Collider Phenomenology

— From basic knowledge to new physics searches

Tao Han

University of Wisconsin – Madison

BUSSTEPP 2010

Univ. of Swansea, Aug. 23–Sept. 3, 2010

Outline:

Lecture I: Colliders and Detectors

Outline:

Lecture I: Colliders and Detectors

Lecture II: Basics Techniques and Tools

Outline:

Lecture I: Colliders and Detectors

Lecture II: Basics Techniques and Tools

Lecture III: (a). An e^+e^- Linear Collider

(b). Perturbative QCD at Hadron Colliders

(c). Hadron Colliders Physics

Outline:

Lecture I: Colliders and Detectors

Lecture II: Basics Techniques and Tools

Lecture III: (a). An e^+e^- Linear Collider

(b). Perturbative QCD at Hadron Colliders

(c). Hadron Colliders Physics

Lecture IV: From Kinematics to Dynamics

Outline:

Lecture I: Colliders and Detectors

Lecture II: Basics Techniques and Tools

Lecture III: (a). An e^+e^- Linear Collider

(b). Perturbative QCD at Hadron Colliders

(c). Hadron Colliders Physics

Lecture IV: From Kinematics to Dynamics

Lecture V: Search for New Physics at Hadron Colliders

Outline:

Lecture I: Colliders and Detectors

Lecture II: Basics Techniques and Tools

Lecture III: (a). An e^+e^- Linear Collider

(b). Perturbative QCD at Hadron Colliders

(c). Hadron Colliders Physics

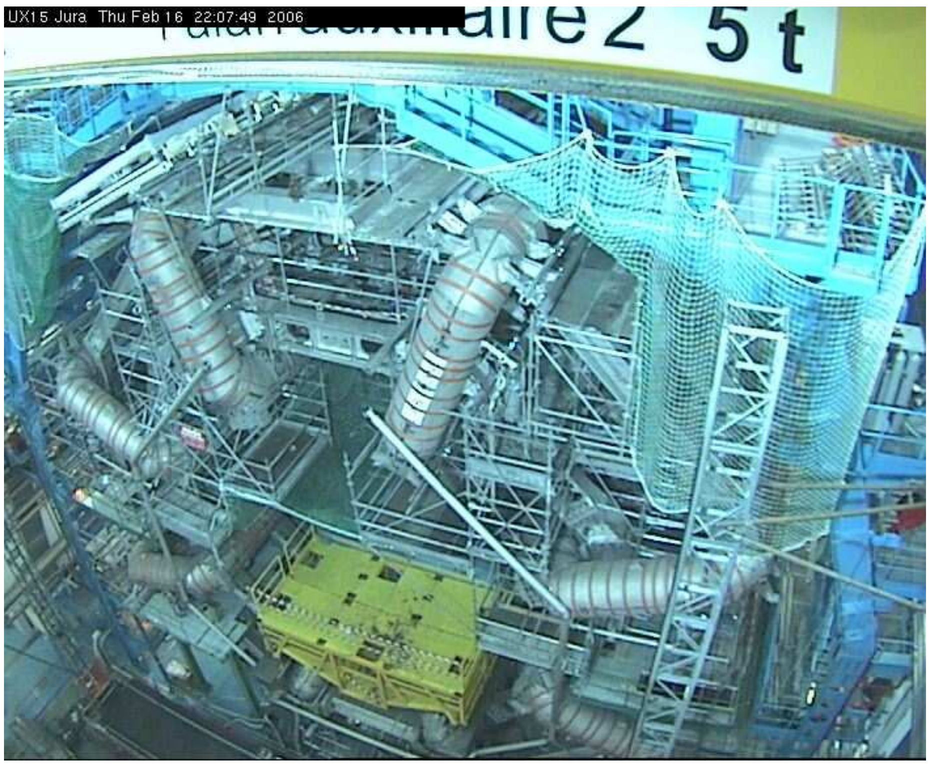
Lecture IV: From Kinematics to Dynamics

Lecture V: Search for New Physics at Hadron Colliders

Main reference: TASI 04 Lecture notes
hep-ph/0508097,
plus the other related lectures in this school.

III(c). Hadron Collider Physics

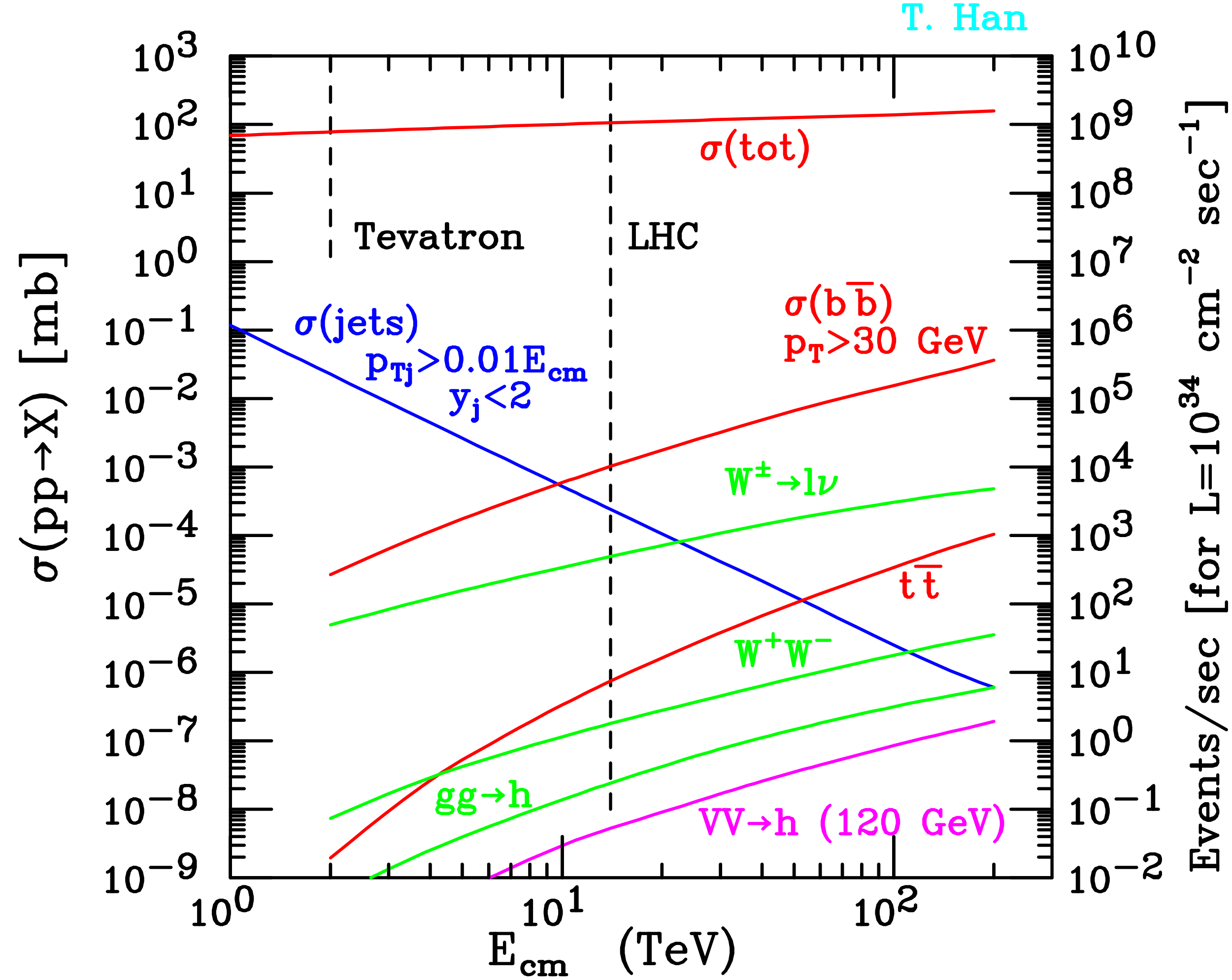
(A). New HEP frontier: the LHC
Major discoveries and excitement ahead ...



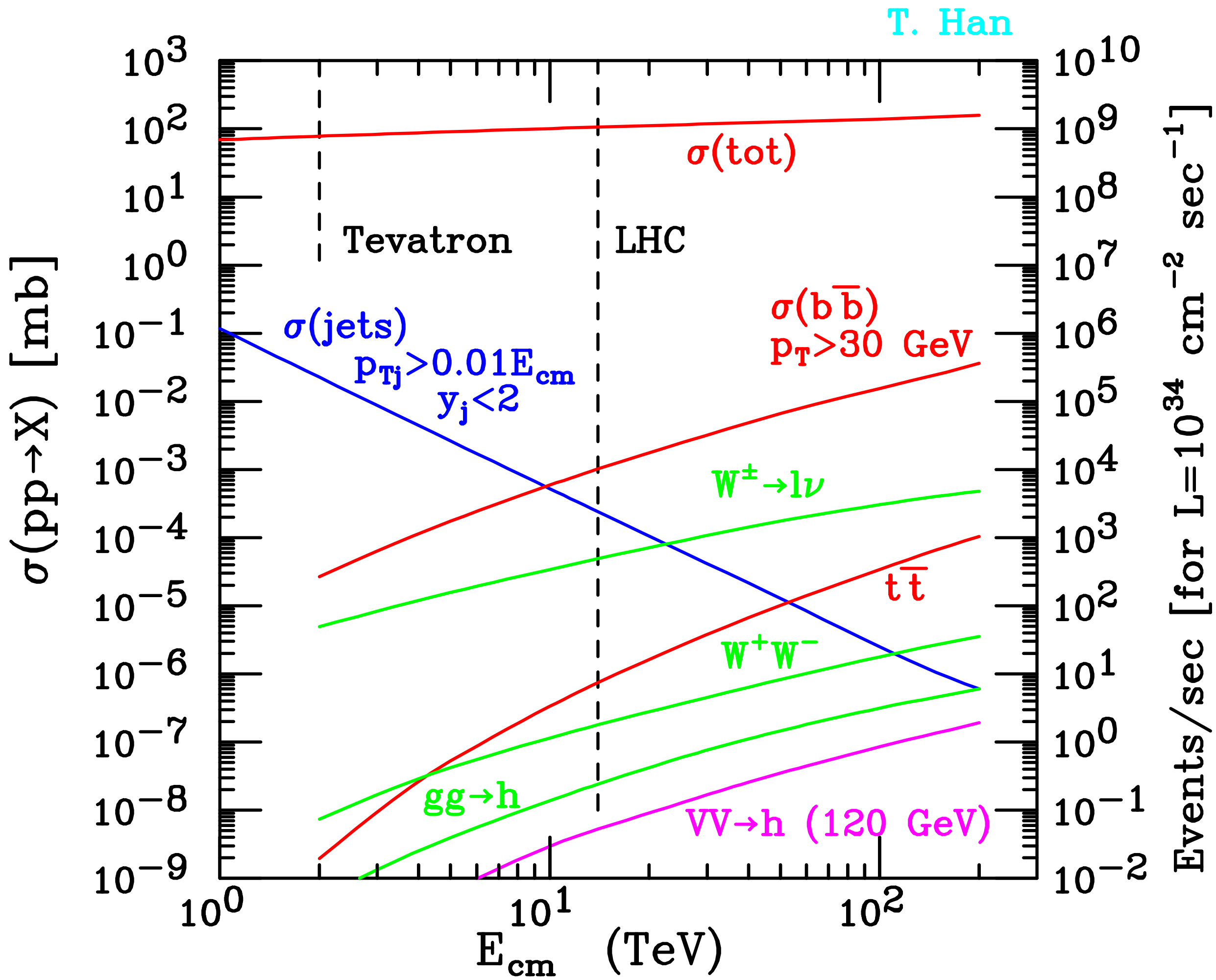
ATLAS (90m underground) CMS

(New mission started in March 2010.)

LHC Event rates for various SM processes:



LHC Event rates for various SM processes:



$10^{34}/cm^2/s \Rightarrow 100\ fb^{-1}/yr.$

Annual yield # of events $= \sigma \times L_{int}$:

10B $W^\pm$; 100M $t\bar{t}$; 10M W^+W^-; 1M H^0...

Great potential to open a new chapter of HEP!

Theoretical challenges:

Unprecedented energy frontier

Theoretical challenges:

Unprecedented energy frontier

(a) Total hadronic cross section: Non-perturbative.

The order of magnitude estimate:

$$\sigma_{pp} = \pi r_{eff}^2 \approx \pi / m_\pi^2 \sim 120 \text{ mb.}$$

Theoretical challenges:

Unprecedented energy frontier

(a) Total hadronic cross section: Non-perturbative.
The order of magnitude estimate:

$$\sigma_{pp} = \pi r_{eff}^2 \approx \pi / m_\pi^2 \sim 120 \text{ mb.}$$

Energy-dependence?

$$\sigma(pp) \begin{cases} \approx 21.7 \; (\frac{s}{\text{GeV}^2})^{0.0808} & \text{Empirical relation} \\ < \frac{\pi}{m_\pi^2} \; \ln^2 \frac{s}{s_0} & \text{Froissart bound.} \end{cases}$$

Theoretical challenges:

Unprecedented energy frontier

(a) Total hadronic cross section: Non-perturbative.
The order of magnitude estimate:

$$\sigma_{pp} = \pi r_{eff}^2 \approx \pi/m_\pi^2 \sim 120 \text{ mb.}$$

Energy-dependence?

$$\sigma(pp) \begin{cases} \approx 21.7\ (\frac{s}{\text{GeV}^2})^{0.0808} & \text{Empirical relation} \\ < \frac{\pi}{m_\pi^2}\ \ln^2 \frac{s}{s_0} & \text{Froissart bound.} \end{cases}$$

(b) Perturbative hadronic cross section:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2)\ \hat{\sigma}_{parton}(s).$$

Theoretical challenges:

Unprecedented energy frontier

(a) Total hadronic cross section: Non-perturbative.
The order of magnitude estimate:

$$\sigma_{pp} = \pi r_{eff}^2 \approx \pi / m_\pi^2 \sim 120 \text{ mb}.$$

Energy-dependence?

$$\sigma(pp) \begin{cases} \approx 21.7 \ (\frac{s}{\text{GeV}^2})^{0.0808} & \text{Empirical relation} \\ < \frac{\pi}{m_\pi^2} \ \ln^2 \frac{s}{s_0} & \text{Froissart bound.} \end{cases}$$

(b) Perturbative hadronic cross section:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2) \ \hat{\sigma}_{parton}(s).$$

- Accurate (higher orders) partonic cross sections $\hat{\sigma}_{parton}(s)$.
- Parton distributions functions to the extreme (density):

$Q^2 \sim (a \ few \ TeV)^2, \quad x \sim 10^{-3} - 10^{-6}.$

Experimental challenges:

- The large rate turns to a hostile environment:
  - ≈ 1 billion event/sec: impossible read-off !
  - ≈ 1 interesting event per $1,000,000$: selection (triggering).

