21.7 \ (\frac{s}{\mathrm{GeV}^2})^{0.0808} & \text{Empirical relation} \\ < \frac{\pi}{m_\pi^2} \ \ln^2 \frac{s}{s_0} & \text{Froissart bound.} \end{cases}$$

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- Accurate (higher orders) partonic cross sections $\hat{\sigma}_{parton}(s)$.
- Parton distributions functions to the extreme (density):

$Q^2 \sim (a\ few\ TeV)^2, \quad x \sim 10^{-3} - 10^{-6}$.

Experimental challenges:

- The large rate turns to a hostile environment:
  - $\approx$ 1 billion event/sec: impossible read-off !
  - $\approx$ 1 interesting event per $1,000,000$: selection (triggering).

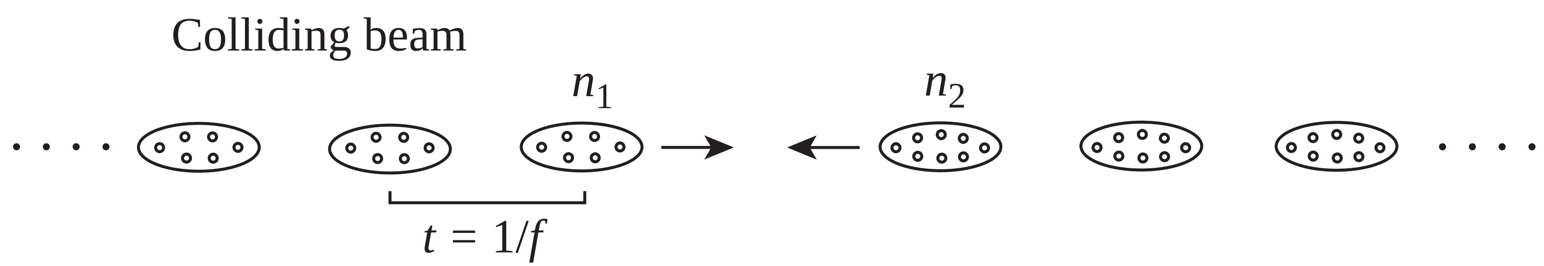
Experimental challenges:

- The large rate turns to a hostile environment:

  ≈ 1 billion event/sec: impossible read-off !

  ≈ 1 interesting event per $1,000,000$: selection (triggering).

  ≈ 25 overlapping events/bunch crossing:



$\Rightarrow$ Severe backgrounds!

Triggering thresholds:

	ATLAS	
Objects	η	p_T (GeV)
μ inclusive	2.4	6 (20)
e/photon inclusive	2.5	17 (26)
Two e's or two photons	2.5	12 (15)
1-jet inclusive	3.2	180 (290)
3 jets	3.2	75 (130)
4 jets	3.2	55 (90)
τ/hadrons	2.5	43 (65)
$\not{E}_T$	4.9	100
Jets+$\not{E}_T$	3.2, 4.9	50,50 (100,100)

($\eta = 2.5 \Rightarrow 10^\circ$; $\eta = 5 \Rightarrow 0.8^\circ$.)

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$(\eta = 2.5 \Rightarrow 10^\circ; \qquad \eta = 5 \Rightarrow 0.8^\circ.)$

With optimal triggering and kinematical selections:

$p_T \geq 30 - 100$ GeV, $\quad |\eta| \leq 3 - 5; \quad \not{E}_T \geq 100$ GeV.

(B). Special kinematics for hadron colliders

Hadron momenta: $P_A = (E_A, 0, 0, p_A), \quad P_B = (E_A, 0, 0, -p_A),$
The parton momenta: $p_1 = x_1 P_A, \quad p_2 = x_2 P_B.$

Then the parton c.m. frame moves randomly, even by event:

$$\beta_{cm} = \frac{x_1 - x_2}{x_1 + x_2}, \quad \text{or :}$$

$$y_{cm} = \frac{1}{2} \ln \frac{1 + \beta_{cm}}{1 - \beta_{cm}} = \frac{1}{2} \ln \frac{x_1}{x_2}, \quad (-\infty < y_{cm} < \infty).$$

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The four-momentum vector transforms as

$$\begin{aligned}\begin{pmatrix} E' \\ p'_z \end{pmatrix} &= \begin{pmatrix} \gamma & -\gamma\ \beta_{cm} \\ -\gamma\ \beta_{cm} & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} \\ &= \begin{pmatrix} \cosh y_{cm} & -\sinh y_{cm} \\ -\sinh y_{cm} & \cosh y_{cm} \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}.\end{aligned}$$

This is often called the “boost”.

One wishes to design final-state kinematics invariant under the boost:
For a four-momentum $p \equiv p^{\mu} = (E, \vec{p})$,

$$\begin{aligned} E_T &= \sqrt{p_T^2 + m^2}, \quad y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}, \\ p^{\mu} &= (E_T \cosh y,\ p_T \sin \phi,\ p_T \cos \phi,\ E_T \sinh y), \\ \frac{d^3\vec{p}}{E} &= p_T dp_T d\phi\ dy = E_T dE_T d\phi\ dy. \end{aligned}$$

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$$

Due to random boost between Lab-frame/c.m. frame event-by-event,

$$
y' = \frac{1}{2}\ln\frac{E'+p'_z}{E'-p'_z} = \frac{1}{2}\ln\frac{(1-\beta_{cm})(E+p_z)}{(1+\beta_{cm})(E-p_z)} = y - y_{cm}.
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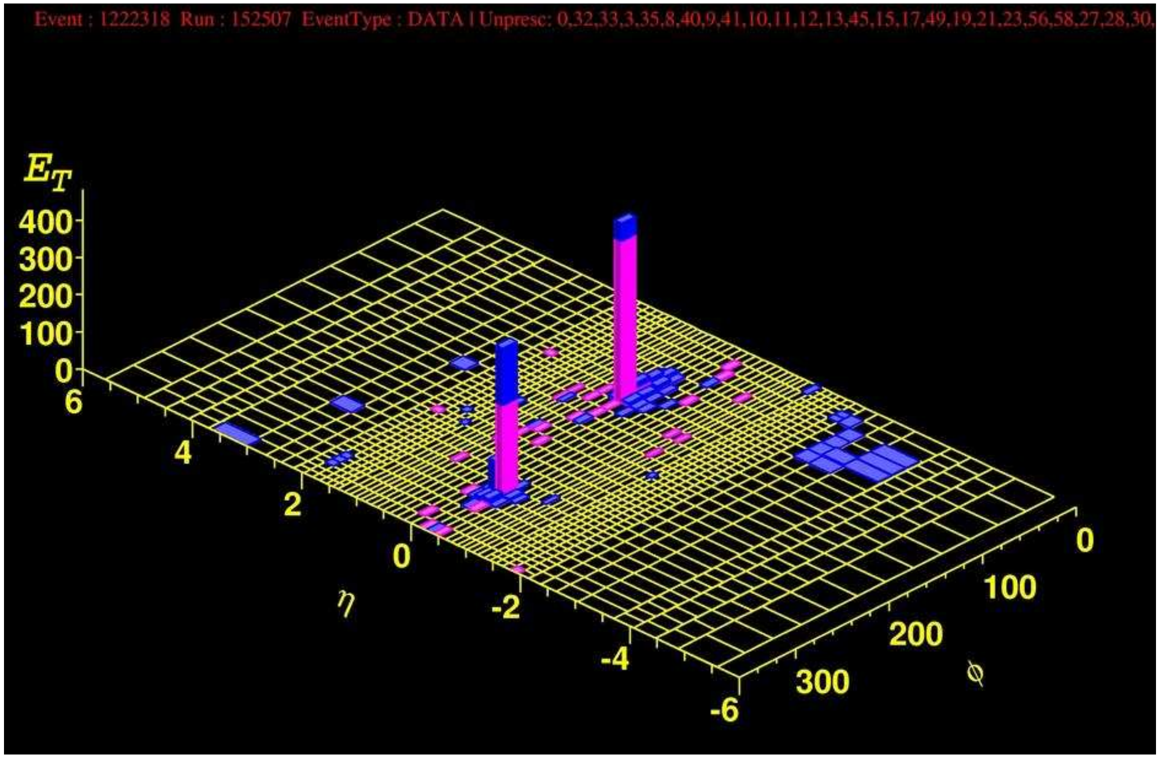
In the massless limit, rapidity $\rightarrow$ pseudo-rapidity:

$$y \rightarrow \eta = \frac{1}{2} \ln \frac{1 + \cos\theta}{1 - \cos\theta} = \ln \cot \frac{\theta}{2}.$$

`Exercise 4.1:  Verify all the above equations.`

The "Lego" plot:



A CDF di-jet event on a lego plot in the $\eta - \phi$ plane.

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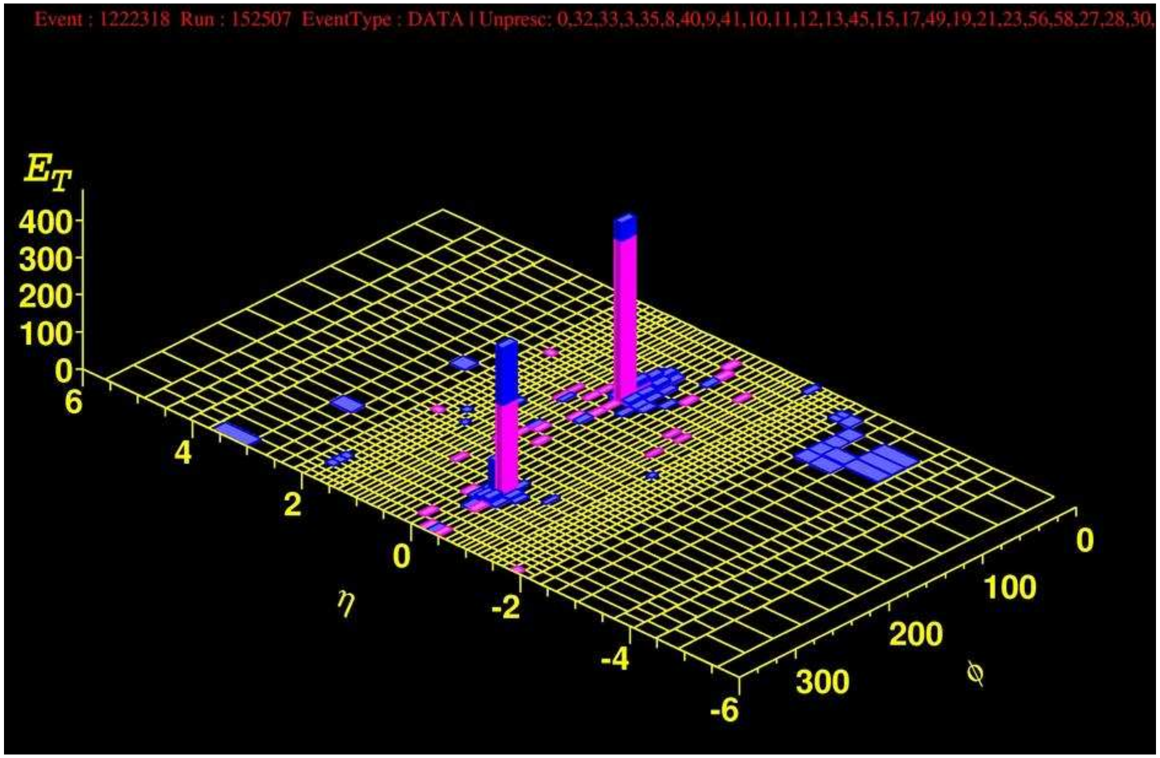

A CDF di-jet event on a lego plot in the $\eta - \phi$ plane.

$\phi, \Delta y = y_2 - y_1$ is boost-invariant.

Thus the “separation” between two particles in an event

$\Delta R = \sqrt{\Delta\phi^2 + \Delta y^2}$ is boost-invariant,

and lead to the “cone definition” of a jet.

(C). Hadron collider status:

The Tevatron rocks, and the LHC delivers !

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At the Tevatron Run II:

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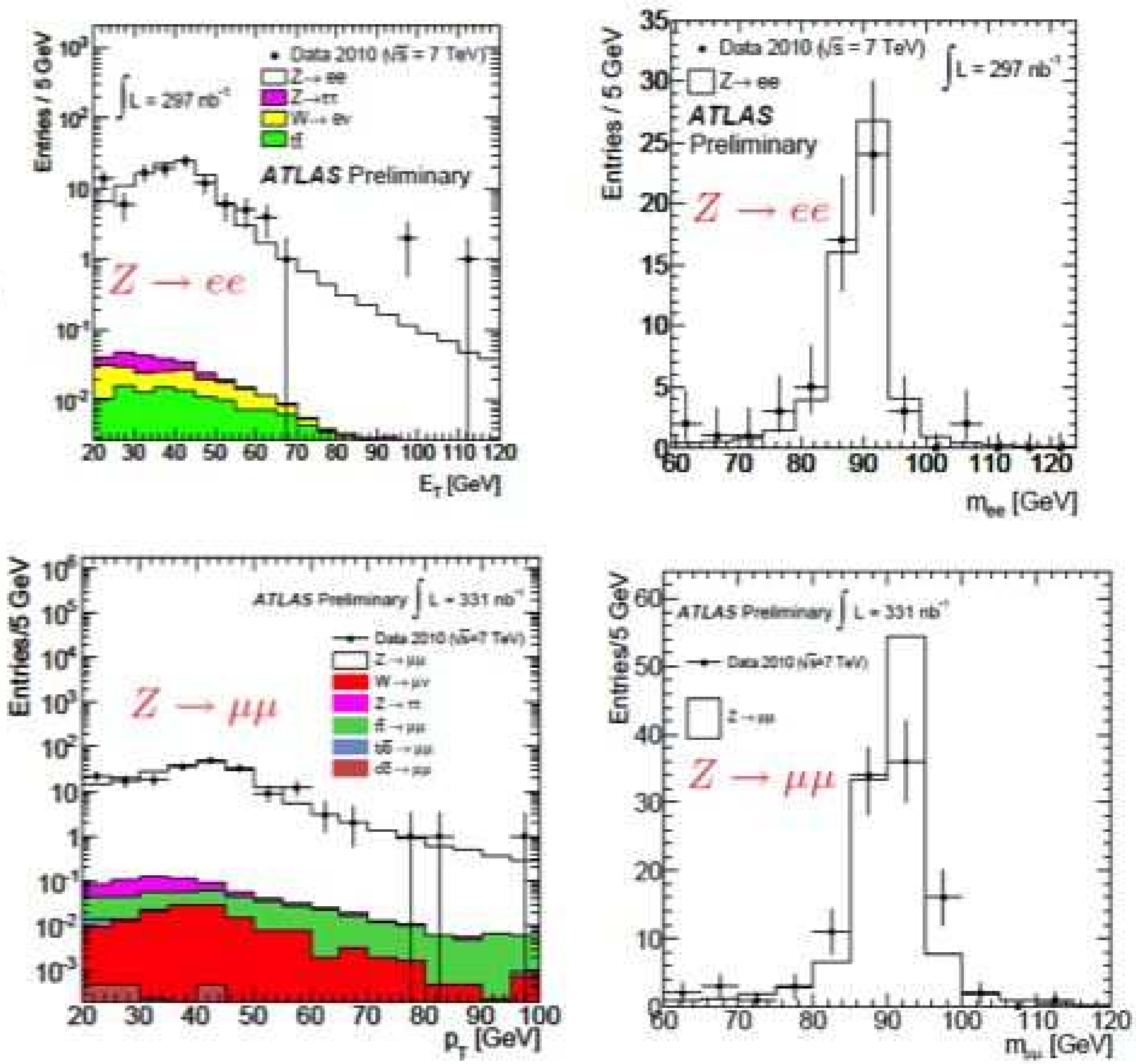
$E_{cm} = 7$ TeV, integrated luminosity 1.5 pb^{-1},
leading the HEP frontier.