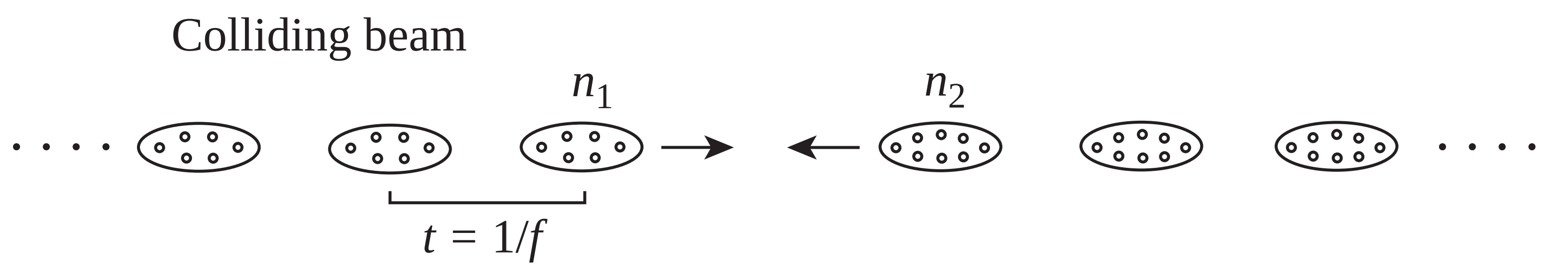
Experimental challenges:

- The large rate turns to a hostile environment:

  ≈ 1 billion event/sec: impossible read-off !

  ≈ 1 interesting event per $1,000,000$: selection (triggering).

  ≈ 25 overlapping events/bunch crossing:



$\Rightarrow$ Severe backgrounds!

Triggering thresholds:

	ATLAS	
Objects	η	p_T (GeV)
μ inclusive	2.4	6 (20)
e/photon inclusive	2.5	17 (26)
Two e's or two photons	2.5	12 (15)
1-jet inclusive	3.2	180 (290)
3 jets	3.2	75 (130)
4 jets	3.2	55 (90)
τ/hadrons	2.5	43 (65)
$\not{E}_T$	4.9	100
Jets+$\not{E}_T$	3.2, 4.9	50,50 (100,100)

($\eta = 2.5 \Rightarrow 10^\circ$; $\eta = 5 \Rightarrow 0.8^\circ$.)

Triggering thresholds:

	ATLAS	
Objects	η	p_T (GeV)
μ inclusive	2.4	6 (20)
e/photon inclusive	2.5	17 (26)
Two e's or two photons	2.5	12 (15)
1-jet inclusive	3.2	180 (290)
3 jets	3.2	75 (130)
4 jets	3.2	55 (90)
τ/hadrons	2.5	43 (65)
$\not{E}_T$	4.9	100
Jets+$\not{E}_T$	3.2, 4.9	50,50 (100,100)

($\eta = 2.5 \Rightarrow 10^\circ$; $\eta = 5 \Rightarrow 0.8^\circ$.)

With optimal triggering and kinematical selections:

$p_T \geq 30 - 100$ GeV, $|\eta| \leq 3 - 5$; $\not{E}_T \geq 100$ GeV.

(B). Special kinematics for hadron colliders

Hadron momenta: $P_A = (E_A, 0, 0, p_A), \quad P_B = (E_A, 0, 0, -p_A),$
The parton momenta: $p_1 = x_1 P_A, \quad p_2 = x_2 P_B.$

Then the parton c.m. frame moves randomly, even by event:

$$\beta_{cm} = \frac{x_1 - x_2}{x_1 + x_2}, \quad \text{or :}$$

$$y_{cm} = \frac{1}{2} \ln \frac{1 + \beta_{cm}}{1 - \beta_{cm}} = \frac{1}{2} \ln \frac{x_1}{x_2}, \quad (-\infty < y_{cm} < \infty).$$

(B). Special kinematics for hadron colliders

Hadron momenta: $P_A = (E_A, 0, 0, p_A), \quad P_B = (E_A, 0, 0, -p_A),$
The parton momenta: $p_1 = x_1 P_A, \quad p_2 = x_2 P_B.$

Then the parton c.m. frame moves randomly, even by event:

$$\beta_{cm} = \frac{x_1 - x_2}{x_1 + x_2}, \quad \text{or :}$$
$$y_{cm} = \frac{1}{2} \ln \frac{1 + \beta_{cm}}{1 - \beta_{cm}} = \frac{1}{2} \ln \frac{x_1}{x_2}, \quad (-\infty < y_{cm} < \infty).$$

The four-momentum vector transforms as

$$\begin{pmatrix} E' \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\ \beta_{cm} \\ -\gamma\ \beta_{cm} & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} = \begin{pmatrix} \cosh y_{cm} & -\sinh y_{cm} \\ -\sinh y_{cm} & \cosh y_{cm} \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}.$$

This is often called the "boost".

One wishes to design final-state kinematics invariant under the boost:
For a four-momentum $p \equiv p^\mu = (E, \vec{p})$,

$$\begin{aligned} E_T &= \sqrt{p_T^2 + m^2}, \quad y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}, \\ p^\mu &= (E_T \cosh y,\ p_T \sin \phi,\ p_T \cos \phi,\ E_T \sinh y), \\ \frac{d^3\vec{p}}{E} &= p_T dp_T d\phi\ dy = E_T dE_T d\phi\ dy. \end{aligned}$$

One wishes to design final-state kinematics invariant under the boost:
For a four-momentum $p \equiv p^\mu = (E, \vec{p})$,

$$
\begin{aligned}
E_T &= \sqrt{p_T^2 + m^2}, \quad y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}, \\
p^\mu &= (E_T \cosh y,\ p_T \sin\phi,\ p_T \cos\phi,\ E_T \sinh y), \\
\frac{d^3\vec{p}}{E} &= p_T dp_T d\phi\ dy = E_T dE_T d\phi\ dy.
\end{aligned}
$$

Due to random boost between Lab-frame/c.m. frame event-by-event,

$$
y' = \frac{1}{2} \ln \frac{E' + p_z'}{E' - p_z'} = \frac{1}{2} \ln \frac{(1 - \beta_{cm})(E + p_z)}{(1 + \beta_{cm})(E - p_z)} = y - y_{cm}.
$$

One wishes to design final-state kinematics invariant under the boost:
For a four-momentum $p \equiv p^\mu = (E, \vec{p})$,

$$\begin{aligned} E_T &= \sqrt{p_T^2 + m^2}, \quad y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}, \\ p^\mu &= (E_T \cosh y,\ p_T \sin\phi,\ p_T \cos\phi,\ E_T \sinh y), \\ \frac{d^3\vec{p}}{E} &= p_T dp_T d\phi\ dy = E_T dE_T d\phi\ dy. \end{aligned}$$

Due to random boost between Lab-frame/c.m. frame event-by-event,

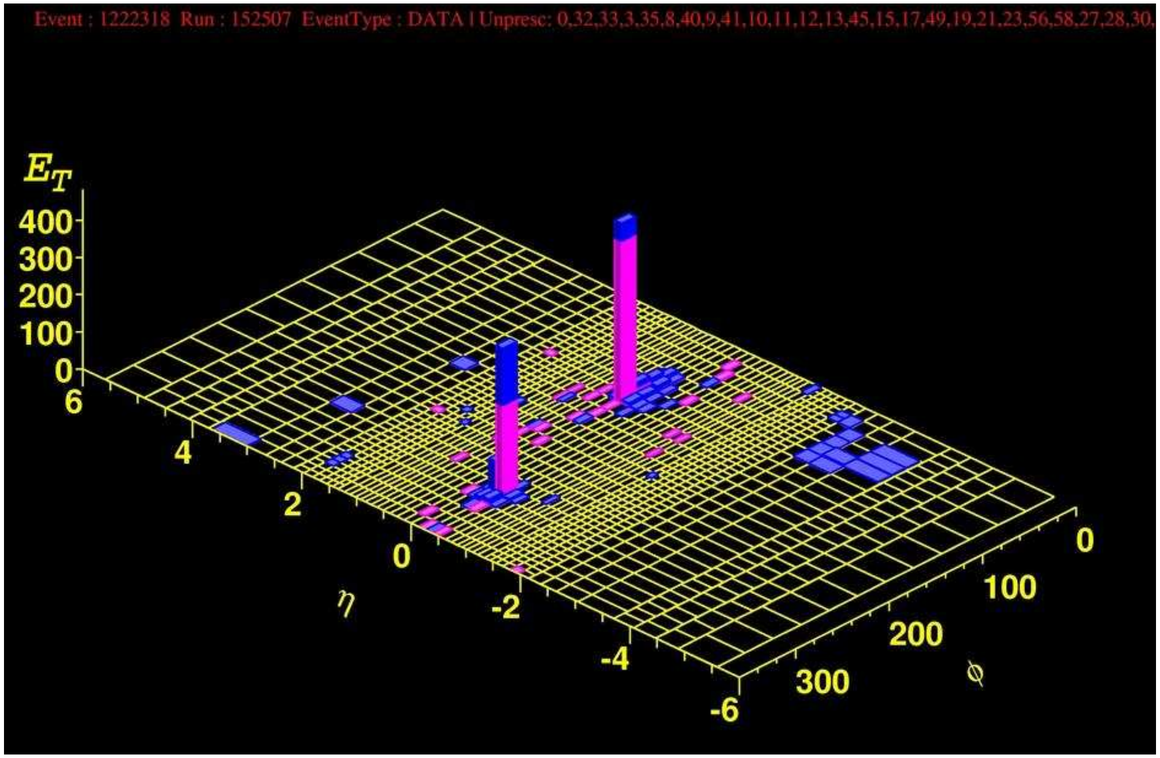
$$y' = \frac{1}{2}\ln\frac{E'+p'_z}{E'-p'_z} = \frac{1}{2}\ln\frac{(1-\beta_{cm})(E+p_z)}{(1+\beta_{cm})(E-p_z)} = y - y_{cm}.$$

In the massless limit, rapidity $\to$ pseudo-rapidity:

$$y \to \eta = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = \ln\cot\frac{\theta}{2}.$$

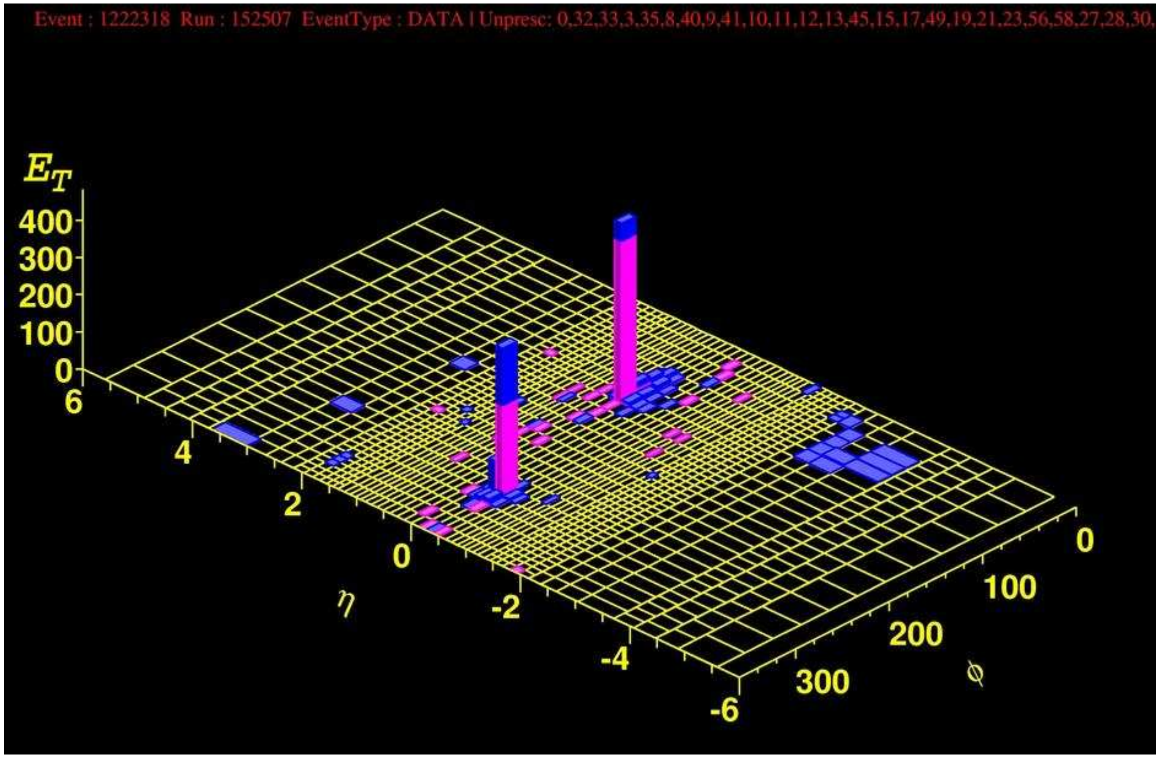
`Exercise 4.1:  Verify all the above equations.`

The "Lego" plot:



A CDF di-jet event on a lego plot in the $\eta - \phi$ plane.

The "Lego" plot:



A CDF di-jet event on a lego plot in the $\eta - \phi$ plane.

$\phi, \Delta y = y_2 - y_1$ is boost-invariant.

Thus the "separation" between two particles in an event

$$\Delta R = \sqrt{\Delta\phi^2 + \Delta y^2}$$ is boost-invariant,

and lead to the "cone definition" of a jet.

(C). Hadron collider status:

The Tevatron rocks, and the LHC delivers !

(C). Hadron collider status:

The Tevatron rocks, and the LHC delivers !

At the Tevatron Run II:

Peak luminosity record high $\approx 2 \times 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$;
Integrated luminosity $5\ \ \mathrm{fb}^{-1}$/expt, still with potential for discovery.

(C). Hadron collider status:

The Tevatron rocks, and the LHC delivers !

At the Tevatron Run II:

Peak luminosity record high $\approx 2 \times 10^{32}$ cm^{-2} s^{-1};
Integrated luminosity 5 fb^{-1}/expt, still with potential for discovery.

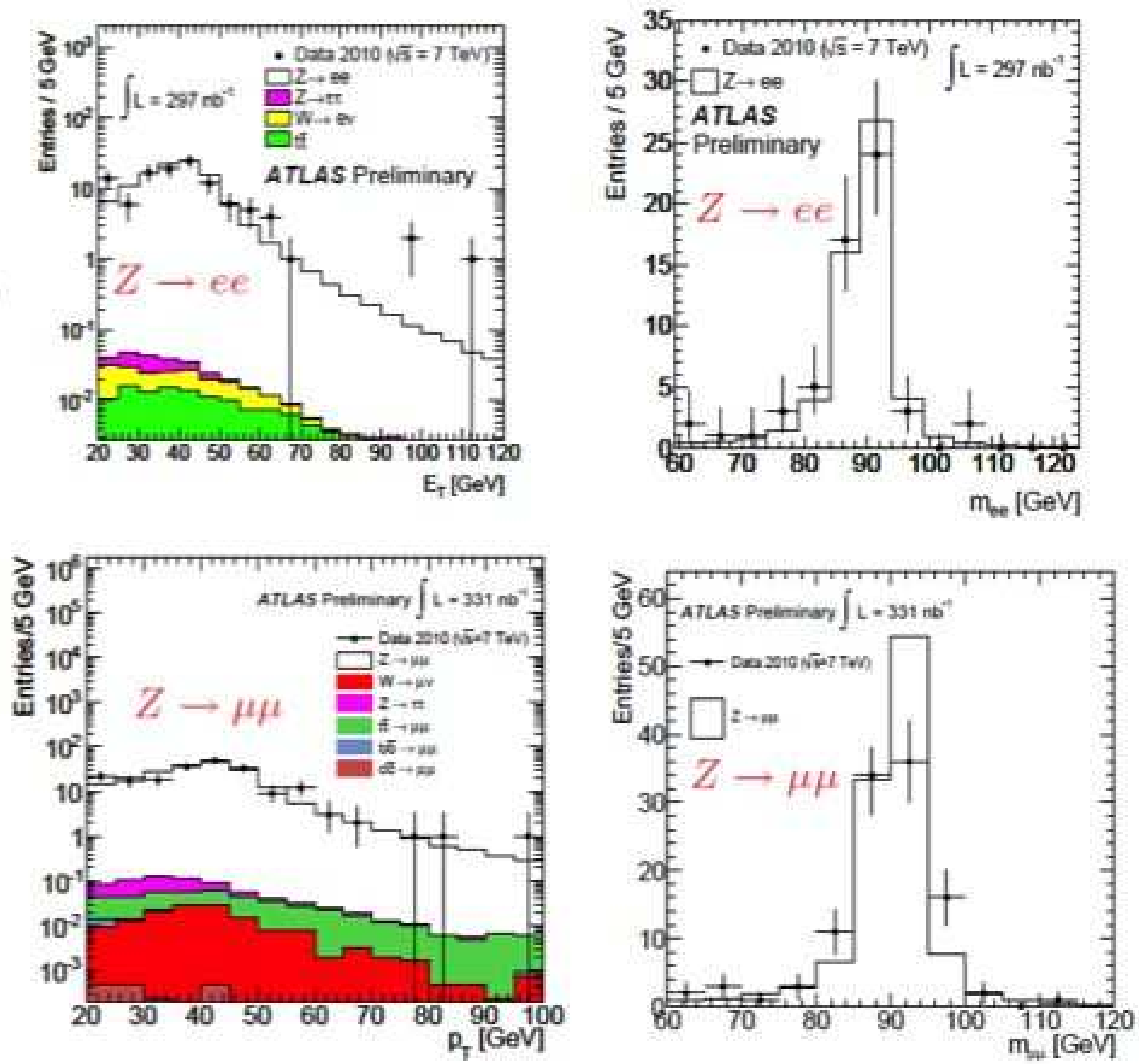
At the LHC:

$E_{cm} = 7$ TeV, integrated luminosity 1.5 pb^{-1},
leading the HEP frontier.

ATLAS Z re-discovery:

Z Selection

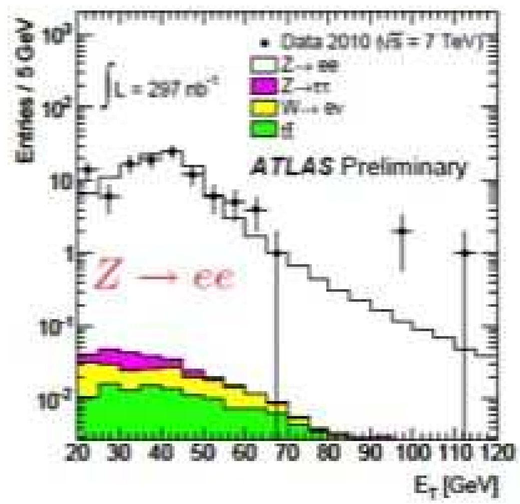
- Two oppositely charged leptons (e/μ).
- Same lepton selection as W analysis except medium electrons.
- Invariant mass $66 < m_Z < 116$ GeV.

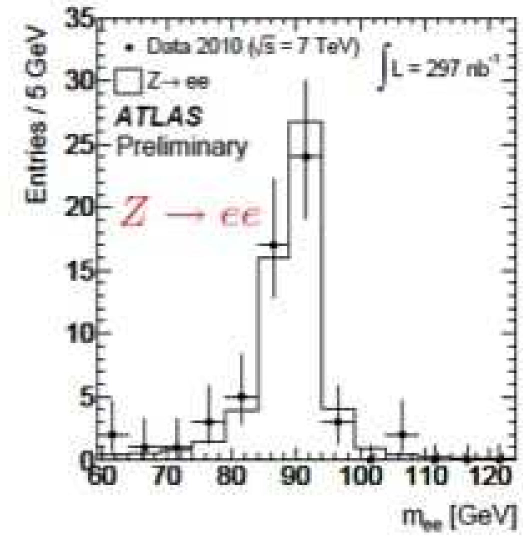


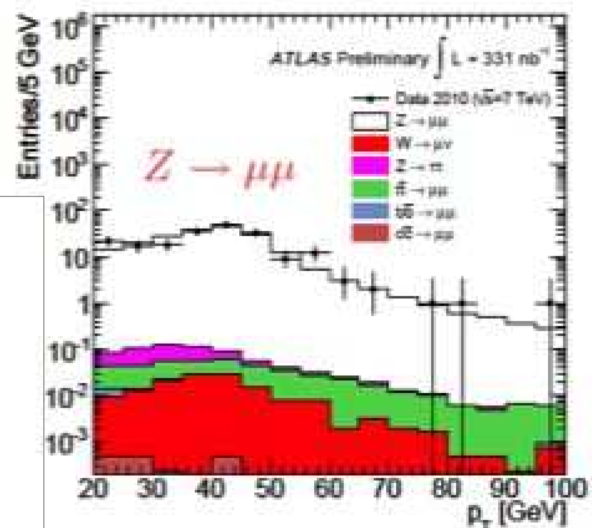
ATLAS Z re-discovery:

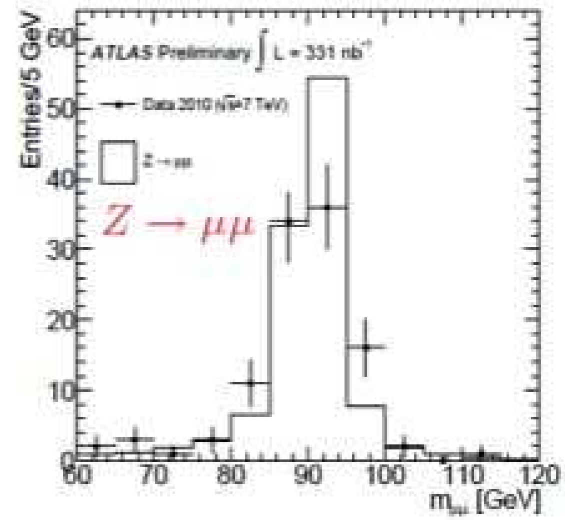
Z Selection

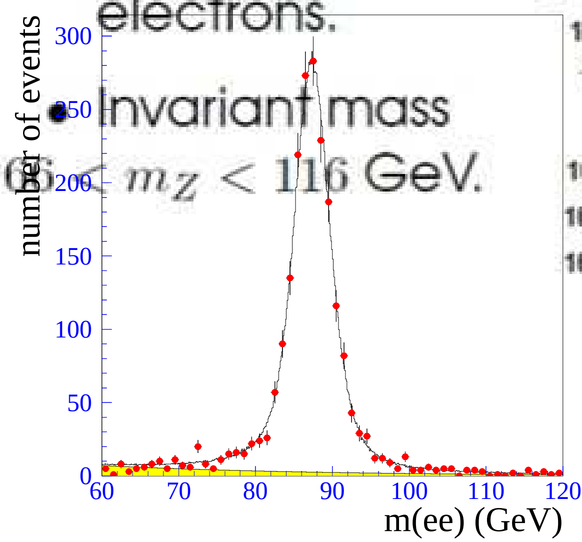
- Two oppositely charged leptons (e/μ).
- Same lepton selection as W analysis except medium electrons.
- Invariant mass $66 < m_Z < 116$ GeV.











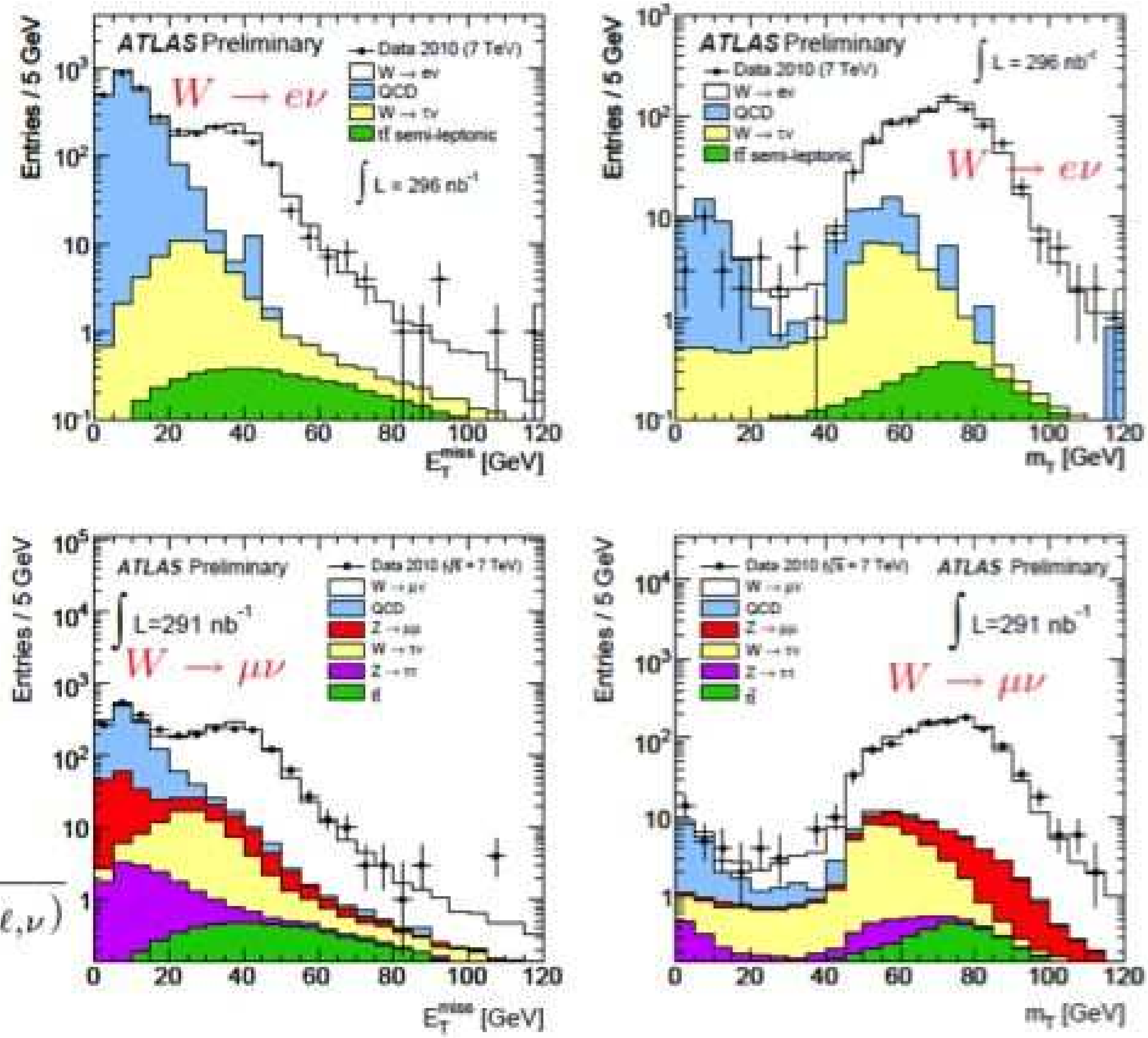
D0 Z events.

ATLAS W re-discovery:

W Selection

- Tight electron.
- Muon with $p_T > 20$ GeV.
- Muon isolation $\sum_{\Delta R<0.4} p_T^{trk}/p_T^{\mu} < 0.2$
- $\not{E}_{\mathrm{T}} > 25$ GeV.
- $m_T > 40$ GeV.