ATLAS Z re-discovery:

Z Selection

- Two oppositely charged leptons (e/μ).
- Same lepton selection as W analysis except medium electrons.
- Invariant mass $66 < m_Z < 116$ GeV.



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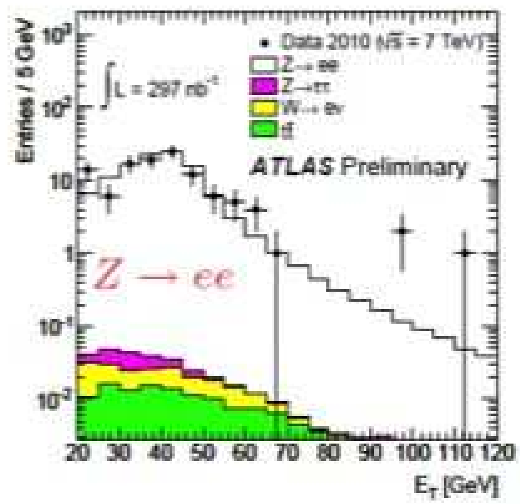

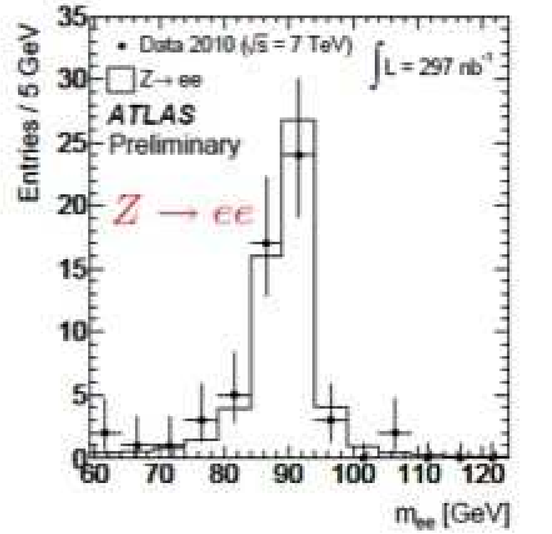

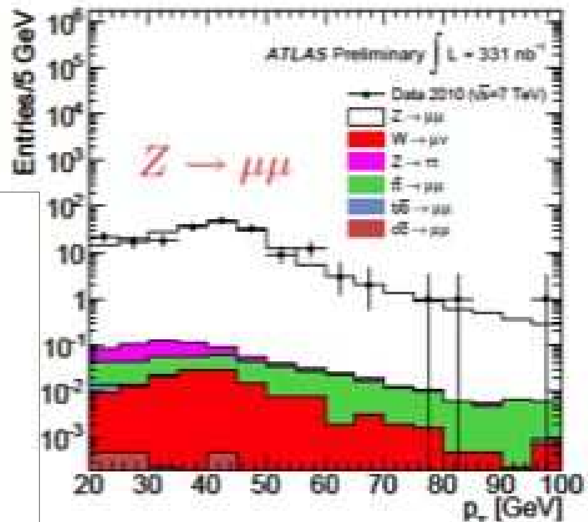

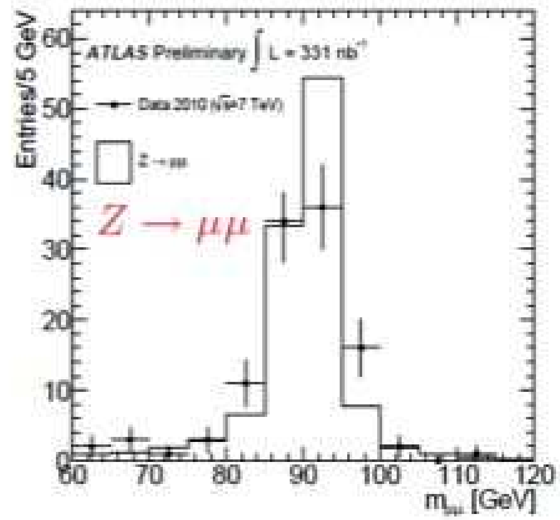

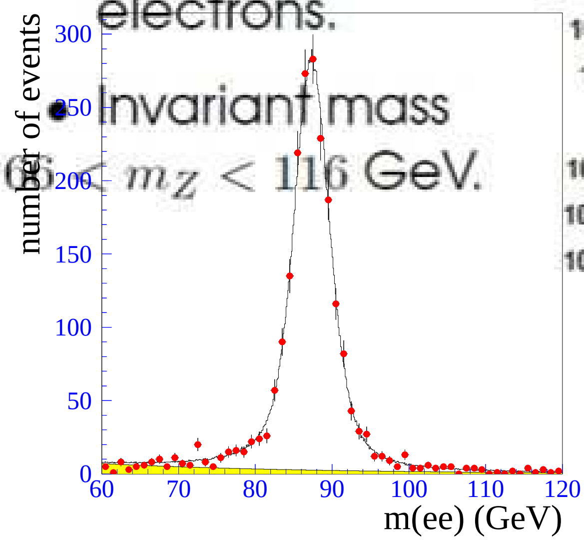

D0 Z events.

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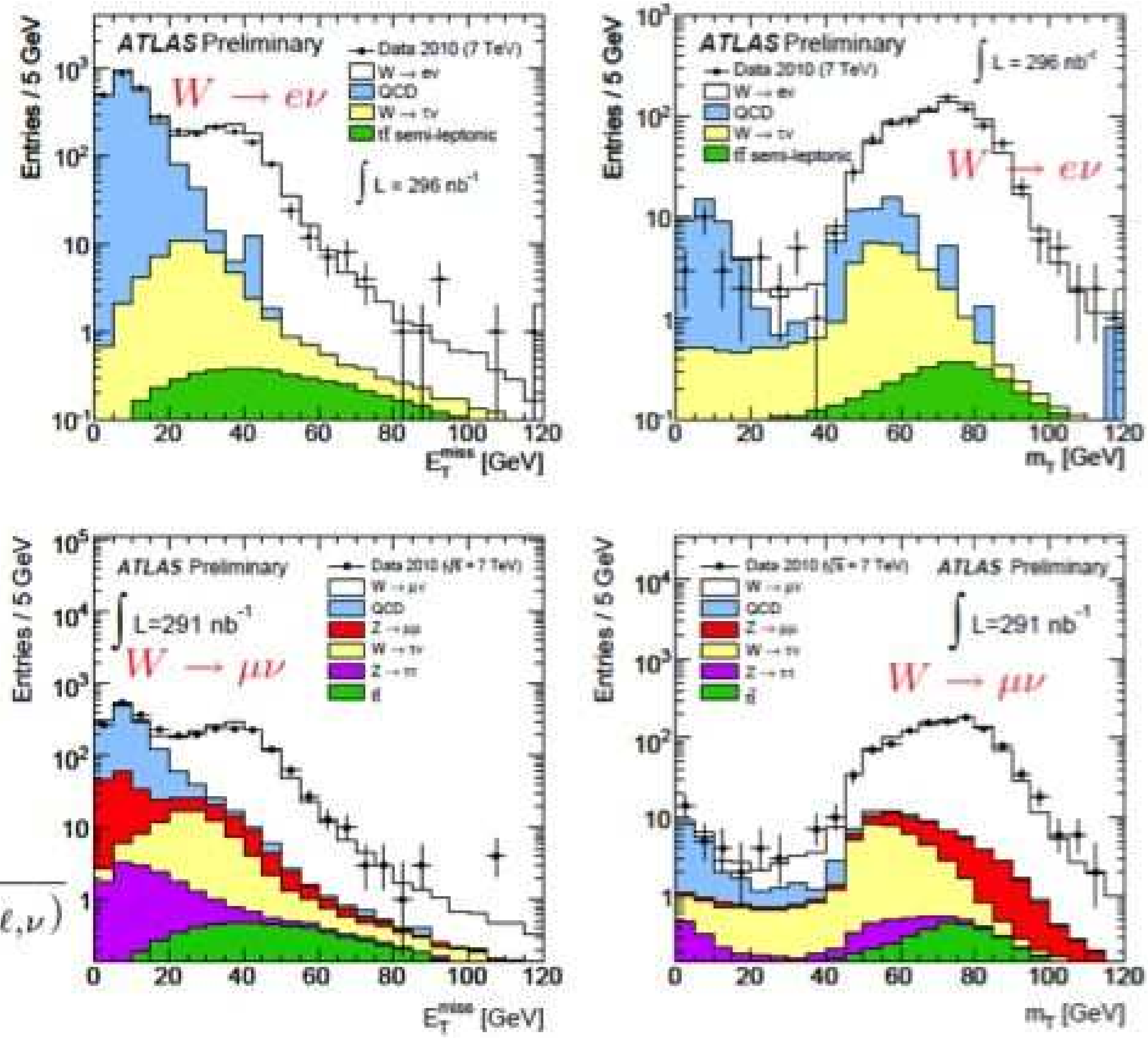
- Tight electron.
- Muon with $p_T > 20$ GeV.
- Muon isolation $\sum_{\Delta R<0.4} p_T^{trk}/p_T^{\mu} < 0.2$
- $\not{E}_{\mathrm{T}} > 25$ GeV.
- $m_T > 40$ GeV.

$$m_T = \sqrt{2p_T^{\ell}p_T^{\nu}(1-\cos(\Delta\phi_{\ell,\nu})}$$



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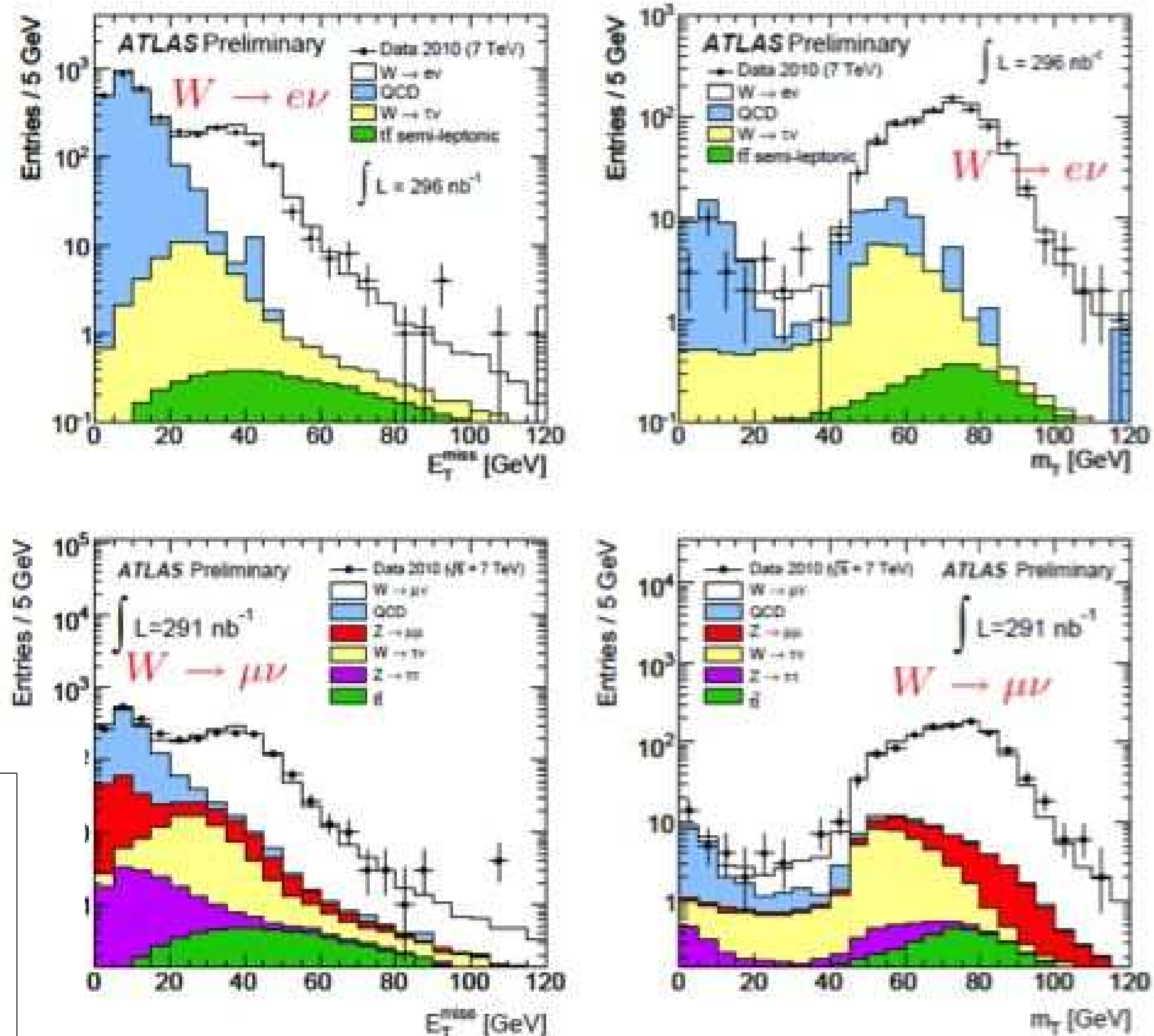
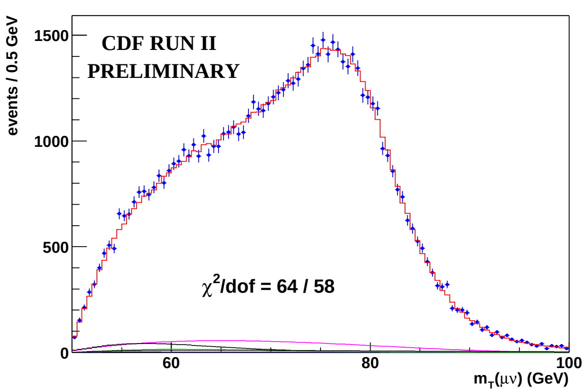
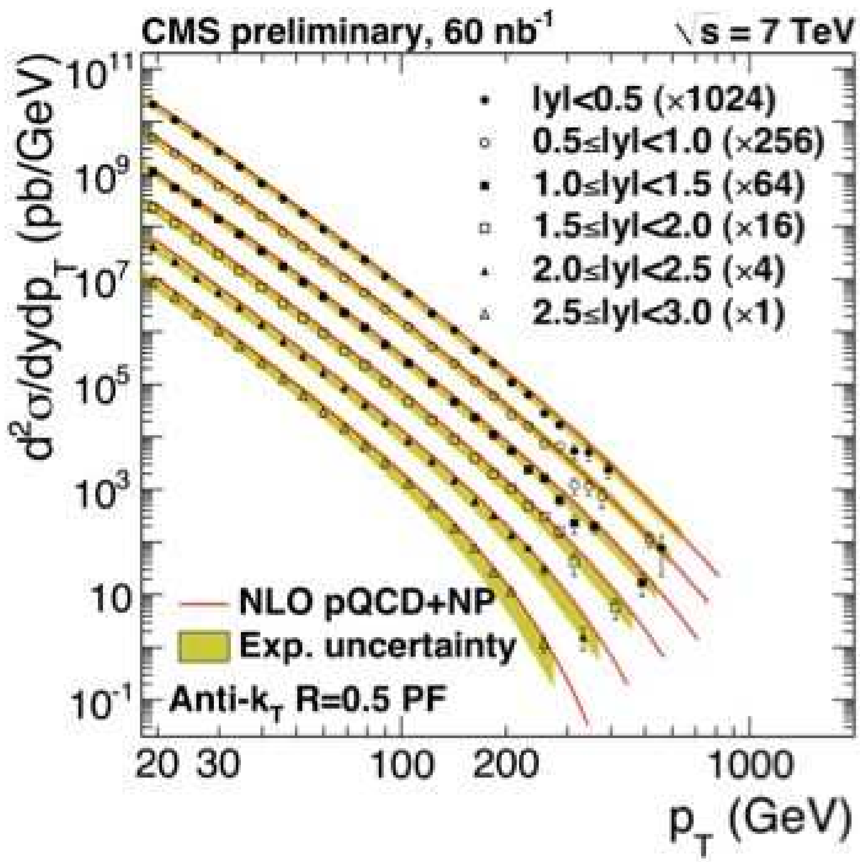


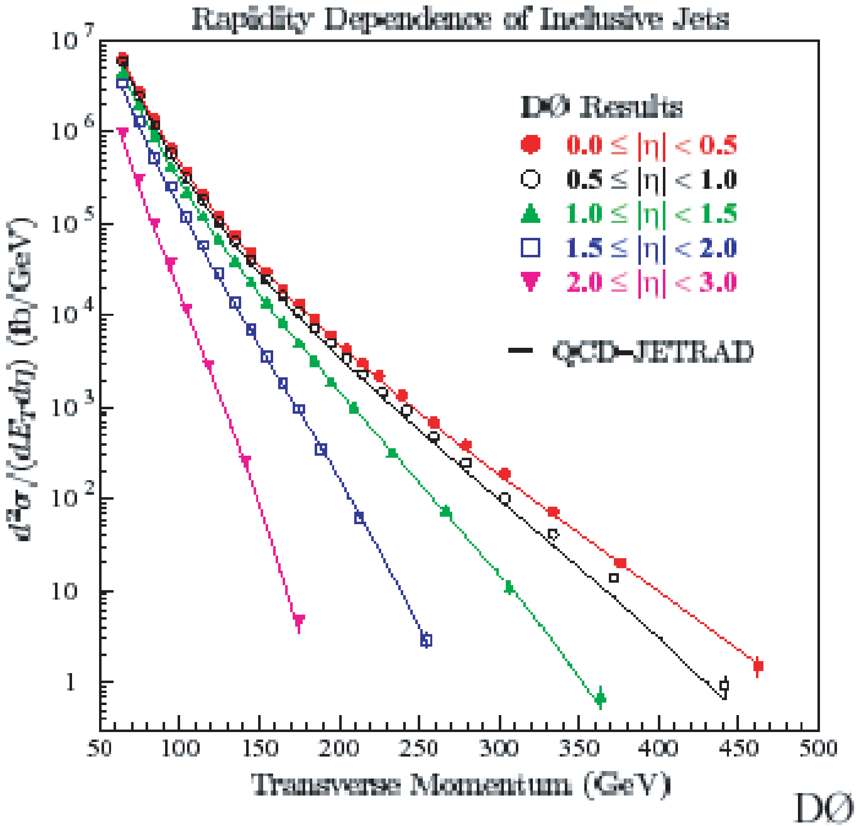


CDF W events.

CMS 1-jet in different rapidities:



D0 1-jet in rapidity ranges:



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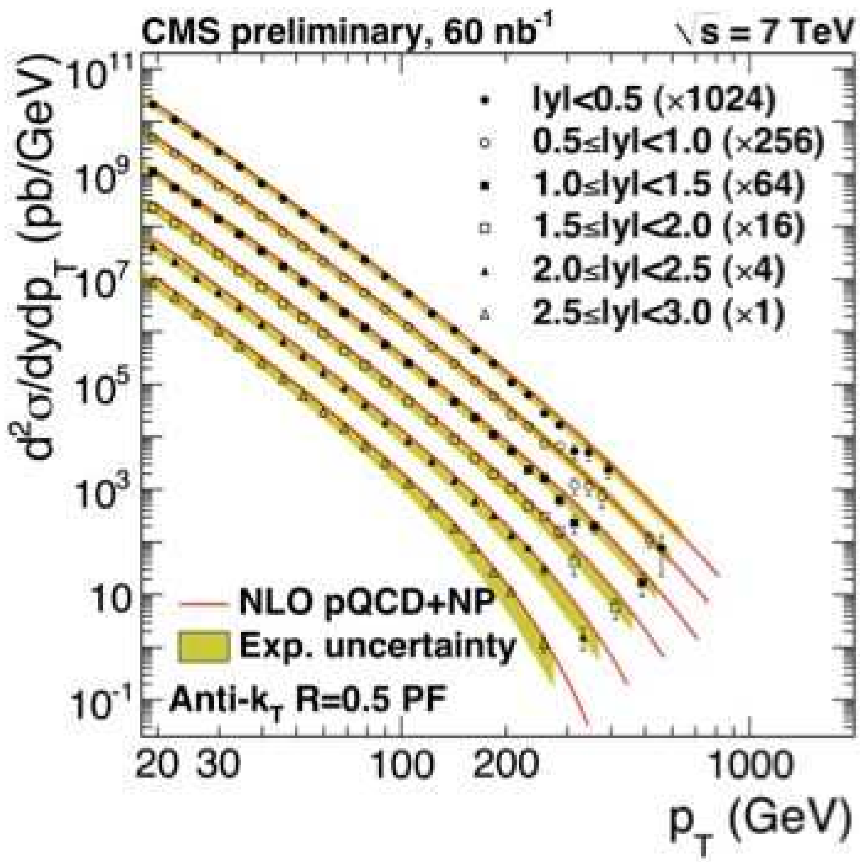
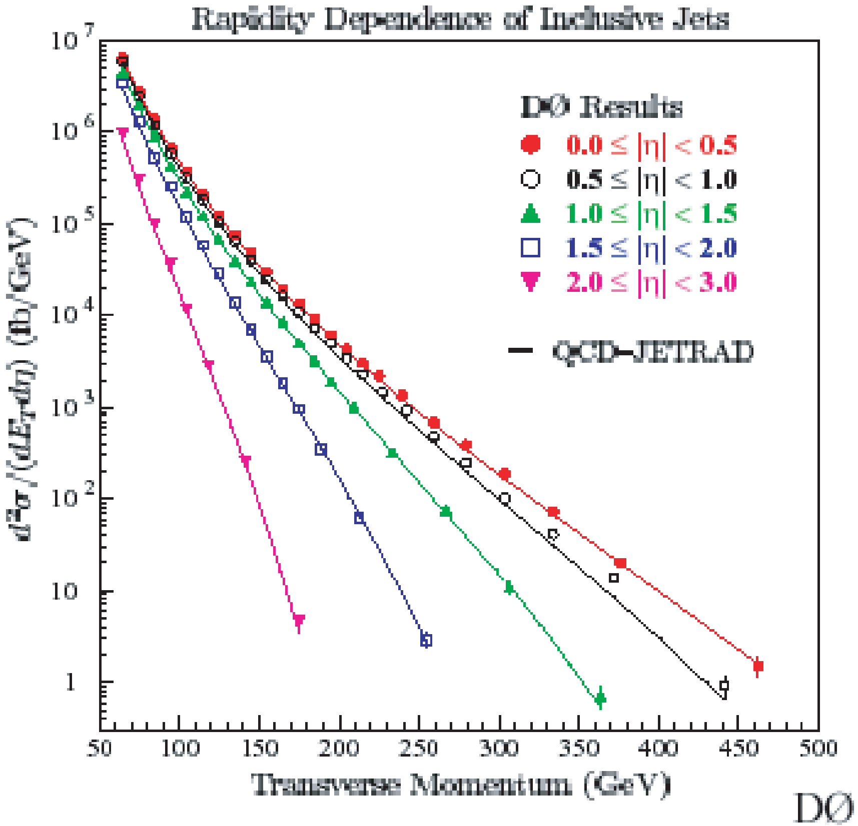
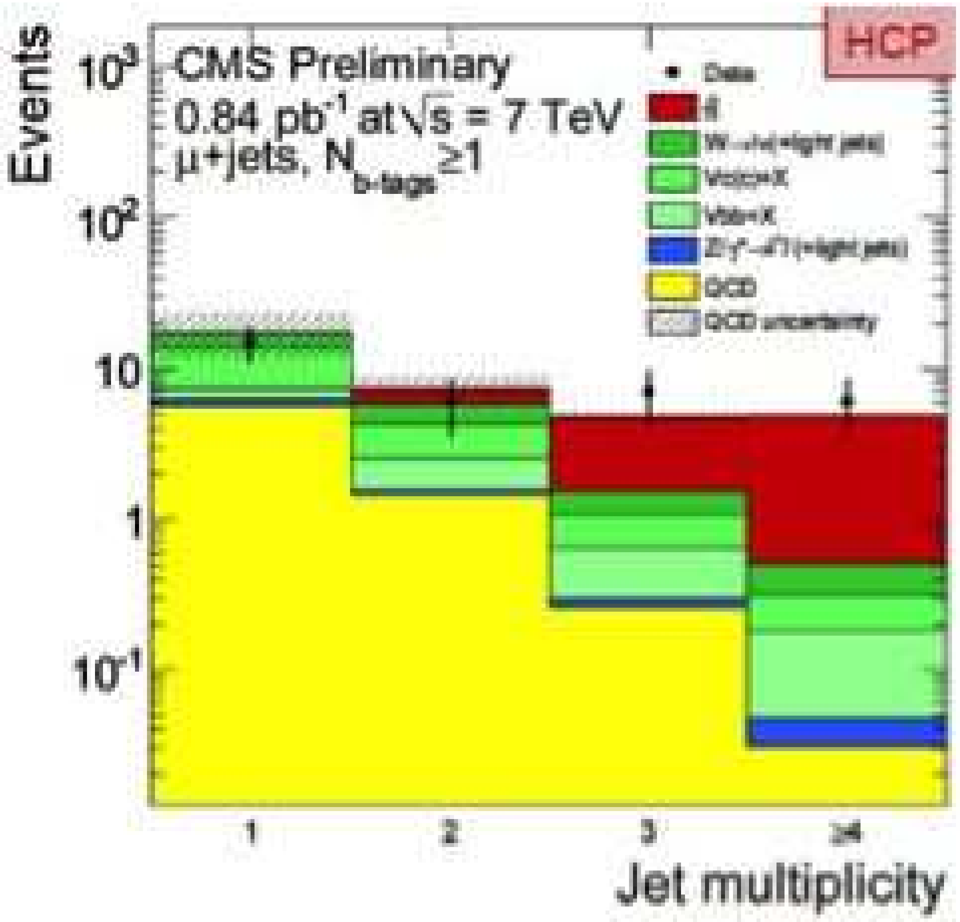


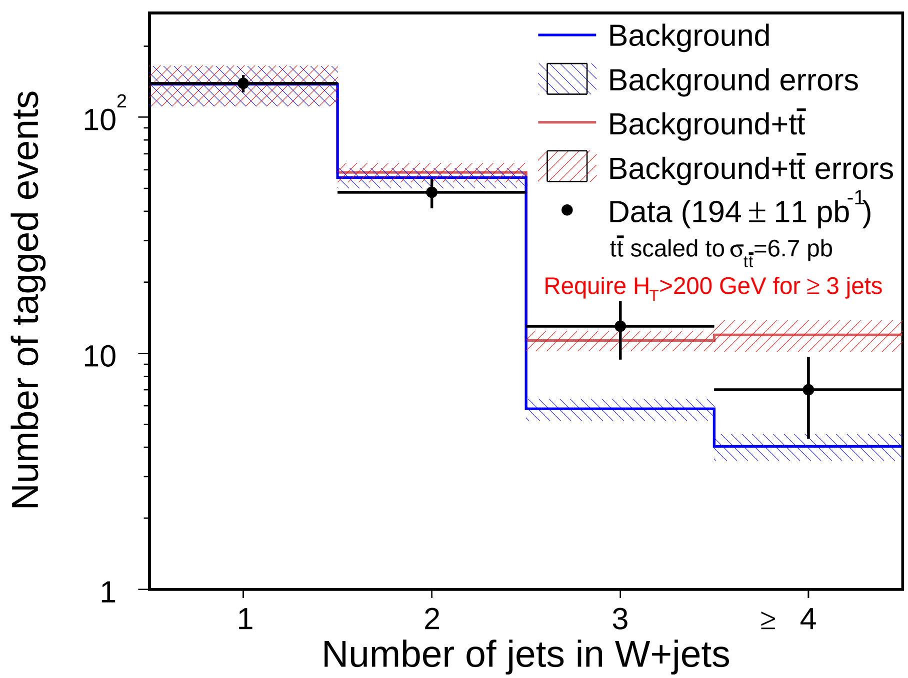


LHC QCD results went BEYOND the Tevatron !