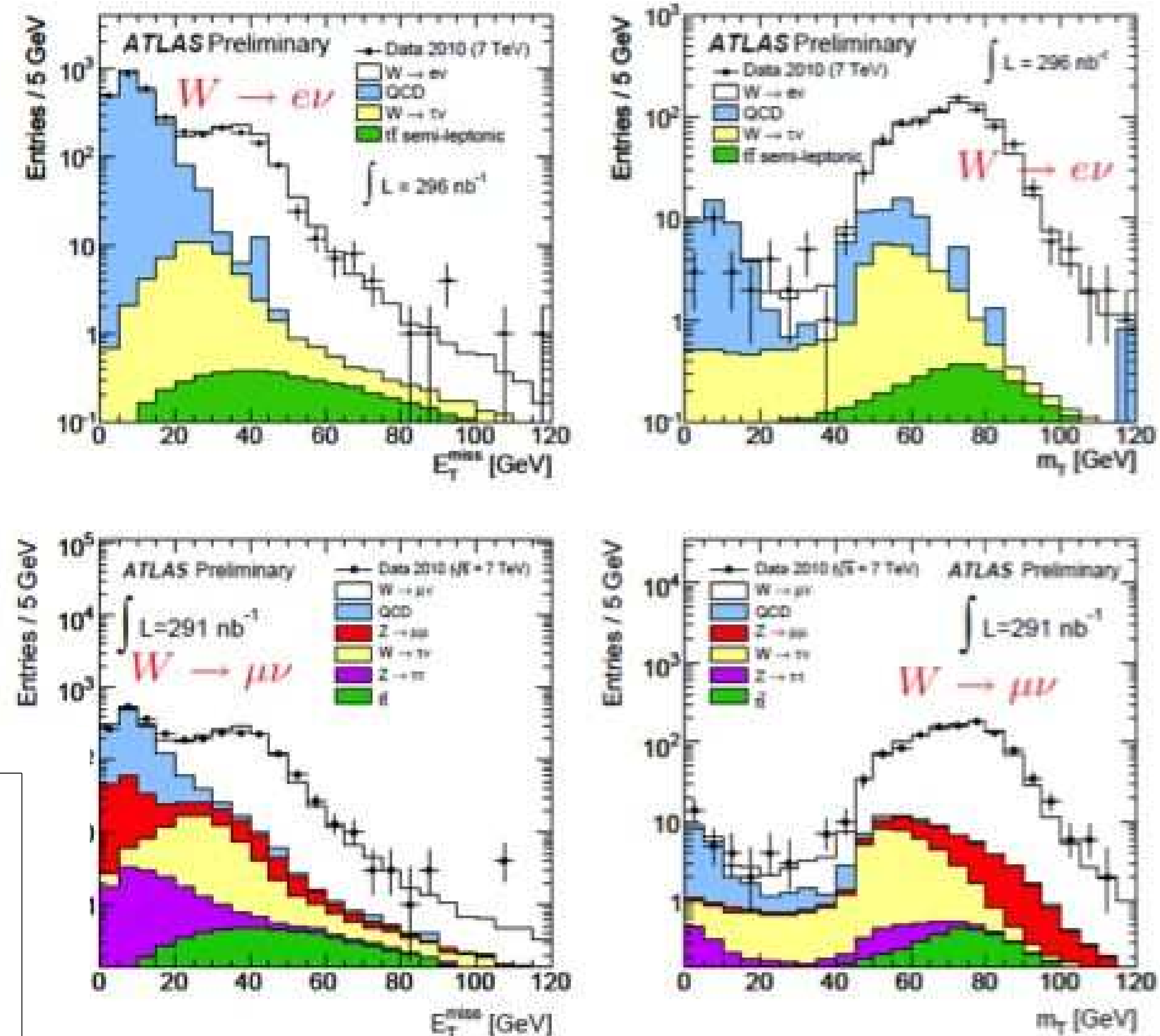
$$m_T = \sqrt{2 p_T^{\ell} p_T^{\nu} (1 - \cos(\Delta\phi_{\ell,\nu})}$$

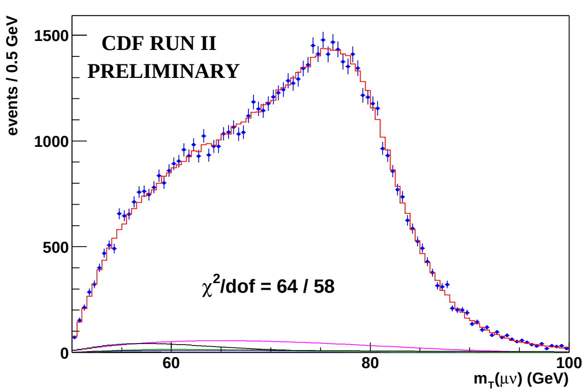


ATLAS W re-discovery:

W Selection

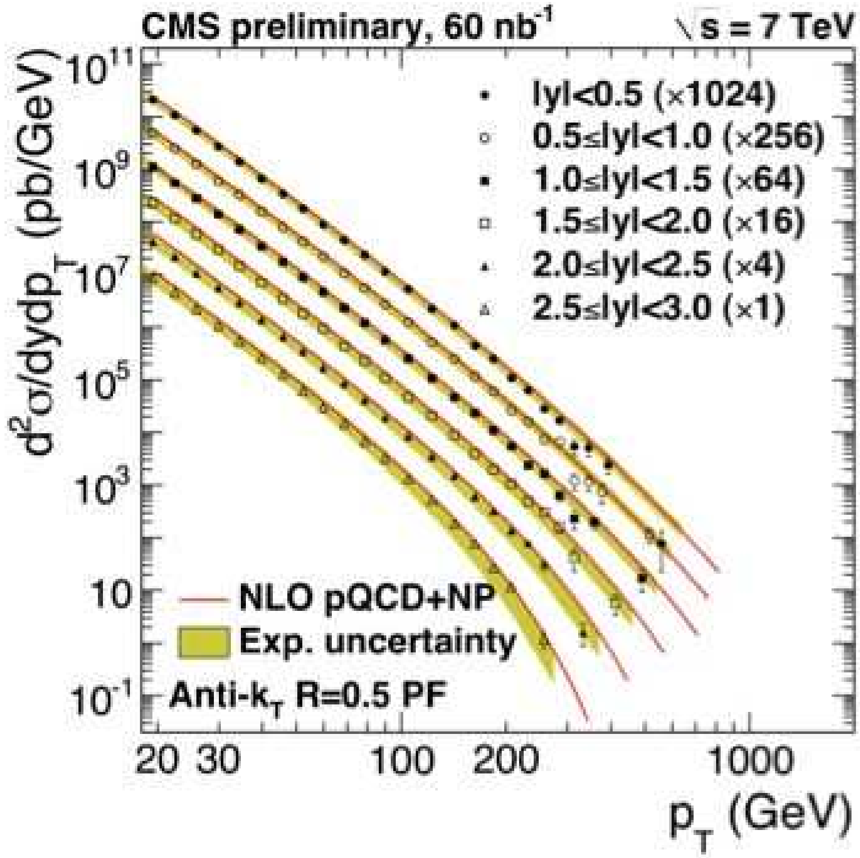
- Tight electron.
- Muon with $p_T > 20$ GeV.
- Muon isolation $\sum_{\Delta R<0.4} p_T^{trk}/p_T^{\mu} < 0.2$

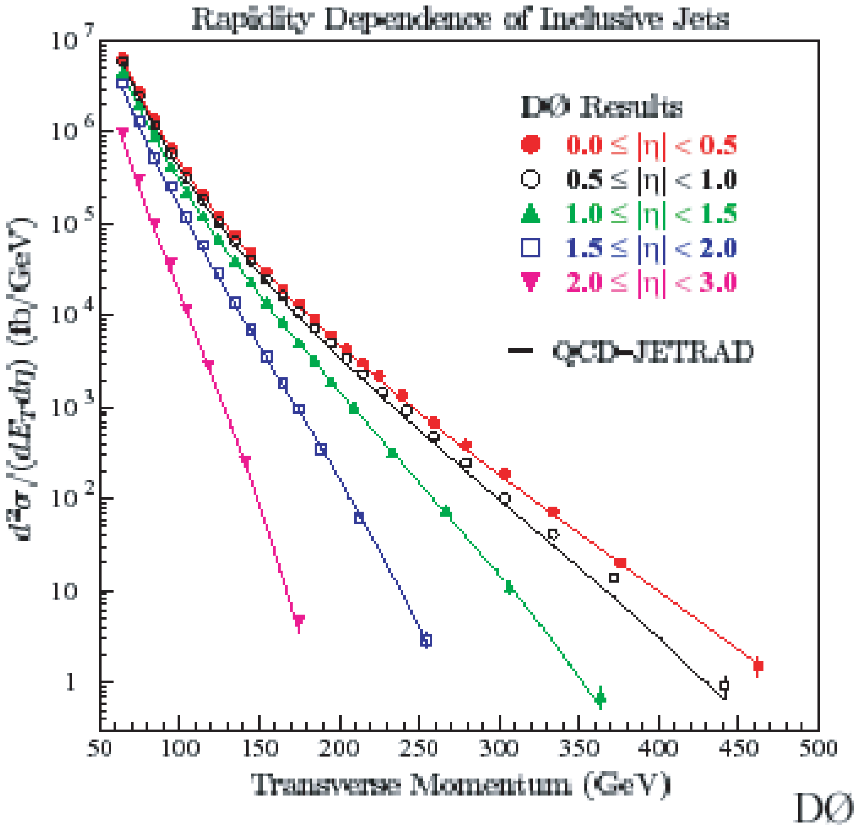




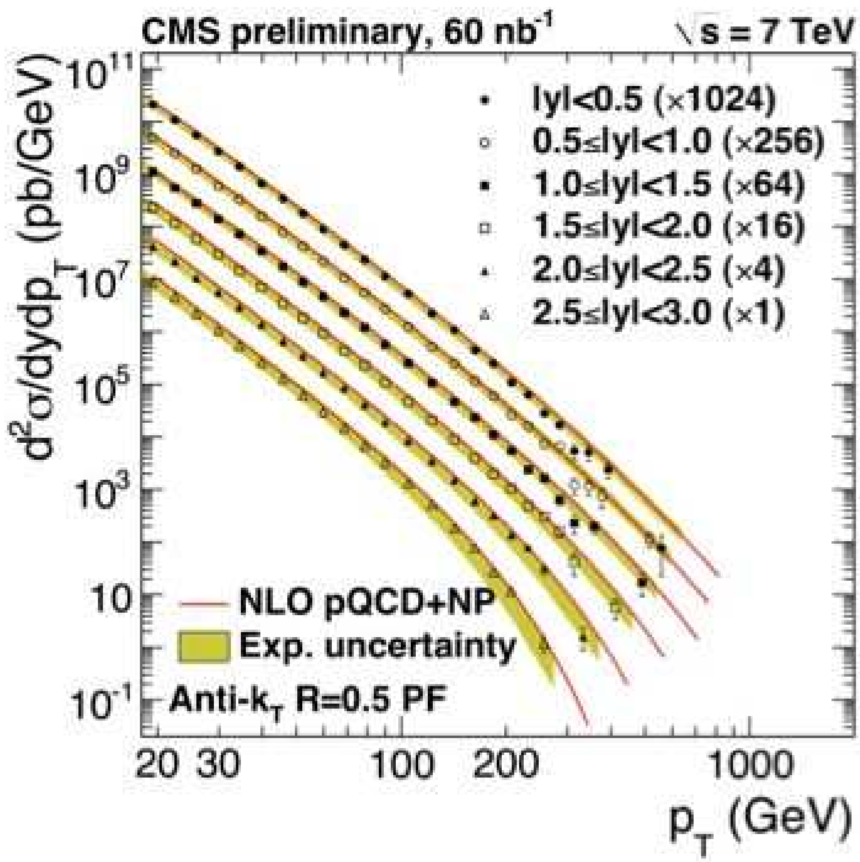
CDF W events.

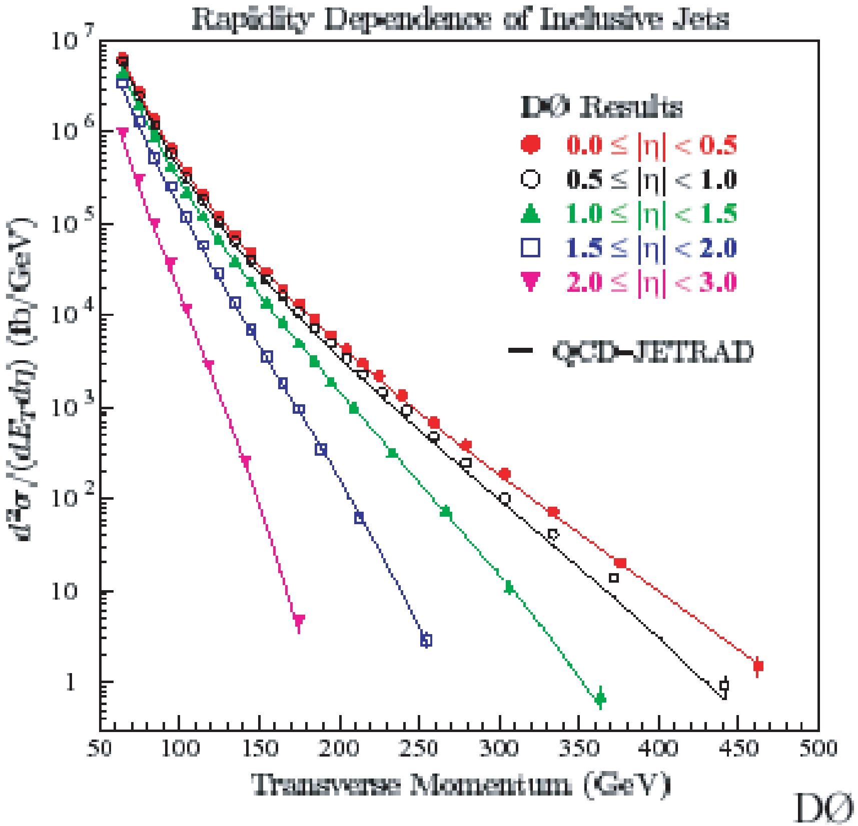
CMS 1-jet in different rapidities: D0 1-jet in rapidity ranges:





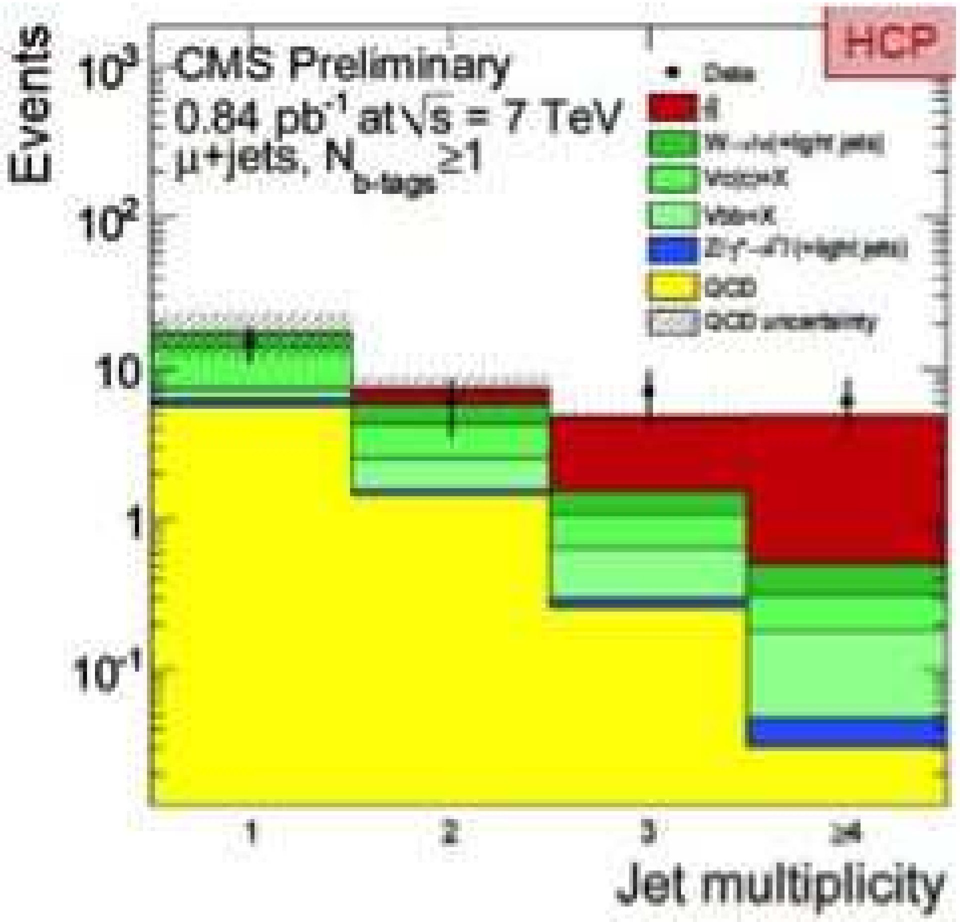
CMS 1-jet in different rapidities: D0 1-jet in rapidity ranges:



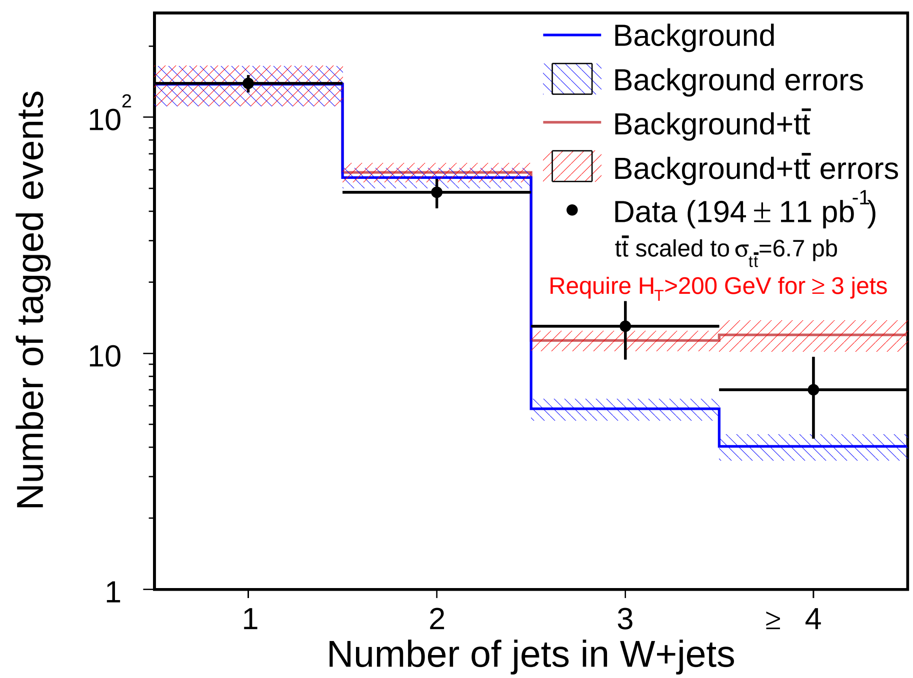


LHC QCD results went BEYOND the Tevatron !

CMS $W+$jets and top events

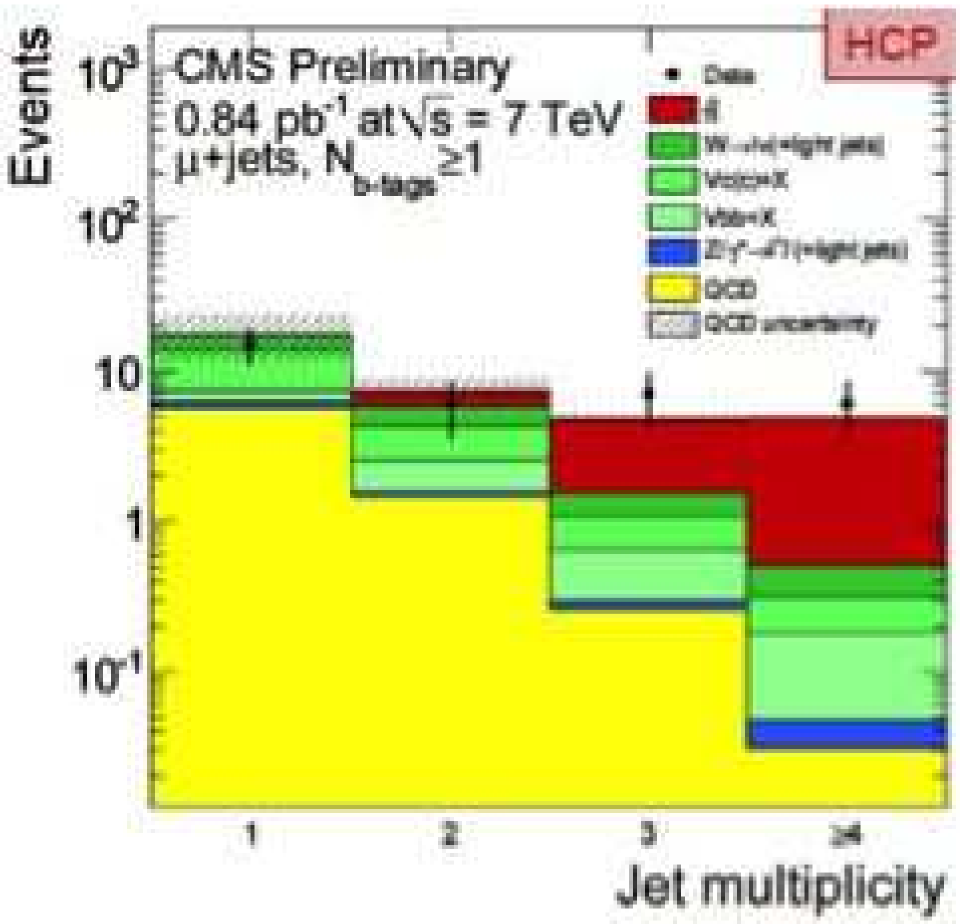


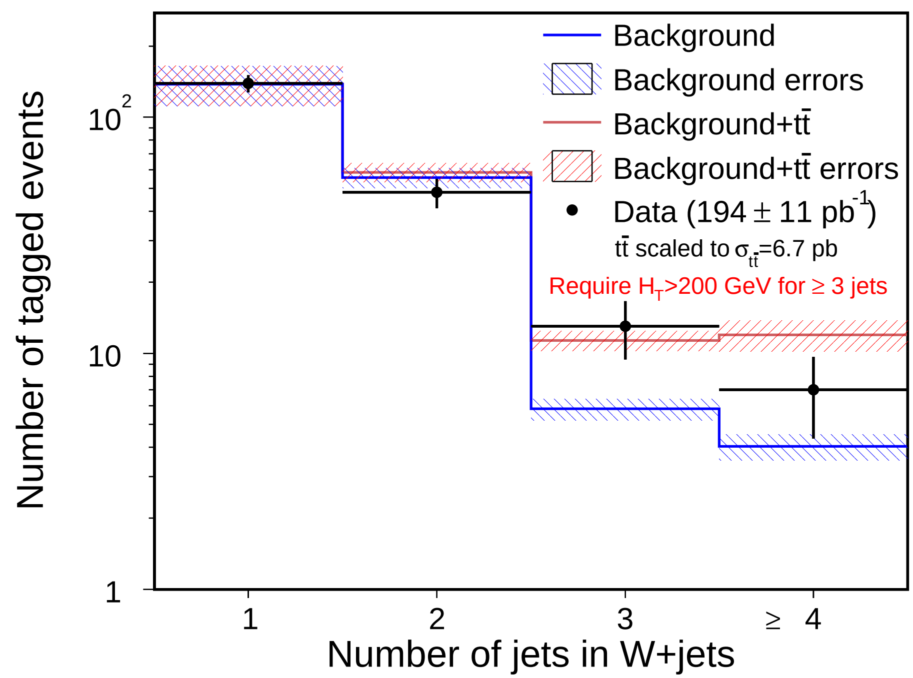
CDF $W+$jets and top



CMS W+jets and top events

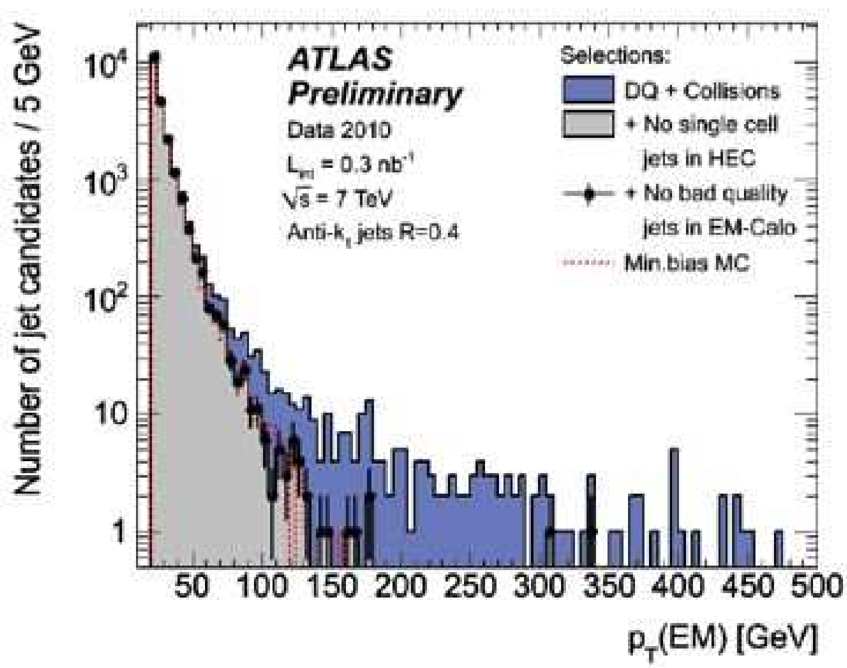
CDF W+jets and top



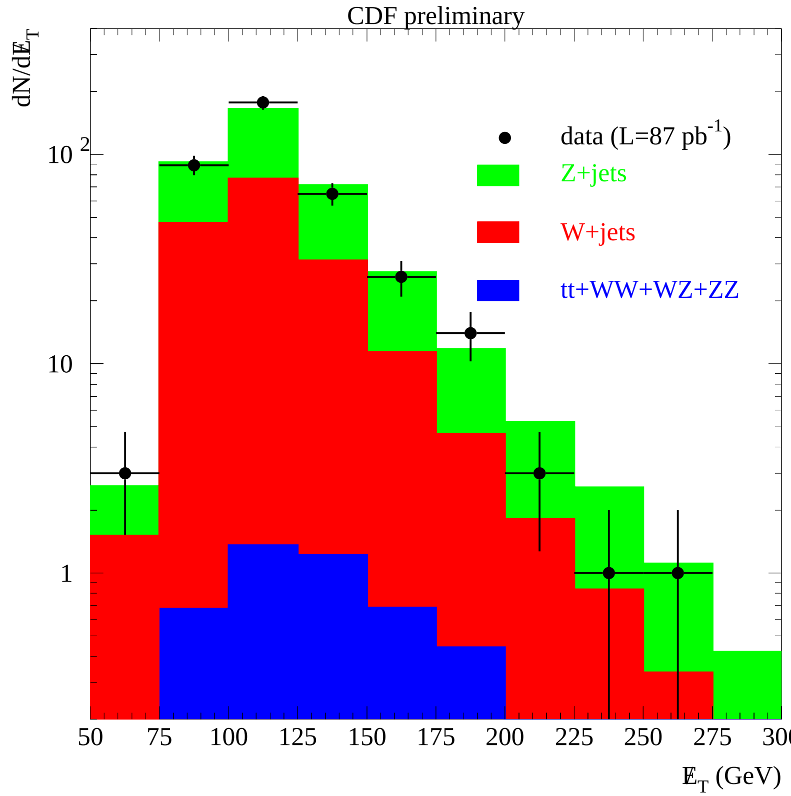


LHC top studies catching up !