CMS W+jets and top events



CDF W+jets and top



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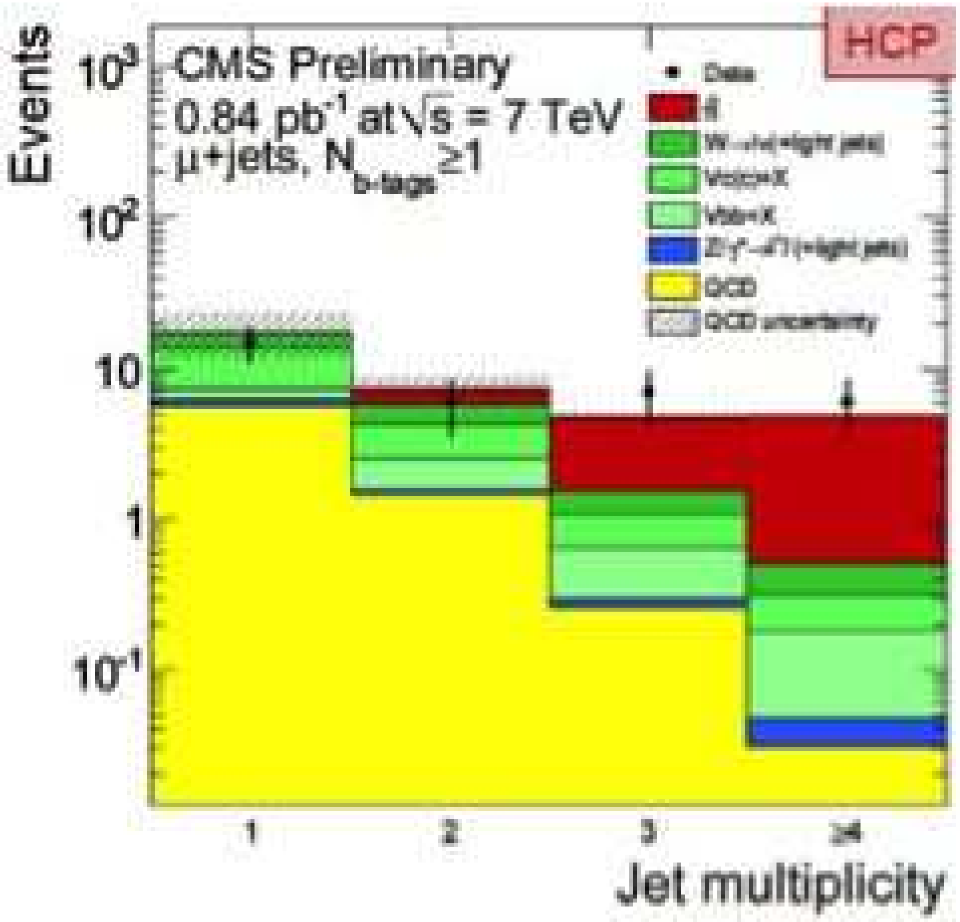
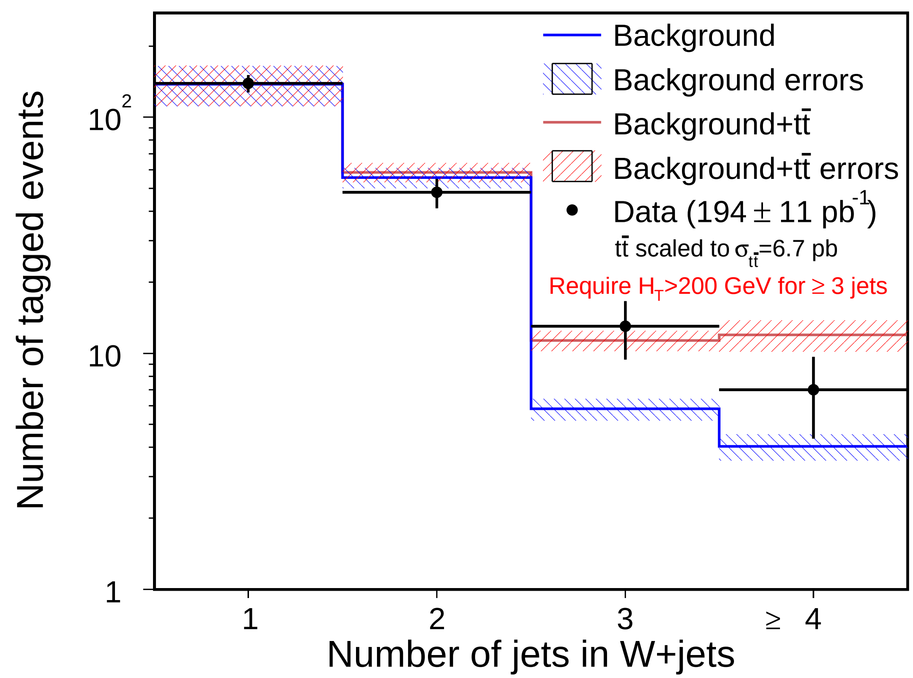
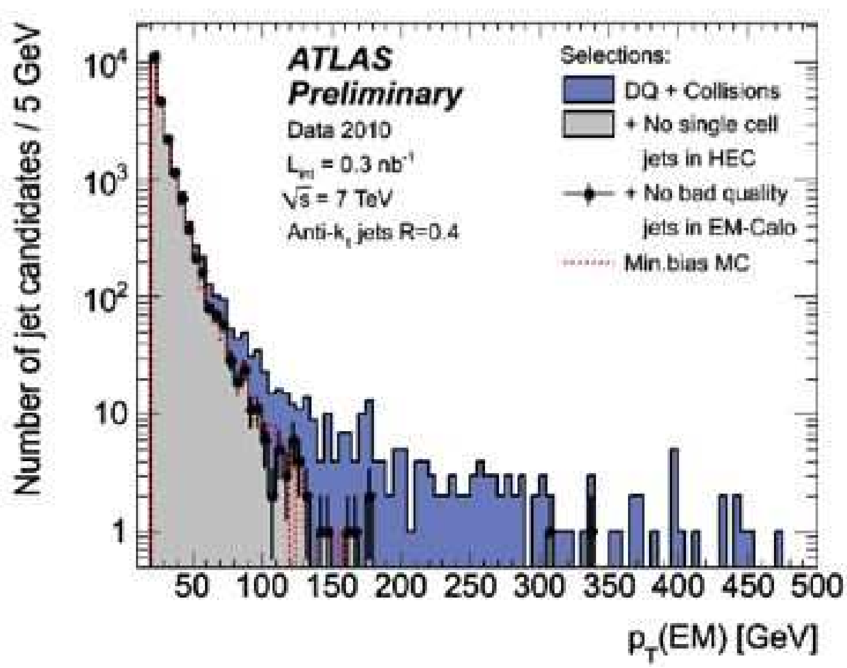


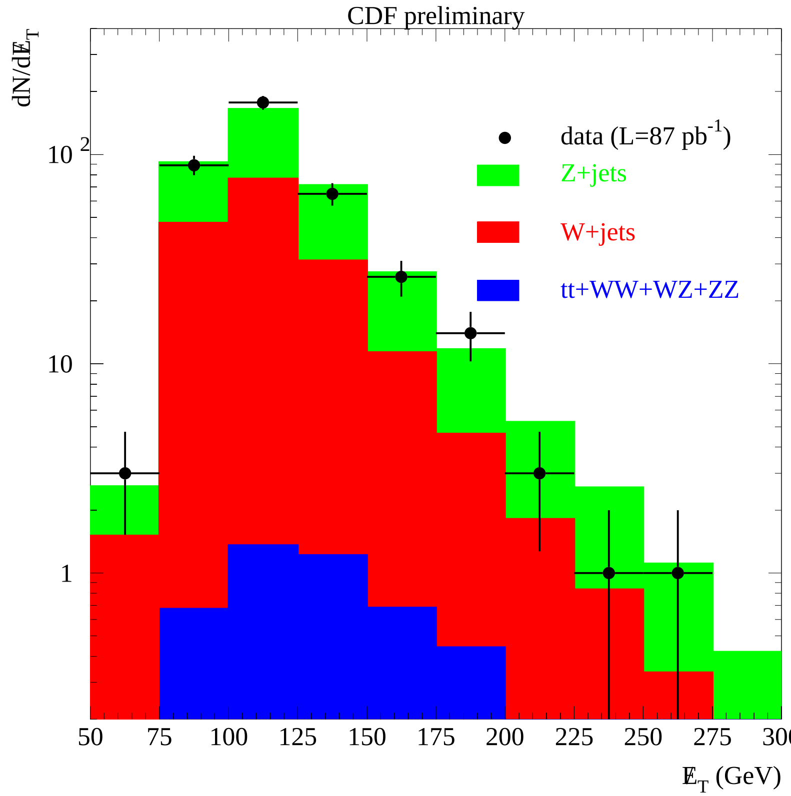


LHC top studies catching up !

ATLAS $\not\!\!E_T$ distribution:



CDF $\not\!\!E_T$ at high end:



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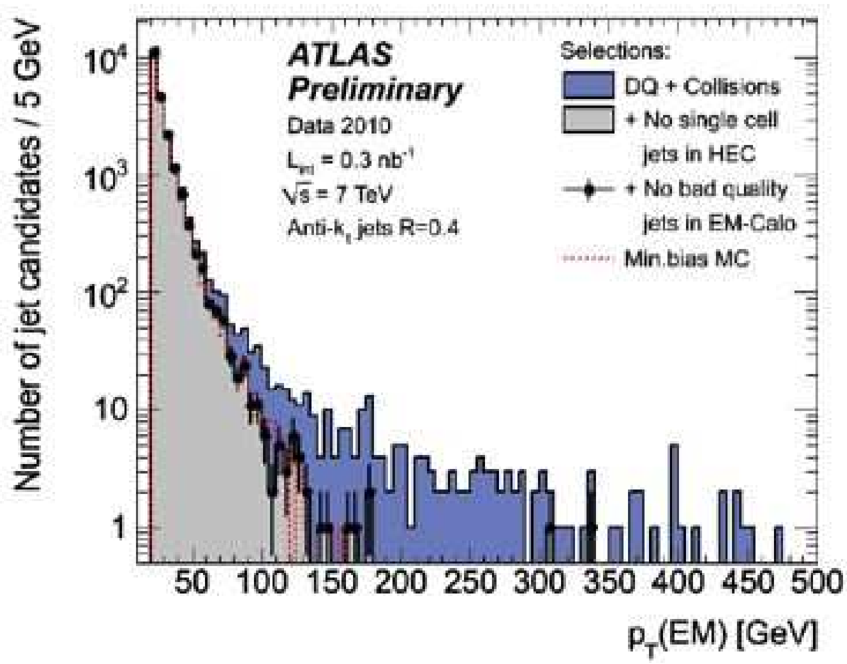

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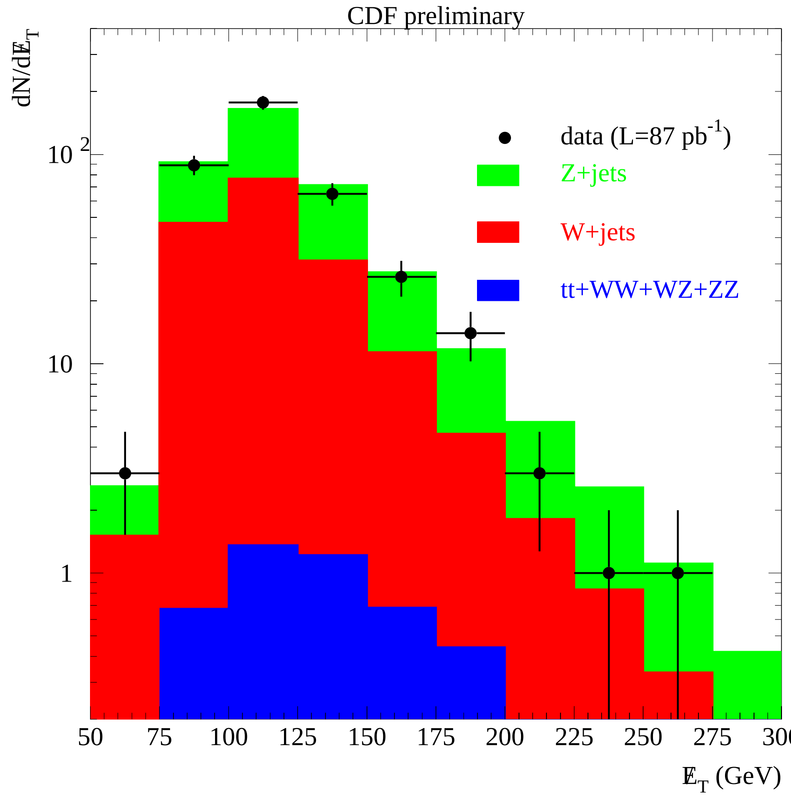
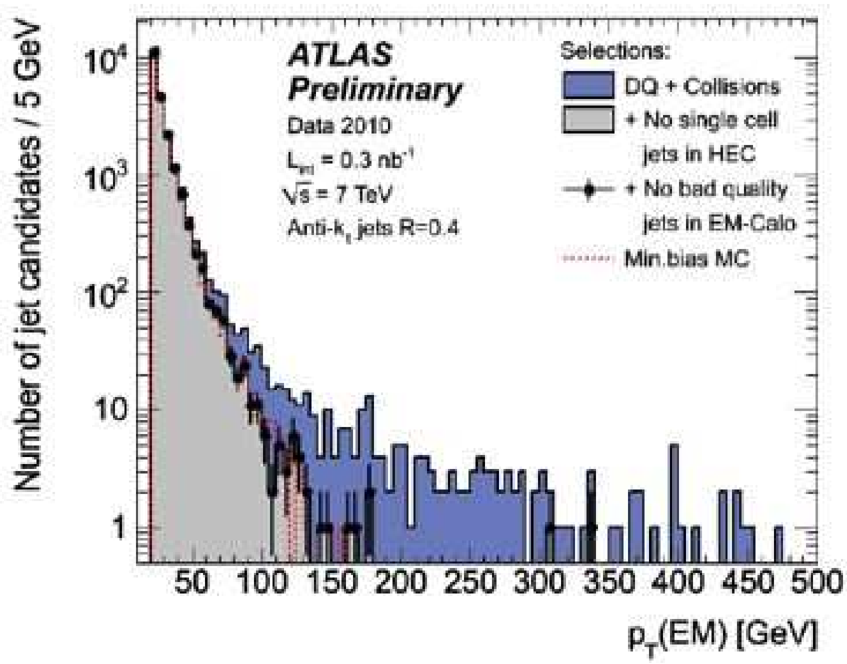
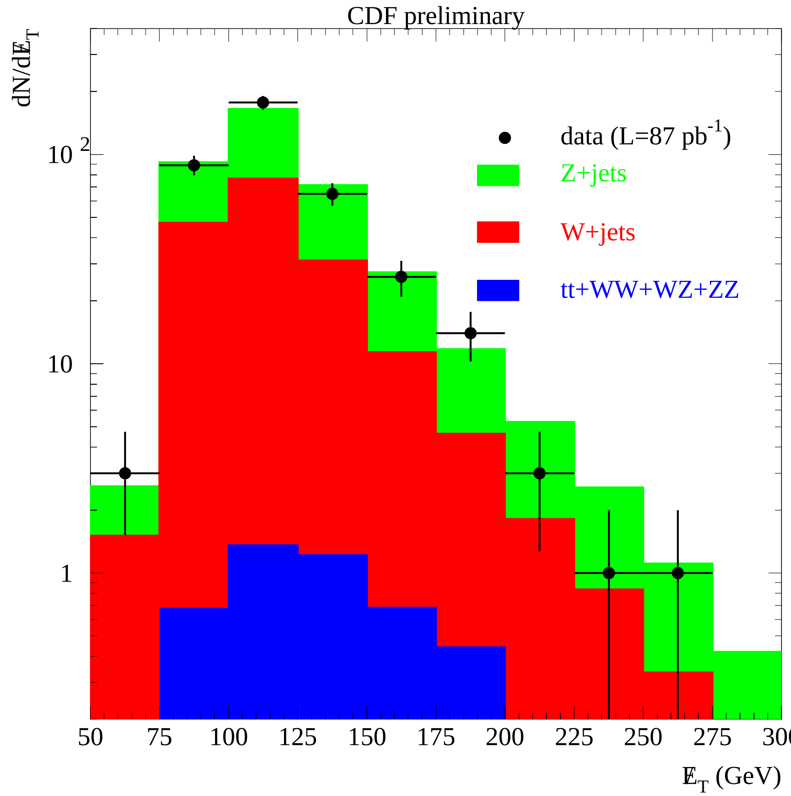