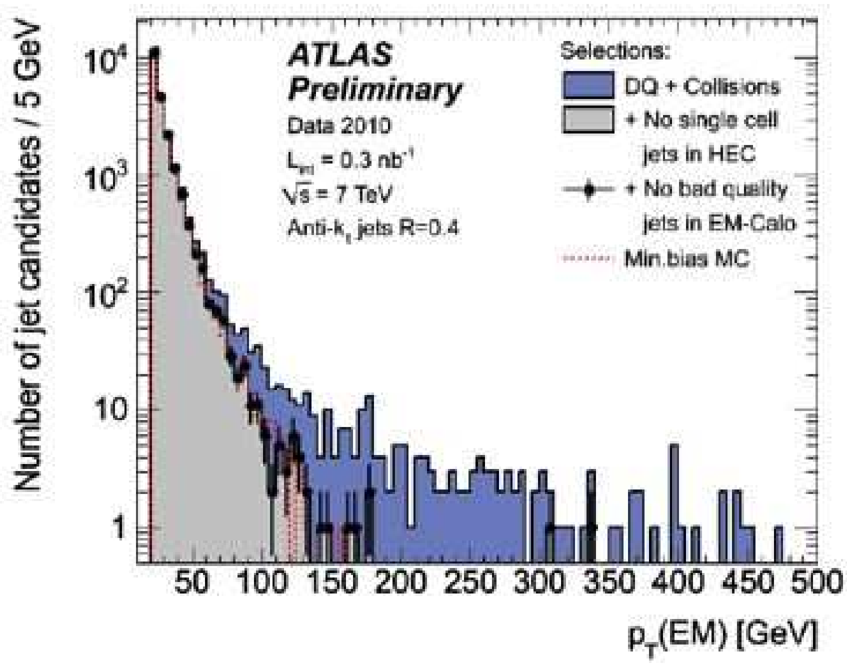
ATLAS $\not\!\!E_T$ distribution:

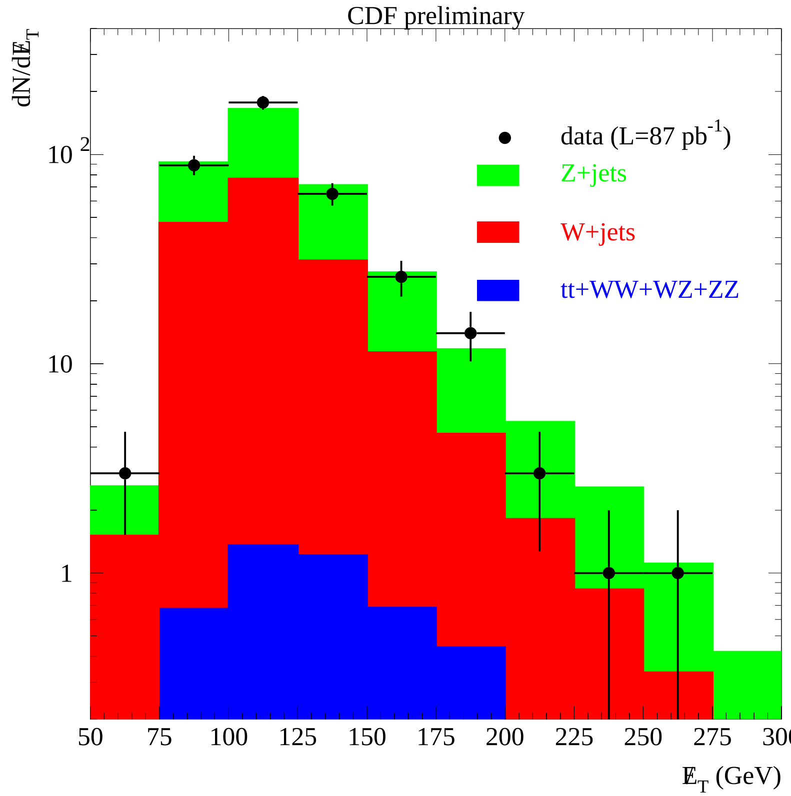


CDF $\not\!\!E_T$ at high end:



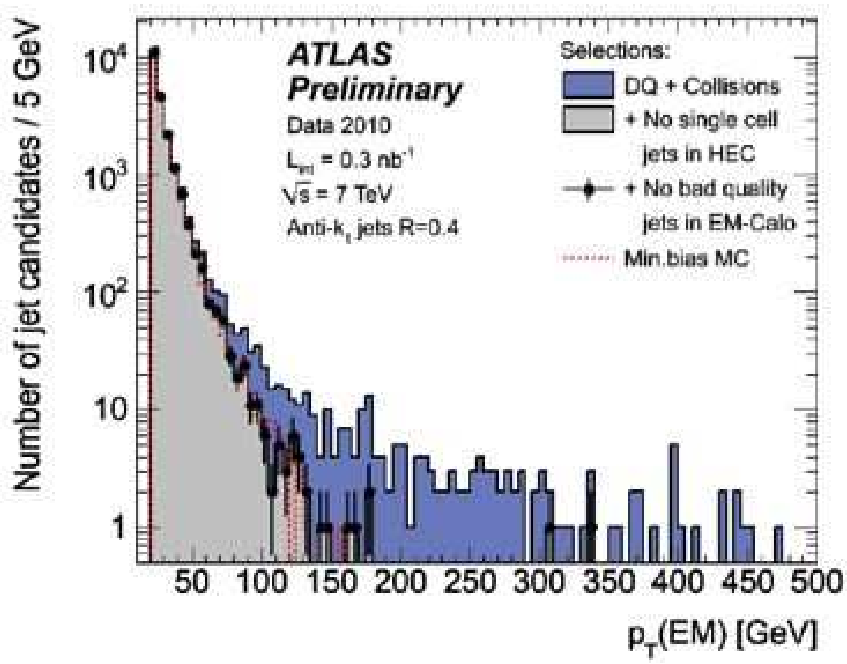
ATLAS $\not\!\!E_T$ distribution: CDF $\not\!\!E_T$ at high end:

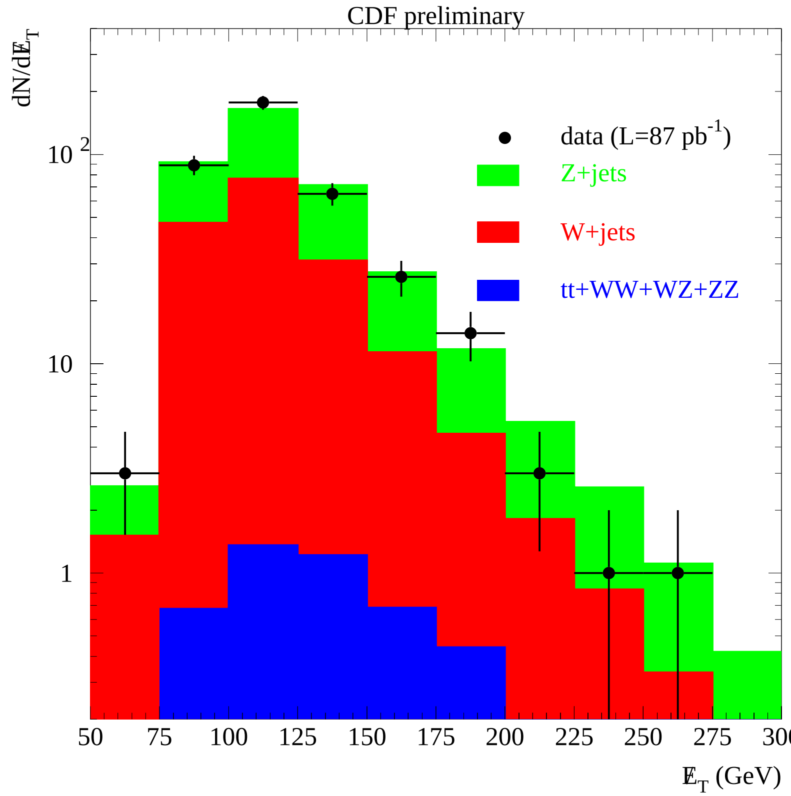




LHC $\not\!\!E_T$ results rapidly improving !

ATLAS $\not\!\!E_T$ distribution: CDF $\not\!\!E_T$ at high end:

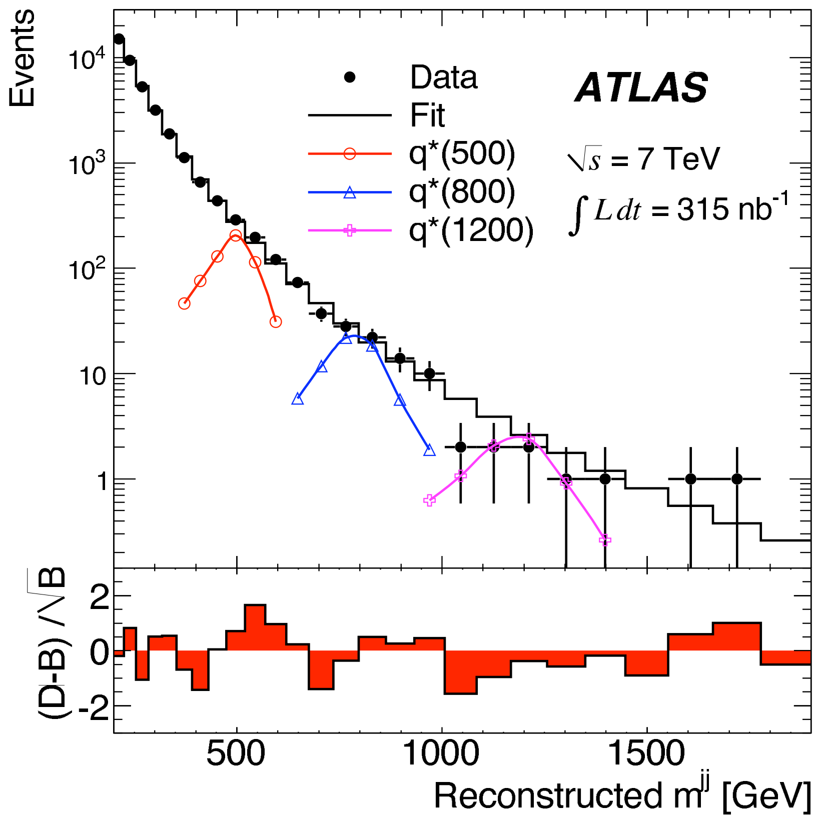


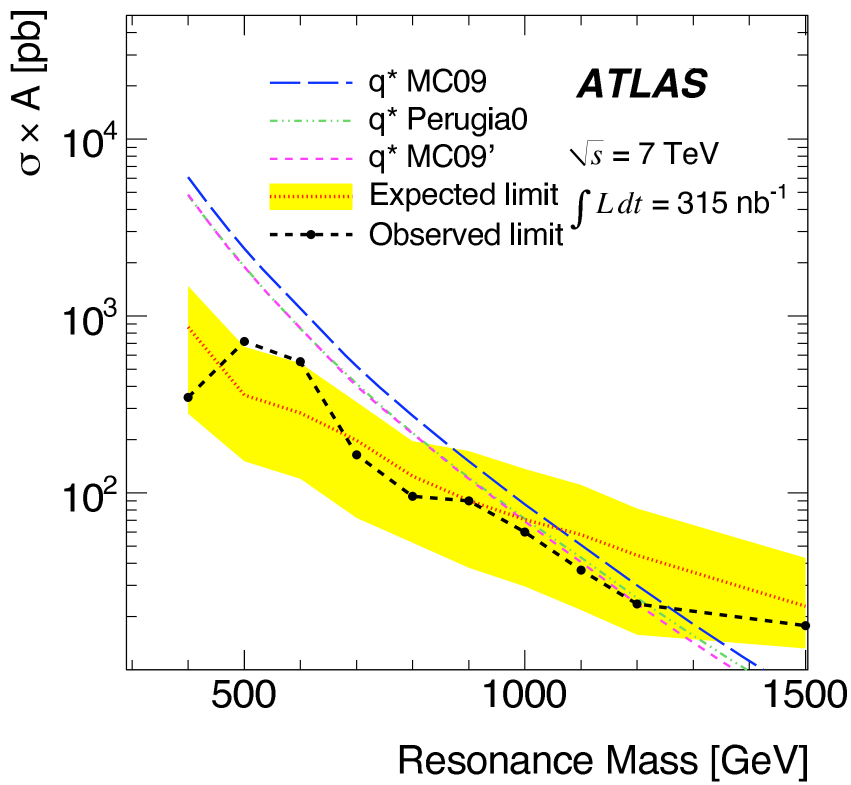


LHC $\not\!\!E_T$ results rapidly improving !

LHC achieved the first crucial step:
The Standard Model rediscovered !

... And have gone on to the physics BSM :

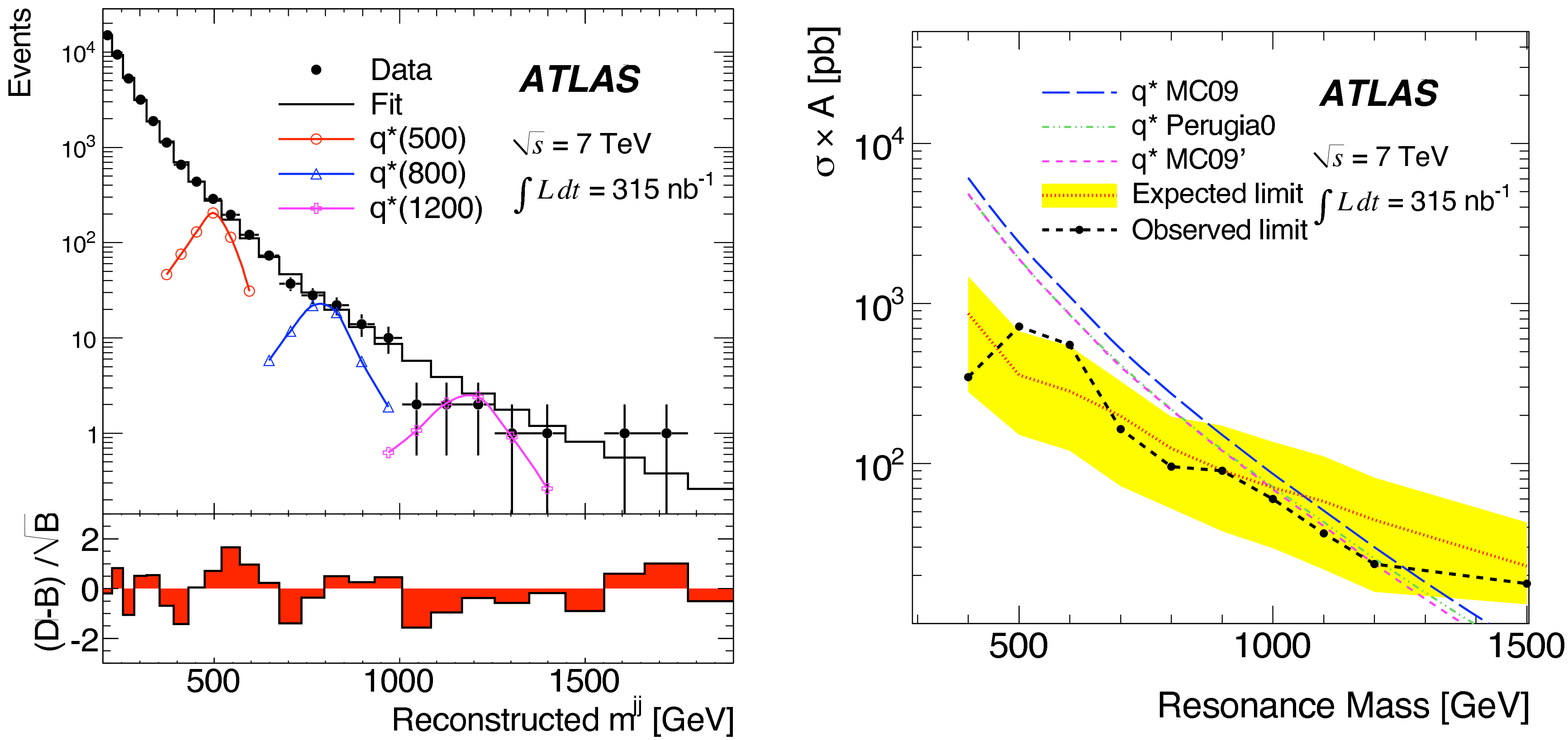




400 GeV $< M_{q*}(jj) <$ 1.26 TeV excluded.

First BSM physics search, beyond the Tevatron reach !

… And have gone on to the physics BSM :



400 GeV $< M_{q*}(jj) <$ 1.26 TeV excluded.

First BSM physics search, beyond the Tevatron reach !

Anxiously waiting for the new excitement …

IV. From Kinematics to Dynamics

(A). Characteristic observables:

Crucial for uncovering new dynamics.

IV. From Kinematics to Dynamics

(A). Characteristic observables: Crucial for uncovering new dynamics.

Selective experimental events

$\Longrightarrow$ Characteristic kinematical observables

(spatial, time, momentaum phase space)

$\Longrightarrow$ Dynamical parameters

(masses, couplings)

IV. From Kinematics to Dynamics

(A). Characteristic observables: Crucial for uncovering new dynamics.

Selective experimental events

$\Longrightarrow$ Characteristic kinematical observables

(spatial, time, momentaum phase space)

$\Longrightarrow$ Dynamical parameters

(masses, couplings)

Energy momentum observables $\Longrightarrow$ mass parameters

Angular observables $\Longrightarrow$ nature of couplings;

Production rates, decay branchings/lifetimes $\Longrightarrow$ interaction strengths.

(B). Kinematical features:

(a). s-channel singularity: bump search we do best.

• invariant mass of two-body $R \to ab: \ m_{ab}^2 = (p_a + p_b)^2 = M_R^2$.

combined with the two-body Jacobian peak in transverse momentum:

$$\frac{d\hat{\sigma}}{dm_{ee}^2 \ dp_{eT}^2} \propto \frac{\Gamma_Z M_Z}{(m_{ee}^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \frac{1}{m_{ee}^2 \sqrt{1 - 4p_{eT}^2/m_{ee}^2}}$$

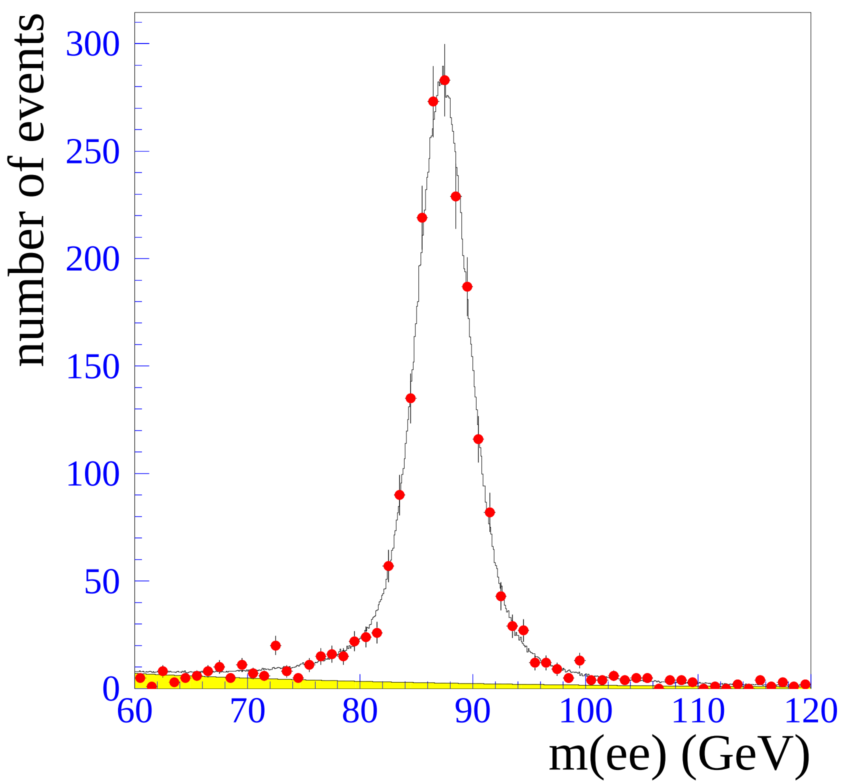
(B). Kinematical features:

(a). s-channel singularity: bump search we do best.

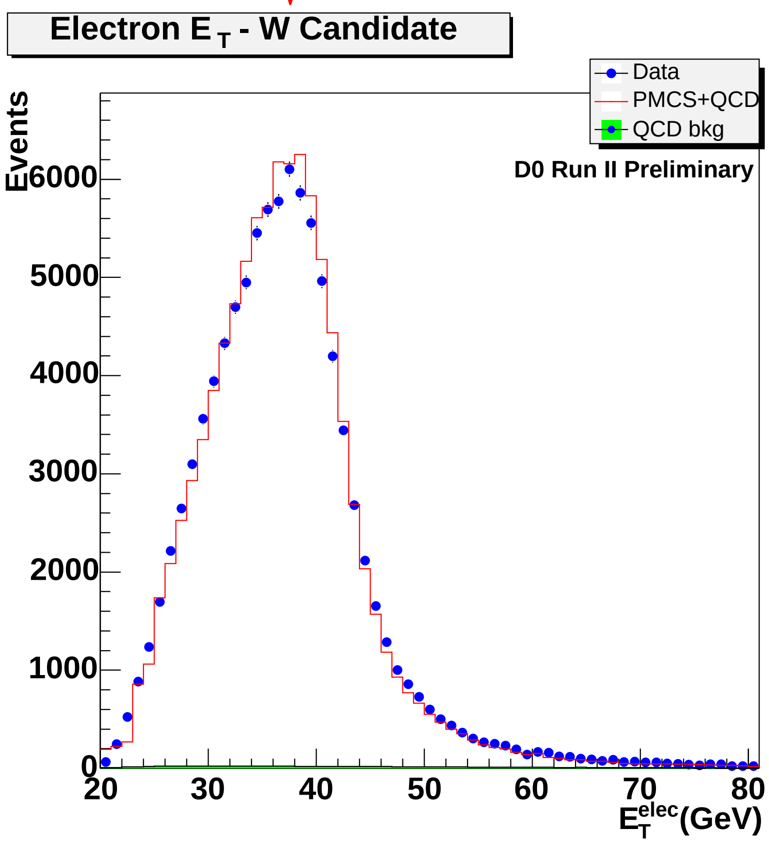
- invariant mass of two-body $R \to ab:\ m_{ab}^2 = (p_a + p_b)^2 = M_R^2$.

combined with the two-body Jacobian peak in transverse momentum:

$$\frac{d\hat{\sigma}}{dm_{ee}^2\ dp_{eT}^2} \propto \frac{\Gamma_Z M_Z}{(m_{ee}^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}\ \frac{1}{m_{ee}^2 \sqrt{1 - 4p_{eT}^2/m_{ee}^2}}$$



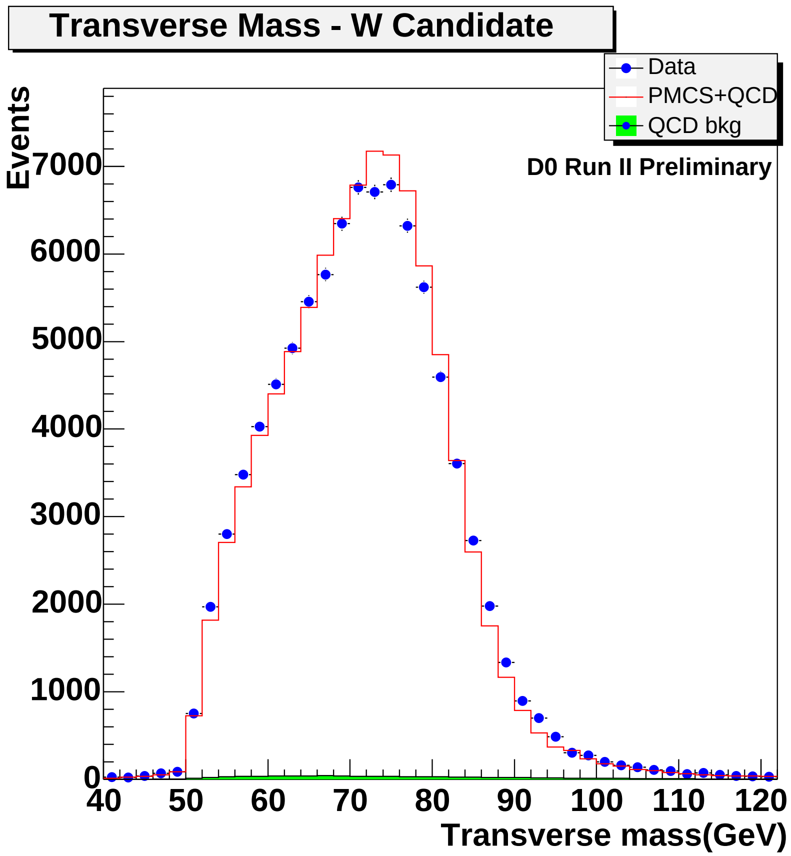
$Z \to e^+e^-$

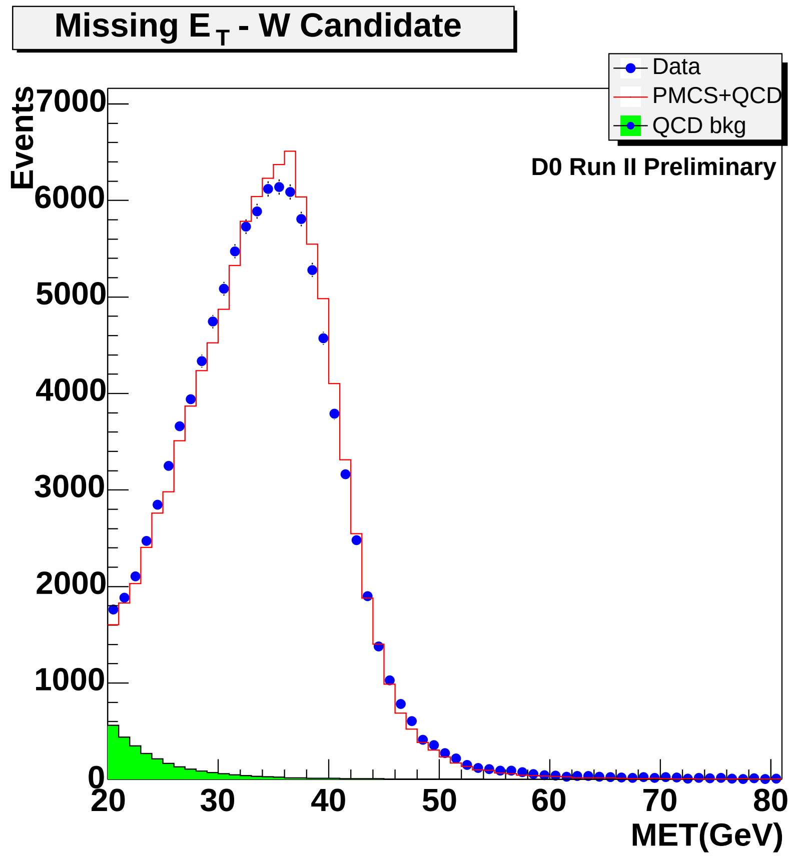


$W \to e\nu$

- "transverse" mass of two-body $W^- \to e^- \bar{\nu}_e$:

$$m_{e\nu\ T}^2 = (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2$$
$$= 2E_{eT}E_T^{miss}(1 - \cos\phi) \le m_{e\nu}^2.$$





If $p_T(W) = 0$, then $m_{e\nu\ T} = 2E_{eT} = 2E_T^{miss}$.

Exercise 5.1: For a two-body final state kinematics, show that

$$\frac{d\hat{\sigma}}{dp_{eT}} = \frac{4p_{eT}}{s\sqrt{1-4p_{eT}^2/s}}\ \frac{d\hat{\sigma}}{d\cos\theta^*}.$$

where $p_{eT} = p_e \sin\theta^*$ is the transverse momentum and θ^* is the polar angle in the c.m. frame. Comment on the apparent singularity at $p_{eT}^2 = s/4$.