LHC $\not\!\!E_T$ results rapidly improving !

ATLAS $\not\!\!E_T$ distribution: CDF $\not\!\!E_T$ at high end:

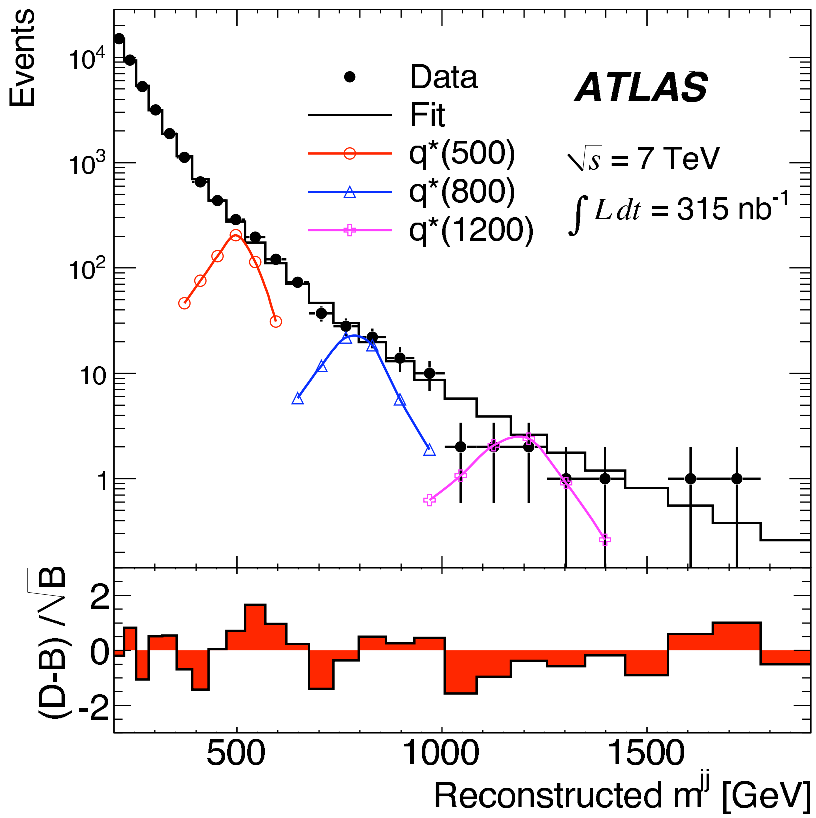


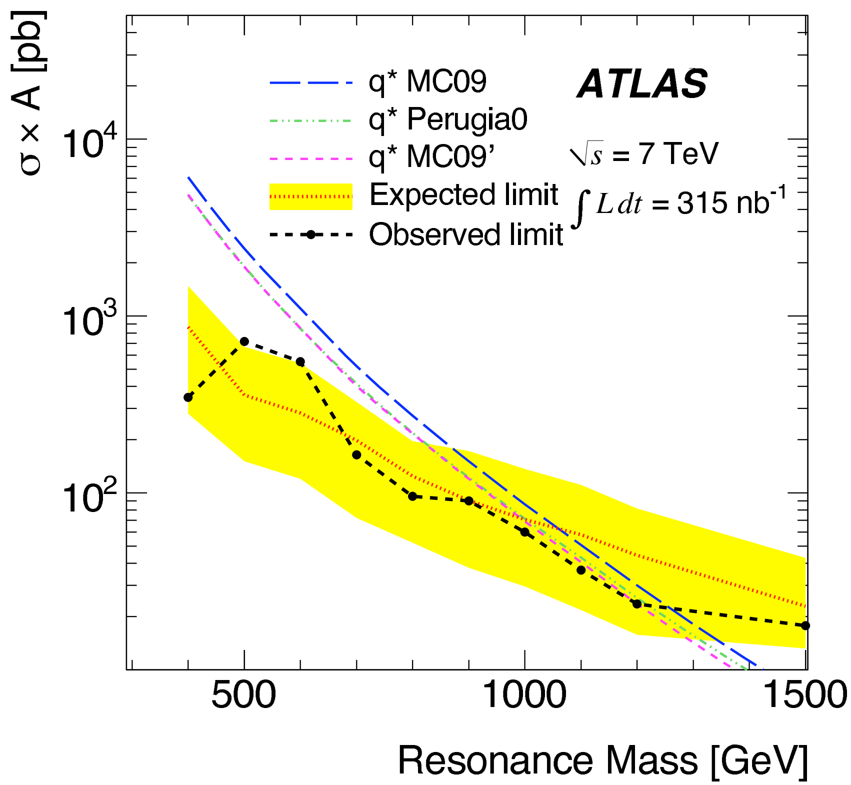


LHC $\not\!\!E_T$ results rapidly improving !

LHC achieved the first crucial step:
The Standard Model rediscovered !

… And have gone on to the physics BSM :





400 GeV $< M_{q*}(jj) <$ 1.26 TeV excluded.

First BSM physics search, beyond the Tevatron reach !

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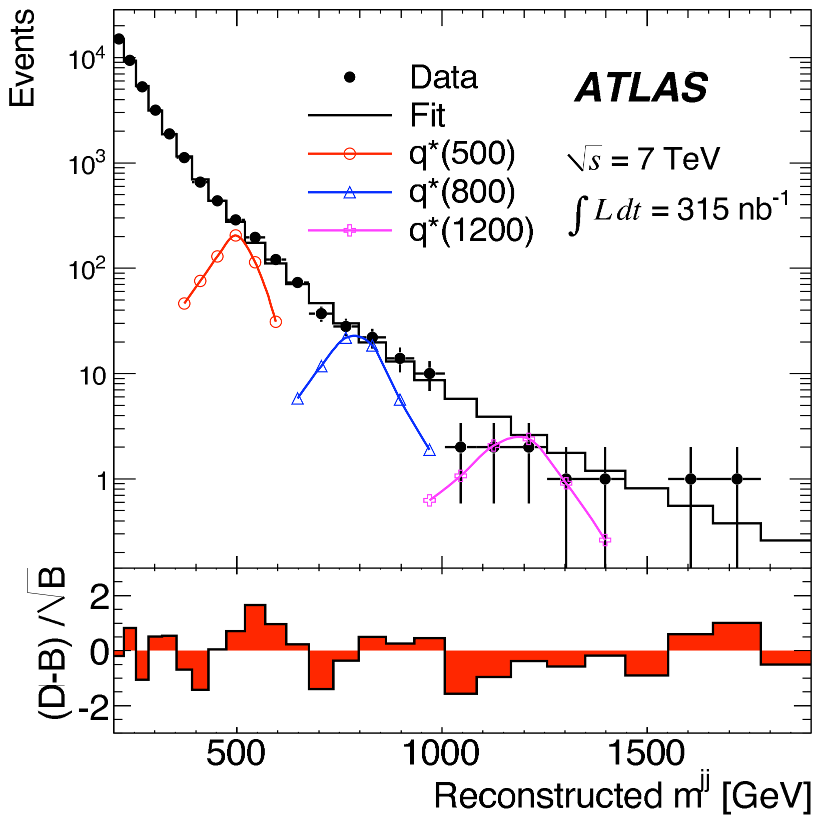

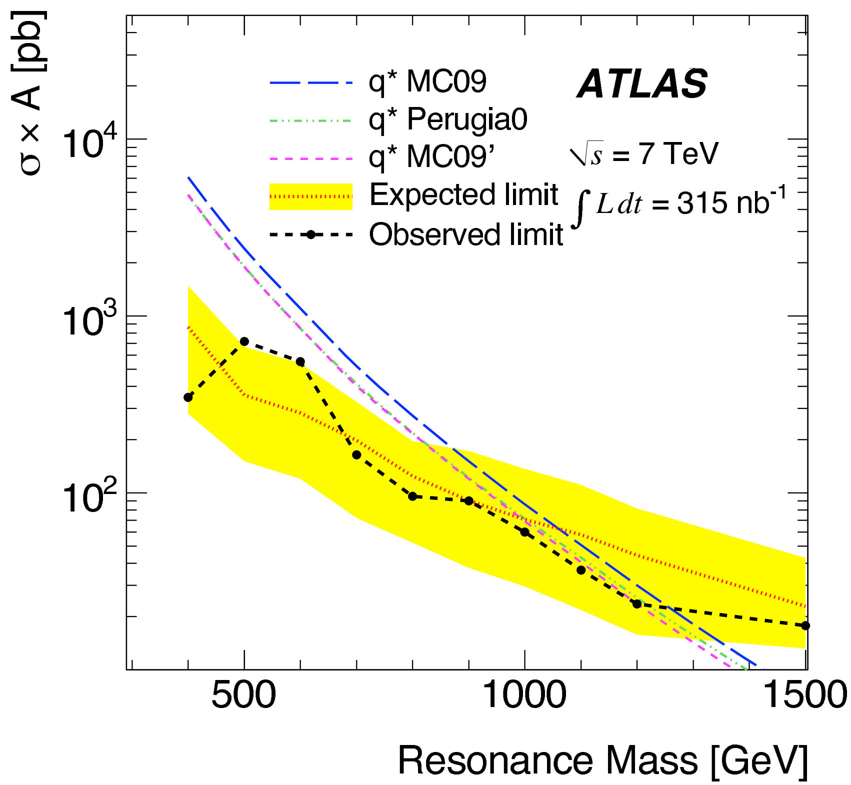

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Anxiously waiting for the new excitement …

IV. From Kinematics to Dynamics

(A). Characteristic observables:
Crucial for uncovering new dynamics.

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Selective experimental events

$\Longrightarrow$ Characteristic kinematical observables

(spatial, time, momentaum phase space)

$\Longrightarrow$ Dynamical parameters

(masses, couplings)

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(spatial, time, momentaum phase space)

$\Longrightarrow$ Dynamical parameters

(masses, couplings)

Energy momentum observables $\Longrightarrow$ mass parameters

Angular observables $\Longrightarrow$ nature of couplings;

Production rates, decay branchings/lifetimes $\Longrightarrow$ interaction strengths.

(B). Kinematical features:

(a). s-channel singularity: bump search we do best.

• invariant mass of two-body $R \to ab$: $m_{ab}^2 = (p_a + p_b)^2 = M_R^2$.

combined with the two-body Jacobian peak in transverse momentum:

$$\frac{d\hat{\sigma}}{dm_{ee}^2 \; dp_{eT}^2} \propto \frac{\Gamma_Z M_Z}{(m_{ee}^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \; \frac{1}{m_{ee}^2 \sqrt{1 - 4p_{eT}^2/m_{ee}^2}}$$

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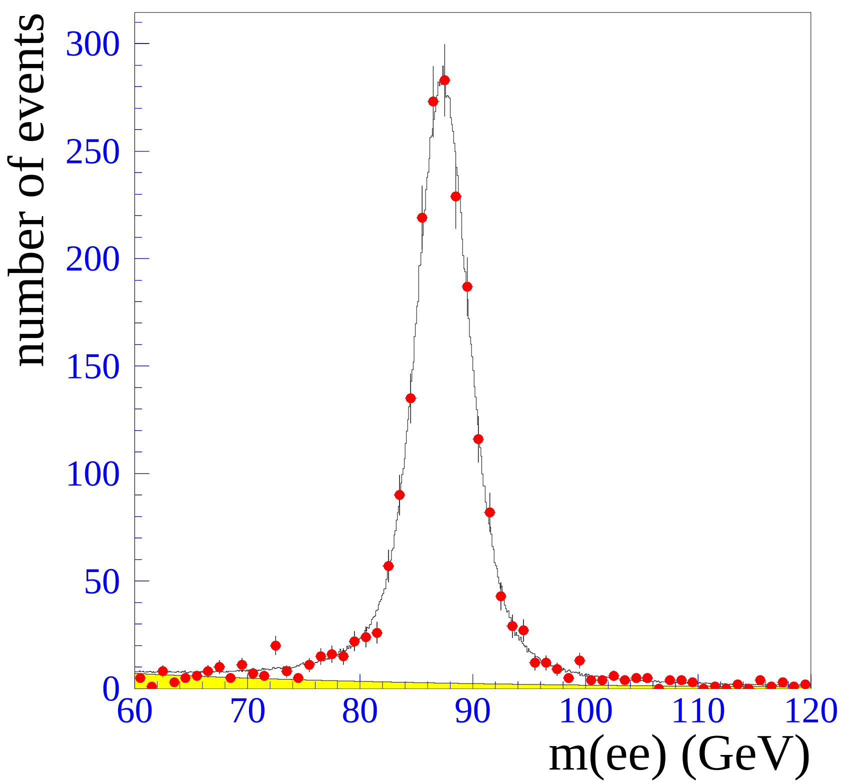
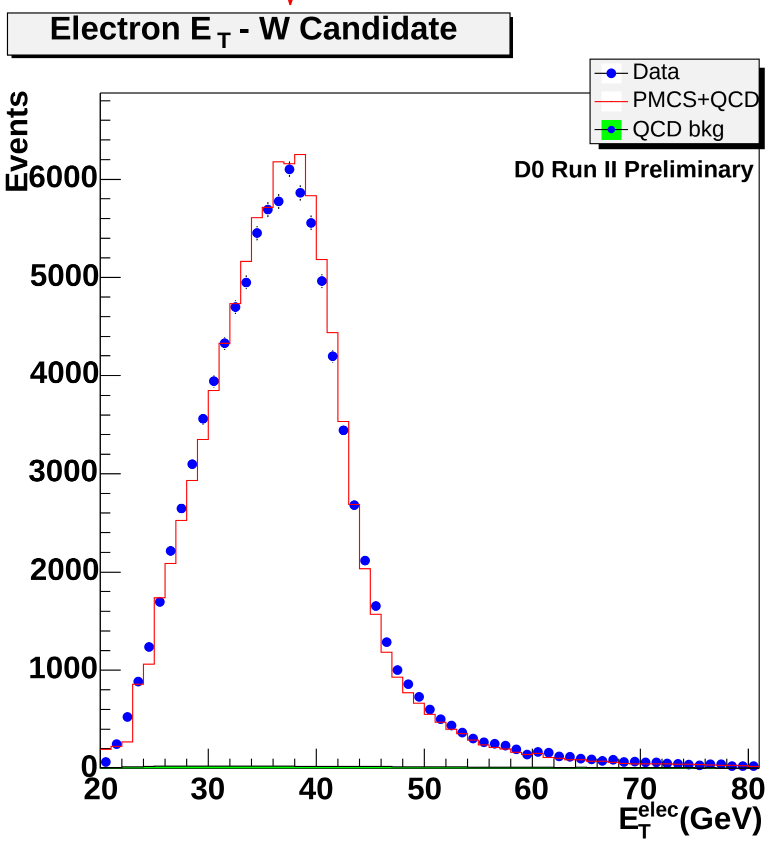


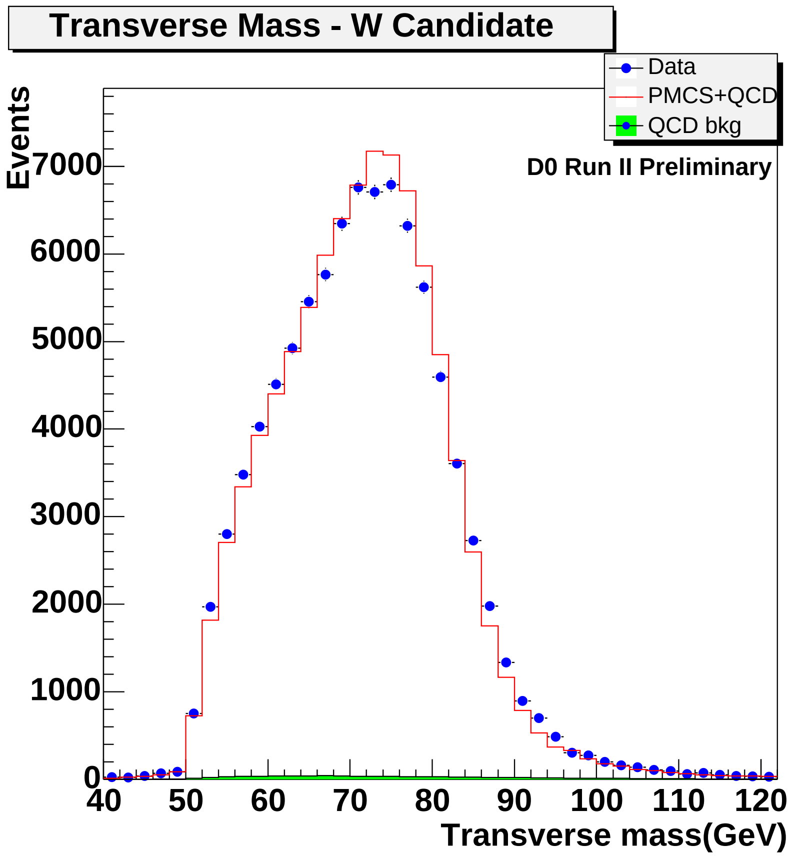
$Z \to e^+e^-$

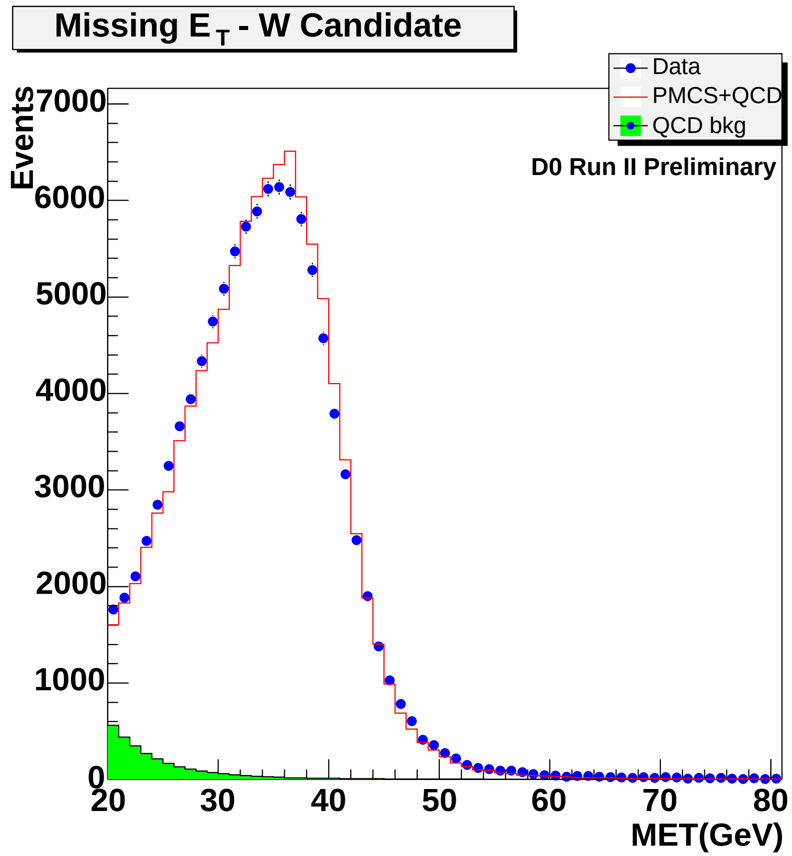


$W \to e\nu$

- "transverse" mass of two-body $W^- \to e^- \bar{\nu}_e$:

$$\begin{aligned} m_{e\nu\ T}^2 &= (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \\ &= 2E_{eT}E_T^{miss}(1 - \cos\phi) \leq m_{e\nu}^2. \end{aligned}$$





If $p_T(W) = 0$, then $m_{e\nu\ T} = 2E_{eT} = 2E_T^{miss}$.

Exercise 5.1: For a two-body final state kinematics, show that

$$\frac{d\hat{\sigma}}{dp_{eT}} = \frac{4p_{eT}}{s\sqrt{1-4p_{eT}^2/s}} \, \frac{d\hat{\sigma}}{d\cos\theta^*}.$$

where $p_{eT} = p_e \sin\theta^*$ is the transverse momentum and θ^* is the polar angle in the c.m. frame. Comment on the apparent singularity at $p_{eT}^2 = s/4$.

Exercise 5.2: Show that for an on-shell decay $W^- \to e^- \bar{\nu}_e$:

$$m_{e\nu\ T}^2 \equiv (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \leq m_{e\nu}^2.$$

Exercise 5.3: Show that if W/Z has some transverse motion, δP_V, then:

$$p'_{eT} \sim p_{eT}\ [1 + \delta P_V/M_V],$$
$$m'^2_{e\nu\ T} \sim m^2_{e\nu\ T}\ [1 - (\delta P_V/M_V)^2],$$
$$m'^2_{ee} = m^2_{ee}.$$

- $H^0 \to W^+W^- \to j_1 j_2\ e^-\bar{\nu}_e$:

cluster transverse mass (I):

$$\begin{aligned} m^2_{WW\ T} &= (E_{W_1T} + E_{W_2T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \\ &= (\sqrt{p^2_{jjT} + M^2_W} + \sqrt{p^2_{e\nu T} + M^2_W})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \leq M^2_H. \end{aligned}$$

where $\vec{p}_T^{\,miss} \equiv \vec{\not{p}}_T = -\sum_{obs}\ \vec{p}_T^{\,obs}$.

- $H^0 \to W^+W^- \to j_1 j_2 \; e^- \bar{\nu}_e$:

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m^2_{WW\ T} &= (E_{W_1T} + E_{W_2T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \\
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\end{aligned}
$$

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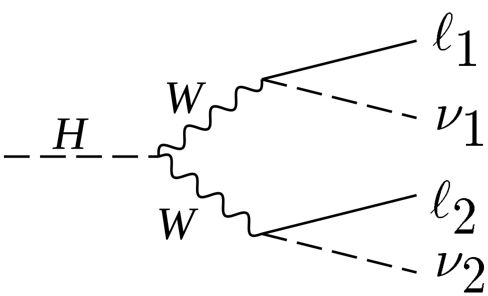


- $H^0 \to W^+W^- \to e^+\nu_e \; e^- \bar{\nu}_e$:

"effecive" transverse mass:

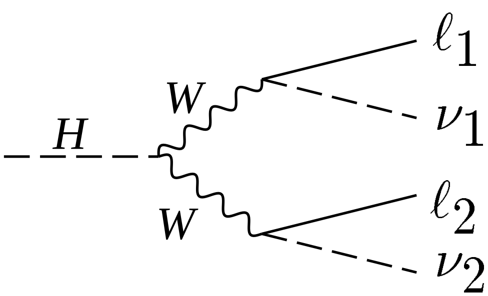
$$
\begin{aligned}
m^2_{eff\ T} &= (E_{e1T} + E_{e2T} + E_T^{miss})^2 - (\vec{p}_{e1T} + \vec{p}_{e2T} + \vec{p}_T^{\,miss})^2 \\
m_{eff\ T} &\approx E_{e1T} + E_{e2T} + E_T^{miss}
\end{aligned}
$$

• $H^0 \to W^+W^- \to j_1 j_2\ e^- \bar{\nu}_e$:

cluster transverse mass (I):

$$m^2_{WW\ T} = (E_{W_1 T} + E_{W_2 T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\ miss})^2$$
$$= (\sqrt{p^2_{jjT} + M^2_W} + \sqrt{p^2_{e\nu T} + M^2_W})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\ miss})^2 \leq M^2_H.$$

where $\vec{p}_T^{\ miss} \equiv \vec{\not{p}}_T = -\sum_{obs}\ \vec{p}_T^{\ obs}$.



• $H^0 \to W^+W^- \to e^+\nu_e\ e^- \bar{\nu}_e$:

"effecive" transverse mass:

$$m^2_{eff\ T} = (E_{e1T} + E_{e2T} + E_T^{miss})^2 - (\vec{p}_{e1T} + \vec{p}_{e2T} + \vec{p}_T^{\ miss})^2$$
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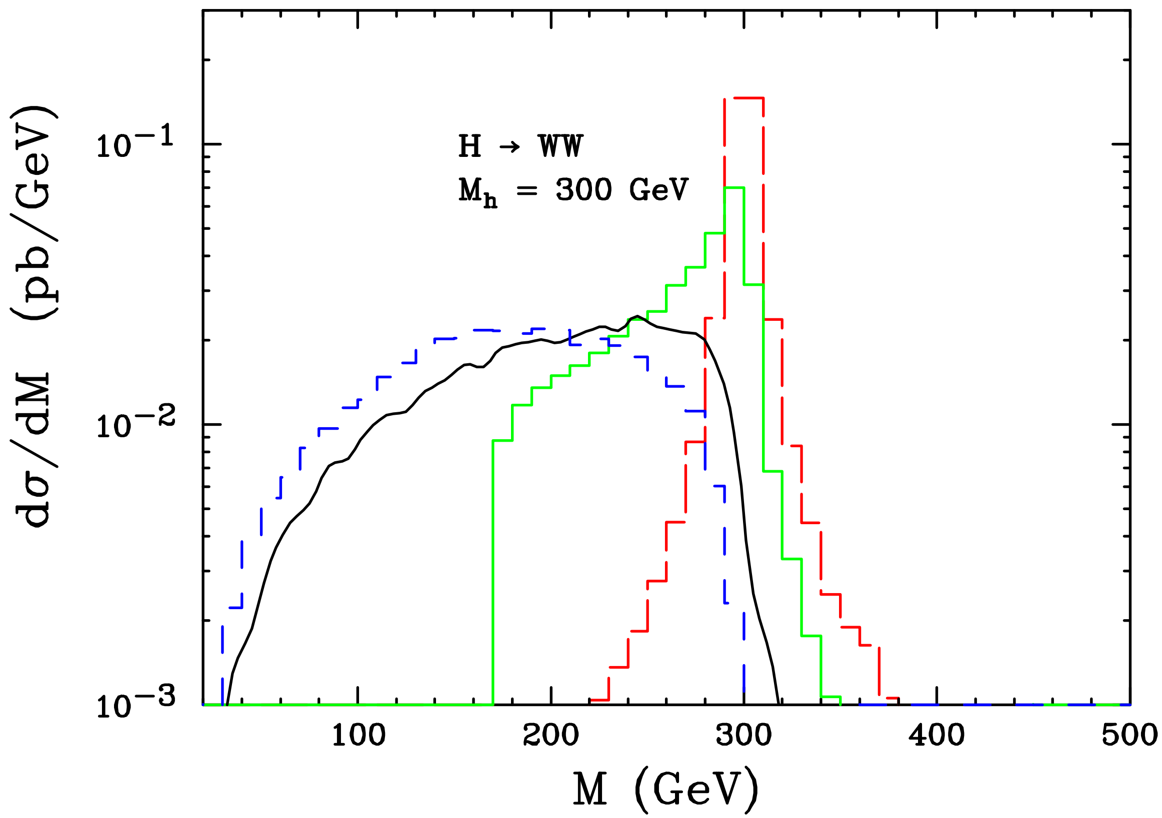
cluster transverse mass (II):

$$m^2_{WW\ C} = \left(\sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not{p}_T\right)^2 - (\vec{p}_{T,\ell\ell} + \vec{\not{p}}_T)^2$$
$$m_{WW\ C} \approx \sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not{p}_T$$



M_{WW} invariant mass (WW fully reconstructable): - - - - - - - - -

$M_{WW,\,T}$ transverse mass (one missing particle ν): ————————

$M_{eff,\,T}$ effetive trans. mass (two missing particles): - - - - - - -

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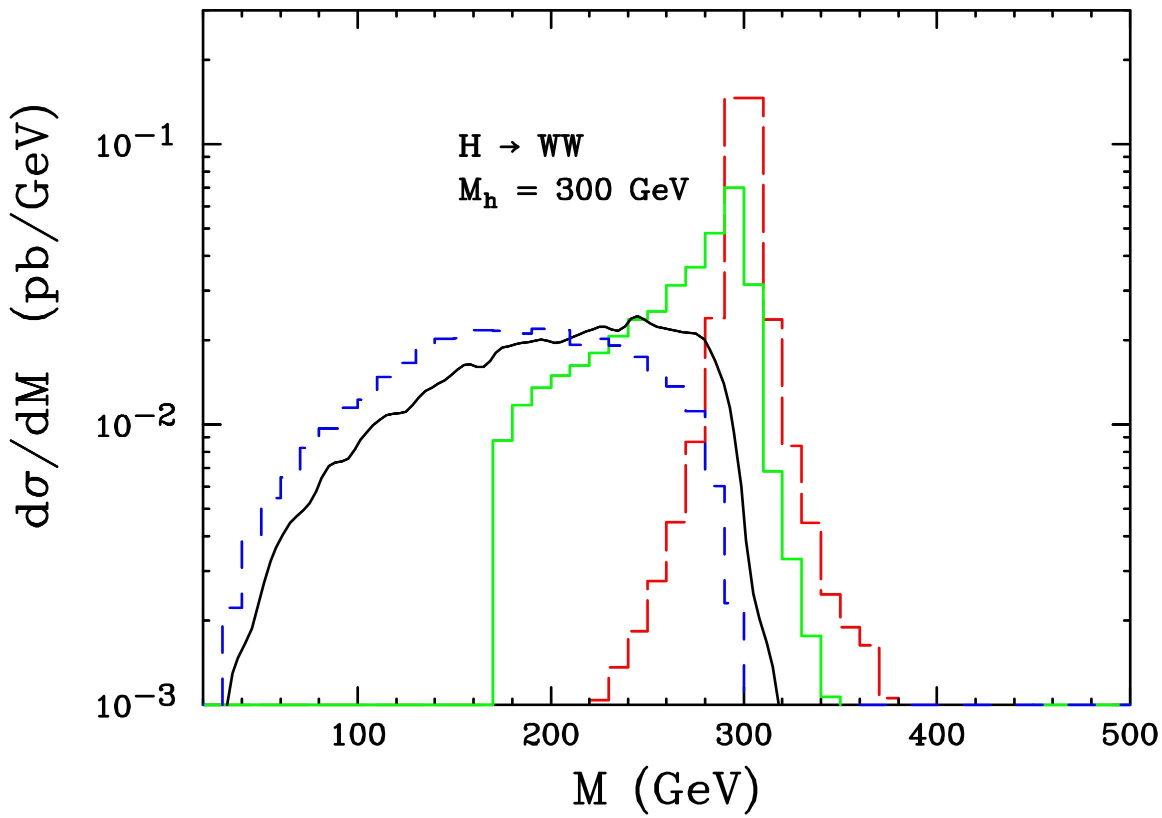

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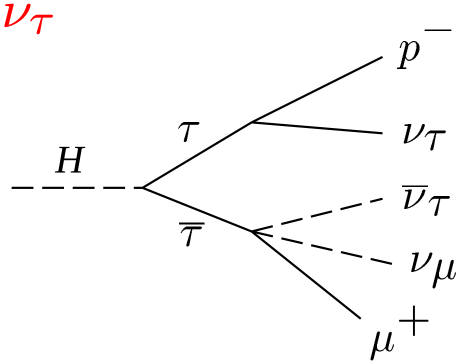
YOU design an optimal variable/observable for the search.

• cluster transverse mass (III):

$$H^0 \to \tau^+\tau^- \to \mu^+ \ \bar{\nu}_\tau \ \nu_\mu, \quad \rho^- \ \nu_\tau$$

A lot more complicated with (many) more $\nu' s$?



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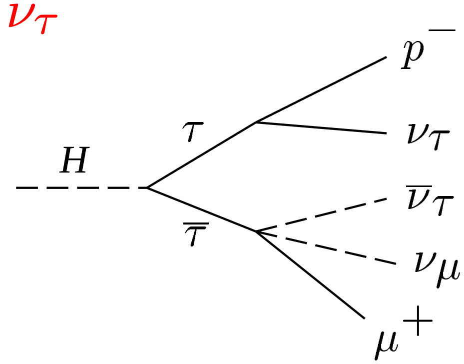

Not really!

$\tau^+\tau^-$ ultra-relativistic, the final states from a τ decay highly collimated:

$$\theta \approx \gamma_\tau^{-1} = m_\tau/E_\tau = 2m_\tau/m_H \approx 1.5^\circ \quad (m_H = 120 \text{ GeV}).$$

We can thus take

$$\begin{aligned} \vec{p}_{\tau^+} &= \vec{p}_{\mu^+} + \vec{p}_+^{\,\nu's}, \quad \vec{p}_+^{\,\nu's} \approx c_+ \vec{p}_{\mu^+}. \\ \vec{p}_{\tau^-} &= \vec{p}_{\rho^-} + \vec{p}_-^{\,\nu's}, \quad \vec{p}_-^{\,\nu's} \approx c_- \vec{p}_{\rho^-}. \end{aligned}$$

where $c_\pm$ are proportionality constants, to be determined.

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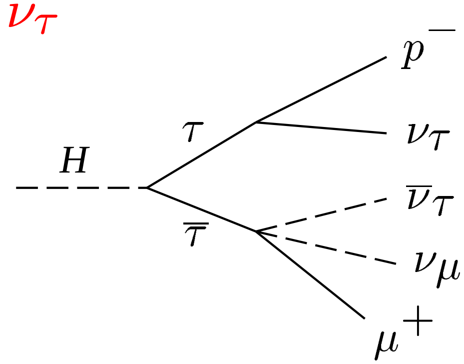

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This is applicable to any decays of fast-moving particles, like

$$T \to Wb \to \ell\nu, \ b.$$

Experimental measurements: p_{ρ^-}, p_{μ^+}, $\not{p}_T$:

$$c_+ (p_{\mu^+})_x + c_- (p_{\rho^-})_x = (\not{p}_T)_x,$$
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Unique solutions for $c_\pm$ exist if

$$(p_{\mu^+})_x/(p_{\mu^+})_y \neq (p_{\rho^-})_x/(p_{\rho^-})_y.$$

Physically, the τ^+ and τ^- should form a finite angle,
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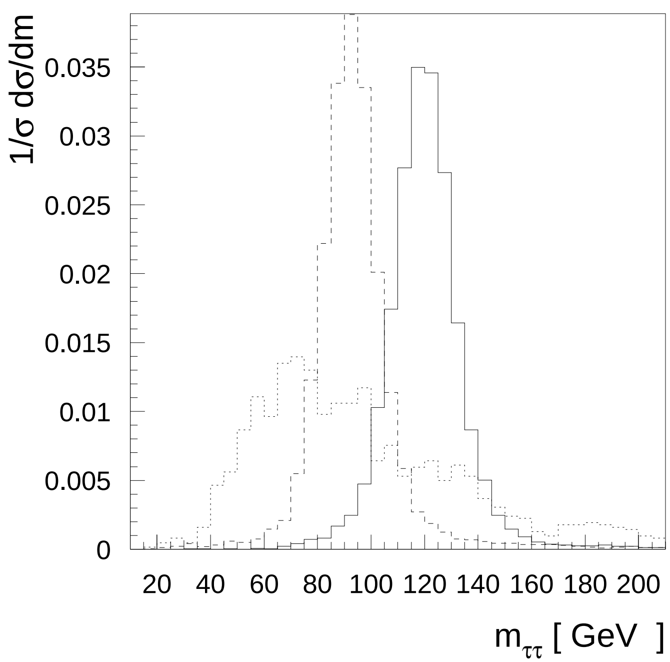

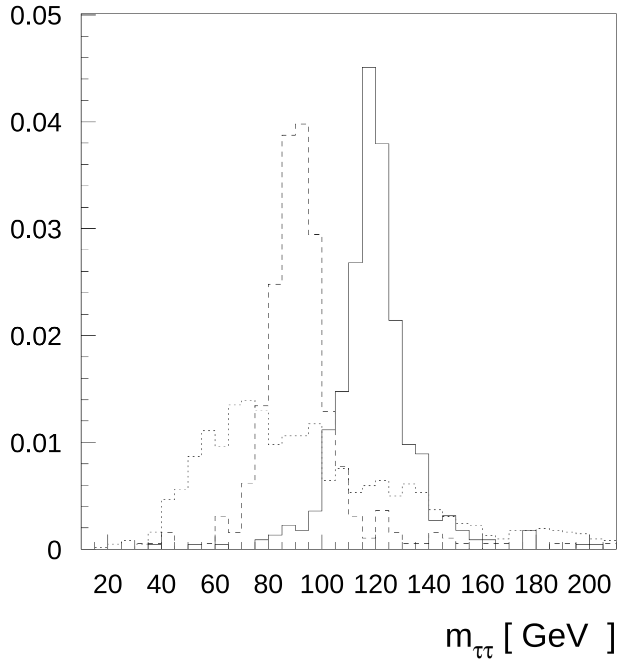

(b). Two-body versus three-body kinematics

- Energy end-point and mass edges:

utilizing the "two-body kinematics"

Consider a simple case:

$$e^+e^- \to \tilde{\mu}_R^+ \ \tilde{\mu}_R^-$$

$$\text{with two}-\text{body decays}: \ \ \tilde{\mu}_R^+ \to \mu^+ \tilde{\chi}_0, \ \ \ \tilde{\mu}_R^- \to \mu^- \tilde{\chi}_0.$$

In the $\tilde{\mu}_R^+$-rest frame: $E_\mu^0 = \frac{M_{\tilde{\mu}_R}^2 - m_\chi^2}{2M_{\tilde{\mu}_R}}$.

In the Lab-frame:

$$(1-\beta)\gamma E_\mu^0 \le E_\mu^{lab} \le (1+\beta)\gamma E_\mu^0$$

$$\text{with} \ \ \beta = \left(1 - 4M_{\tilde{\mu}_R}^2/s\right)^{1/2}, \ \ \gamma = (1-\beta)^{-1/2}.$$

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Mass edge: $m_{\mu^+\mu^-}^{max} = \sqrt{s} - 2m_\chi$.

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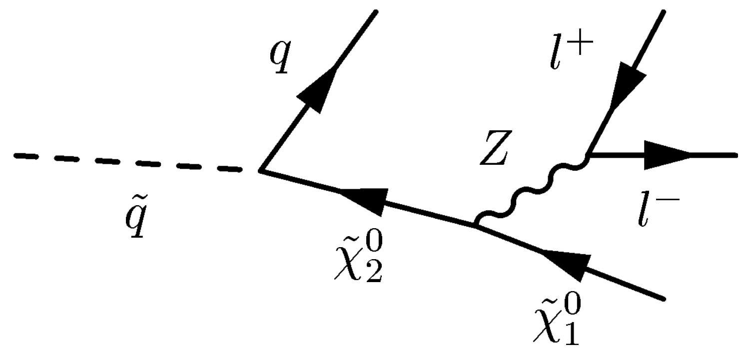
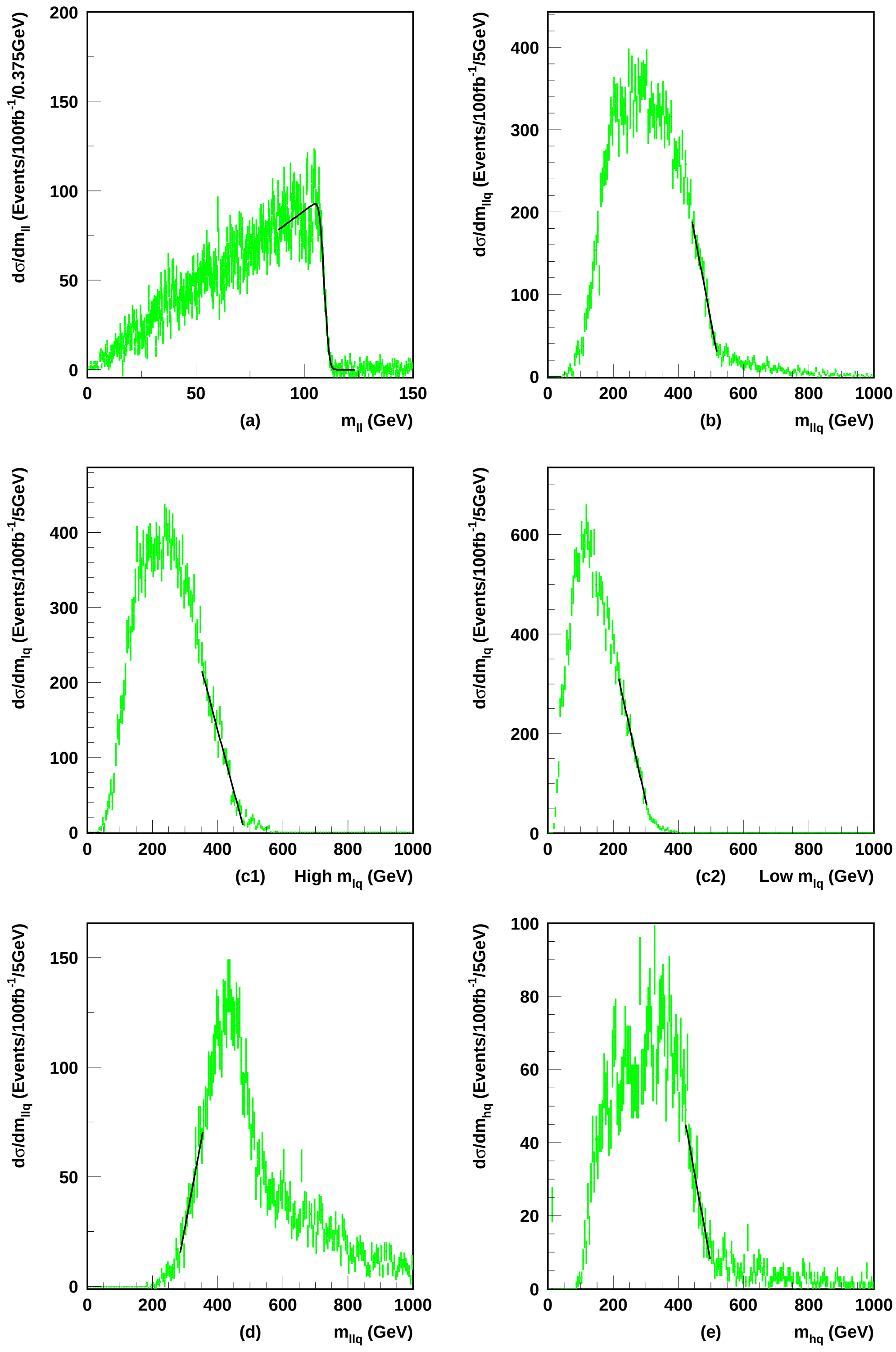
Same idea can be applied to hadron colliders ...

Consider a squark cascade decay:



1st edge : $M^{max}(\ell\ell) \approx M_{\chi_2^0} - M_{\chi_1^0}$;

2nd edge : $M^{max}(\ell\ell j) \approx M_{\tilde{q}} - M_{\chi_1^0}$.

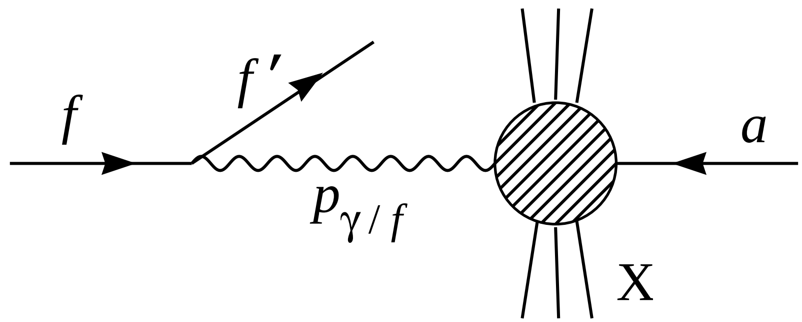


dσ/dm_ll (Events/100fb^-1/0.375GeV)
m_ll (GeV)
(a)
dσ/dm_llq (Events/100fb^-1/5GeV)
m_llq (GeV)
(b)
dσ/dm_lq (Events/100fb^-1/5GeV)
(c1)
High m_lq (GeV)
dσ/dm_lq (Events/100fb^-1/5GeV)
(c2)
Low m_lq (GeV)
dσ/dm_llq (Events/100fb^-1/5GeV)
(d)
m_llq (GeV)
dσ/dm_hq (Events/100fb^-1/5GeV)
(e)
m_hq (GeV)

(c). t-channel singularity: splitting.

- Gauge boson radiation off a fermion:

The familiar Weizsäcker-Williams approximation



$$\sigma(fa \to f'X) \approx \int dx\ dp_T^2\ P_{\gamma/f}(x, p_T^2)\ \sigma(\gamma a \to X),$$

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{E^2}{m_e^2},$$

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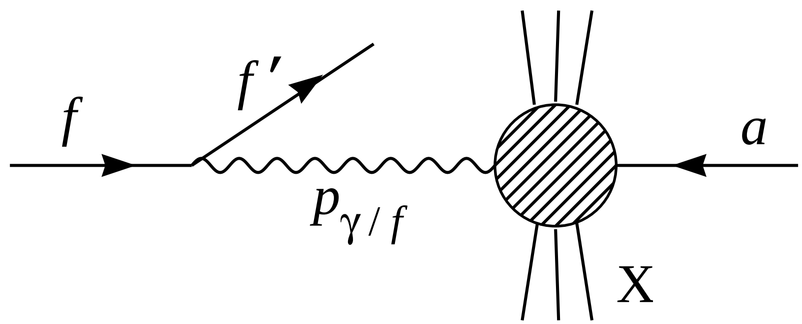

$$\sigma(fa \to f'X) \;\approx\; \int dx\, dp_T^2\, P_{\gamma/f}(x, p_T^2)\, \sigma(\gamma a \to X),$$
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† The kernel is the same as $q \to qg^*$ $\Rightarrow$ generic for parton splitting;

† The high energy enhancement $\ln(E/m_e)$ reflects the collinear behavior.

- Generalize to massive gauge bosons:

$$P_{V/f}^{T}(x,p_T^2) = \frac{g_V^2+g_A^2}{8\pi^2}\,\frac{1+(1-x)^2}{x}\,\frac{p_T^2}{(p_T^2+(1-x)M_V^2)^2},$$

$$P_{V/f}^{L}(x,p_T^2) = \frac{g_V^2+g_A^2}{4\pi^2}\,\frac{1-x}{x}\,\frac{(1-x)M_V^2}{(p_T^2+(1-x)M_V^2)^2}.$$

- Generalize to massive gauge bosons:

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$$P^L_{V/f}(x,p_T^2) = \frac{g_V^2+g_A^2}{4\pi^2}\,\frac{1-x}{x}\,\frac{(1-x)M_V^2}{(p_T^2+(1-x)M_V^2)^2}.$$

Special kinematics for massive gauge boson fusion processes:

For the accompanying jets,

At low-p_{jT},

$$\left.\begin{array}{l} p_{jT}^2 \approx (1-x)M_V^2 \\ E_j \sim (1-x)E_q \end{array}\right\} \textit{forward jet tagging}$$

At high-p_{jT},

$$\left.\begin{array}{l} \frac{d\sigma(V_T)}{dp_{jT}^2} \propto 1/p_{jT}^2 \\ \frac{d\sigma(V_L)}{dp_{jT}^2} \propto 1/p_{jT}^4 \end{array}\right\} \textit{central jet vetoing}$$

has become important tools for Higgs searches, single-top signal etc.

(C). Charge forward-backward asymmetry A_{FB}:

The coupling vertex of a vector boson V_μ to an arbitrary fermion pair f

$$i \sum_{\tau}^{L,R} g_\tau^f \, \gamma^\mu \, P_\tau \qquad \rightarrow \qquad \text{crucial to probe chiral structures.}$$

The parton-level forward-backward asymmetry is defined as

$$A_{FB}^{i,f} \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4}\mathcal{A}_i \mathcal{A}_f,$$

$$\mathcal{A}_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}.$$

where N_F (N_B) is the number of events in the forward (backward) direction defined in the parton c.m. frame relative to the initial-state fermion $\vec{p}_i$.

At hadronic level:

$$A_{FB}^{\mathsf{LHC}} = \frac{\int dx_1 \sum_q A_{FB}^{q,f} \Big(P_q(x_1) P_{\bar{q}}(x_2) - P_{\bar{q}}(x_1) P_q(x_2) \Big) \, \mathsf{sign}(x_1 - x_2)}{\int dx_1 \sum_q \Big(P_q(x_1) P_{\bar{q}}(x_2) + P_{\bar{q}}(x_1) P_q(x_2) \Big)}.$$

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Perfectly fine for Z/Z'-type:

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$.

In pp collisions, however, what is the direction of $\vec{p}_{quark}$?

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It is the boost-direction of $\ell^+\ell^-$.

How about $W^{\pm}/W'^{\pm}(\ell^{\pm}\nu)$-type?

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$,
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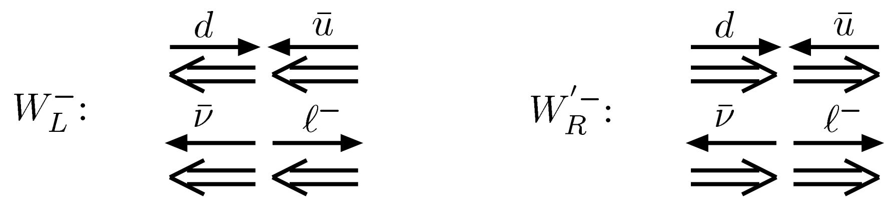

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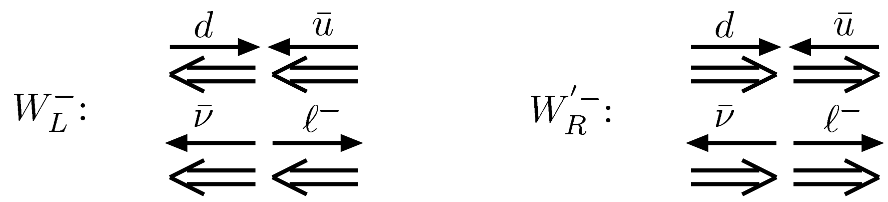
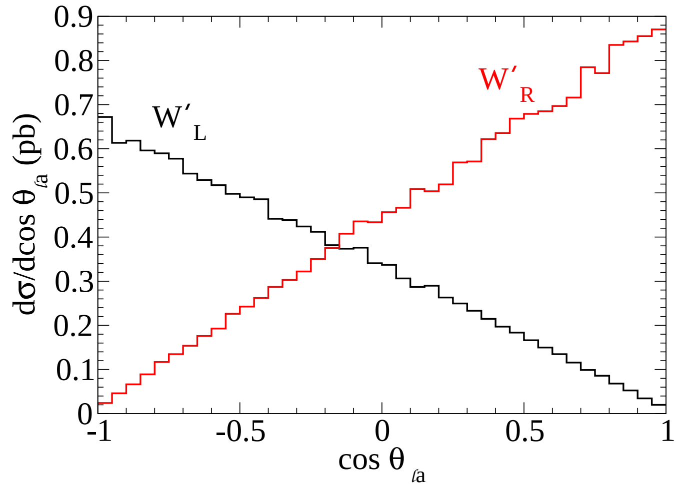


(a) (b)

In $p\bar{p}$ collisions: (1). a reconstructable system; (2). with spin correlation:
Only tops: $W' \to t\bar{b} \to \ell^{\pm}\nu\ \bar{b}$:



(D). CP asymmetries A_{CP}:

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Definition: A_{CP} vanishes if CP-violation interactions do not exist (for the relevant particles involved).

This is meant to be in contrast to an observable:
that'd be *modified* by the presence of CP-violation,
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Two ways:

a). Compare the rates between a process and its CP-conjugate process:

$$\frac{R(i \to f) - R(\bar{i} \to \bar{f})}{R(i \to f) + R(\bar{i} \to \bar{f})}, \quad \text{e.g.} \quad \frac{\Gamma(t \to W^+ q) - \Gamma(\bar{t} \to W^- \bar{q})}{\Gamma(t \to W^+ q) + \Gamma(\bar{t} \to W^- \bar{q})}.$$

b). Construct a CP-odd kinematical variable for an initially CP-eigenstate:

$$\mathcal{M} \sim M_1 + M_2 \sin\theta,$$
$$A_{CP} = \sigma^F - \sigma^B = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

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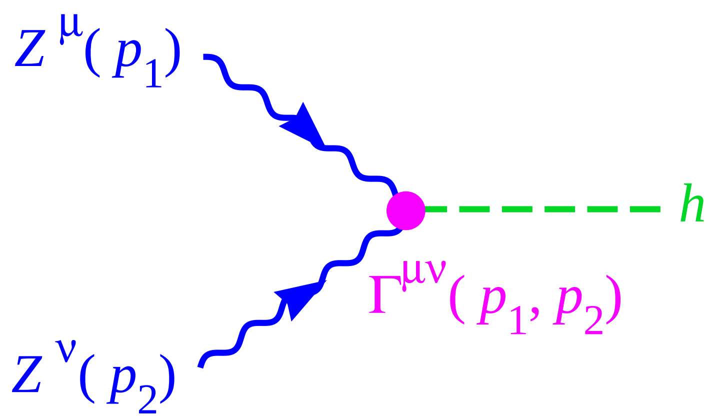
E.g. 1: $H \to Z(p_1) Z^*(p_2) \to e^+(q_1) e^-(q_2),\ \mu^+\mu^-$



$$\Gamma^{\mu\nu}(p_1,p_2) = i\frac{2}{v}\ h[a\ M_Z^2 g^{\mu\nu} + b\ (p_1^\mu p_2^\nu - p_1 \cdot p_2 g^{\mu\nu}) + \tilde{b}\ \epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}]$$

$a = 1,\ b = \tilde{b} = 0$ for SM.

In general, a, b, $\tilde{b}$ complex form factors,

describing new physics at a higher scale.

For $H \to Z(p_1)Z^*(p_2) \to e^+(q_1)e^-(q_2),\ \mu^+\mu^-$, define:

$$O_{CP} \sim (\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2),$$

$$\text{or}\ \ \cos\theta = \frac{(\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2)}{|\vec{p}_1 - \vec{p}_2||\vec{q}_1 \times \vec{q}_2)|}.$$

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E.g. 2: $H \to t(p_t)\bar{t}(p_{\bar{t}}) \to e^+(q_1)\nu_1 b_1,\ e^-(q_2)\nu_2 b_2.$

$$-\frac{m_t}{v}\bar{t}(a + b\gamma^5)t\ H$$
$$O_{CP} \sim (\vec{p}_t - \vec{p}_{\bar{t}}) \cdot (\vec{p}_{e^+} \times \vec{p}_{e^-}).$$

thus define an asymmetry angle.