Exercise 5.2: Show that for an on-shell decay $W^- \to e^-\bar{\nu}_e$:

$$m_{e\nu\ \ T}^2 \equiv (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \le m_{e\nu}^2.$$

Exercise 5.3: Show that if W/Z has some transverse motion, δP_V, then:

$$p'_{eT} \sim p_{eT}\ [1 + \delta P_V/M_V],$$
$$m'^2_{e\nu\ \ T} \sim m^2_{e\nu\ \ T}\ [1 - (\delta P_V/M_V)^2],$$
$$m'^2_{ee} = m^2_{ee}.$$

• $H^0 \to W^+W^- \to j_1 j_2 \; e^- \bar{\nu}_e$:

cluster transverse mass (I):

$$
\begin{aligned}
m^2_{WW\ T} &= (E_{W_1 T} + E_{W_2 T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \\
&= (\sqrt{p^2_{jjT} + M_W^2} + \sqrt{p^2_{e\nu T} + M_W^2})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \leq M_H^2.
\end{aligned}
$$

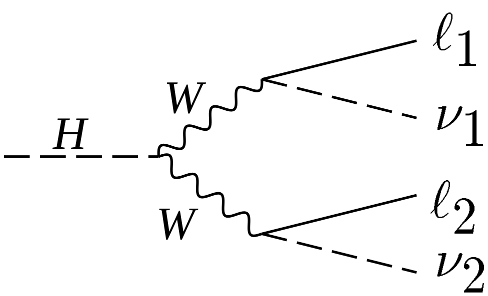
where $\vec{p}_T^{\,miss} \equiv \vec{\not{p}}_T = -\sum_{obs} \; \vec{p}_T^{\,obs}$.

• $H^0 \to W^+W^- \to j_1 j_2\ e^- \bar{\nu}_e$:

cluster transverse mass (I):

$$m^2_{WW\ T} = (E_{W_1 T} + E_{W_2 T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2$$
$$= (\sqrt{p^2_{jjT} + M^2_W} + \sqrt{p^2_{e\nu T} + M^2_W})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \le M^2_H.$$

where $\vec{p}_T^{\,miss} \equiv \vec{\not{p}}_T = -\sum_{obs}\ \vec{p}_T^{\,obs}$.



• $H^0 \to W^+W^- \to e^+\nu_e\ e^-\bar{\nu}_e$:

"effecive" transverse mass:

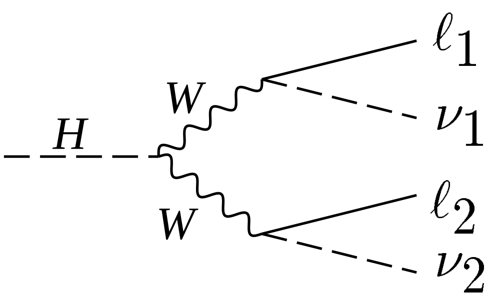
$$m^2_{eff\ T} = (E_{e1T} + E_{e2T} + E_T^{miss})^2 - (\vec{p}_{e1T} + \vec{p}_{e2T} + \vec{p}_T^{\,miss})^2$$
$$m_{eff\ T} \approx E_{e1T} + E_{e2T} + E_T^{miss}$$

- $H^0 \to W^+W^- \to j_1 j_2\ e^-\bar{\nu}_e$:

cluster transverse mass (I):

$$m^2_{WW\ T} = (E_{W_1 T} + E_{W_2 T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2$$
$$= \left(\sqrt{p^2_{jjT} + M_W^2} + \sqrt{p^2_{e\nu T} + M_W^2}\right)^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \leq M_H^2.$$

where $\vec{p}_T^{\,miss} \equiv \vec{\not{p}}_T = -\sum_{obs}\ \vec{p}_T^{\,obs}$.



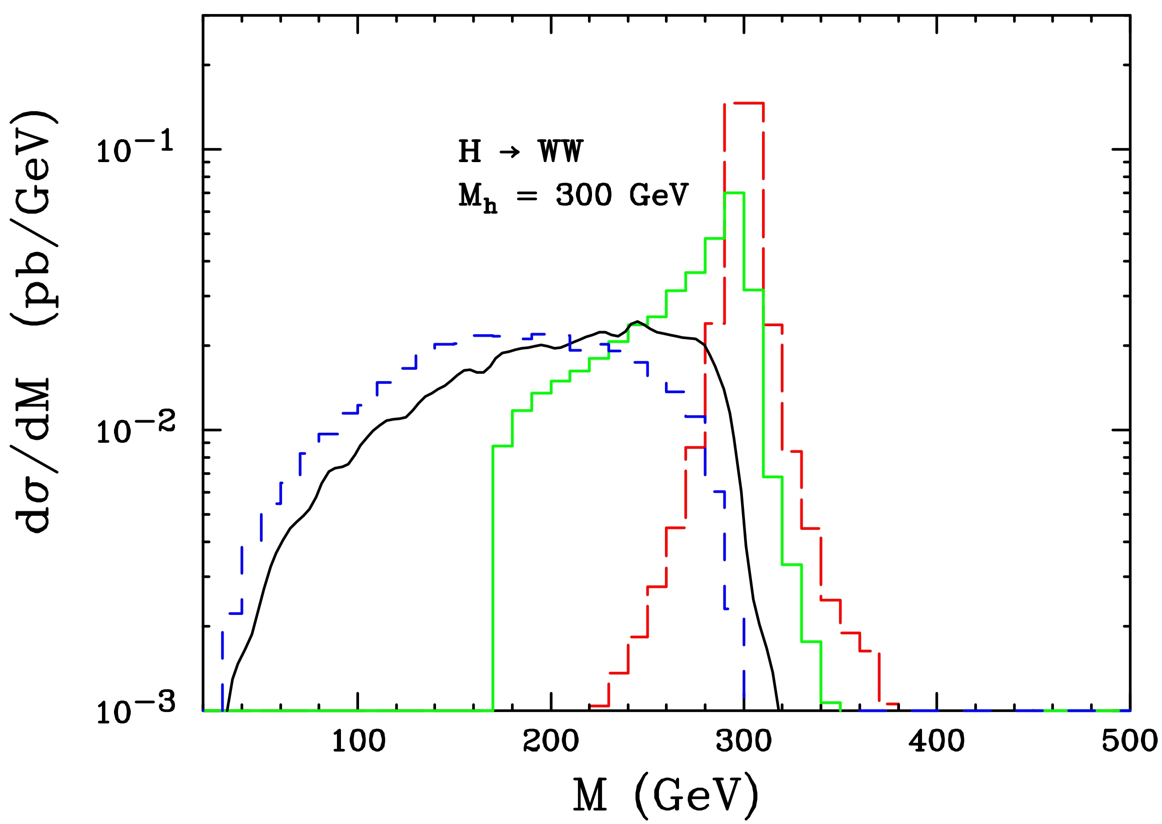
- $H^0 \to W^+W^- \to e^+\nu_e\ e^-\bar{\nu}_e$:

"effecive" transverse mass:

$$m^2_{eff\ T} = (E_{e1T} + E_{e2T} + E_T^{miss})^2 - (\vec{p}_{e1T} + \vec{p}_{e2T} + \vec{p}_T^{\,miss})^2$$
$$m_{eff\ T} \approx E_{e1T} + E_{e2T} + E_T^{miss}$$

cluster transverse mass (II):

$$m^2_{WW\ C} = \left(\sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not{p}_T\right)^2 - (\vec{p}_{T,\ell\ell} + \vec{\not{p}}_T)^2$$
$$m_{WW\ C} \approx \sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not{p}_T$$

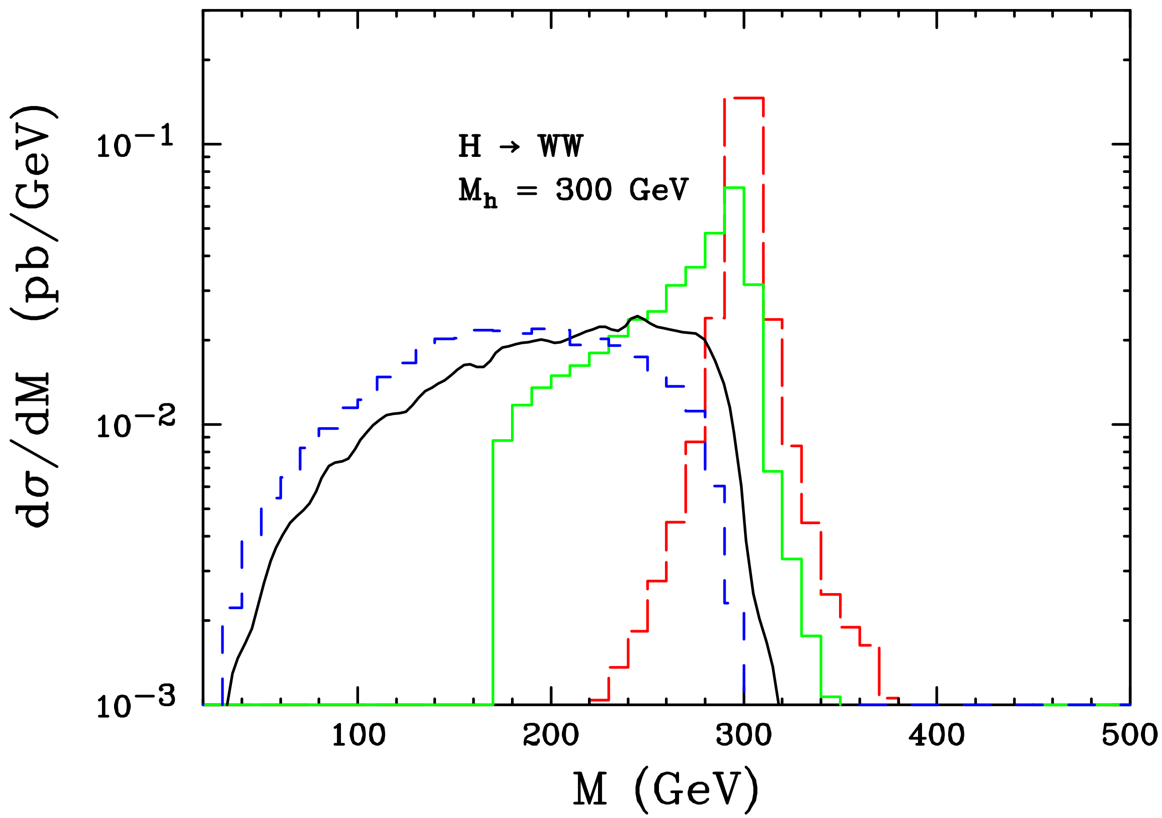


M_{WW} invariant mass (WW fully reconstructable): - - - - - - - - -

$M_{WW,\,T}$ transverse mass (one missing particle ν): ——————

$M_{eff,\,T}$ effetive trans. mass (two missing particles): - - - - - - -

$M_{WW,\,C}$ cluster trans. mass (two missing particles): ——————



M_{WW} invariant mass (WW fully reconstructable): - - - - - - - - -

$M_{WW,\,T}$ transverse mass (one missing particle ν): ————————

$M_{eff,\,T}$ effetive trans. mass (two missing particles): - - - - - - - -

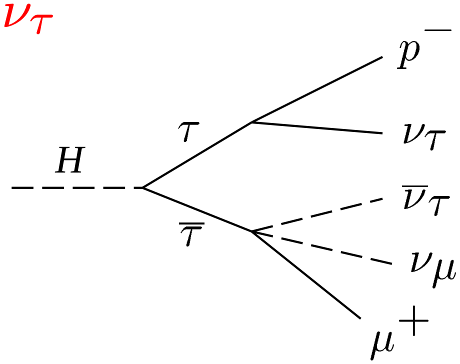
$M_{WW,\,C}$ cluster trans. mass (two missing particles): ————————

YOU design an optimal variable/observable for the search.

- cluster transverse mass (III):

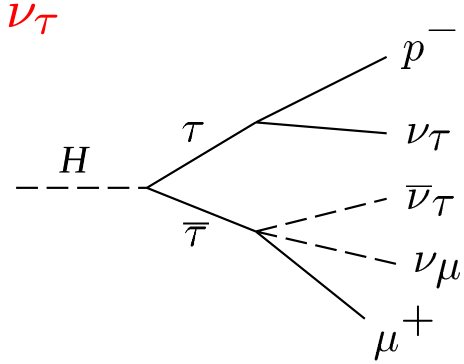
$$H^0 \to \tau^+\tau^- \to \mu^+ \; \bar{\nu}_\tau \; \nu_\mu, \quad \rho^- \; \nu_\tau$$

A lot more complicated with (many) more $\nu's$?



• cluster transverse mass (III):

$$H^0 \to \tau^+\tau^- \to \mu^+ \, \bar{\nu}_\tau \, \nu_\mu, \quad \rho^- \, \nu_\tau$$



A lot more complicated with (many) more $\nu' s$?

Not really!

$\tau^+\tau^-$ ultra-relativistic, the final states from a τ decay highly collimated:

$$\theta \approx \gamma_\tau^{-1} = m_\tau / E_\tau = 2m_\tau / m_H \approx 1.5^\circ \quad (m_H = 120 \text{ GeV}).$$

We can thus take

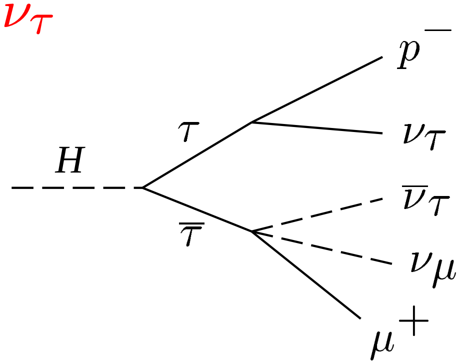
$$\begin{aligned} \vec{p}_{\tau^+} &= \vec{p}_{\mu^+} + \vec{p}_+^{\,\nu' s}, \quad \vec{p}_+^{\,\nu' s} \approx c_+ \vec{p}_{\mu^+}. \\ \vec{p}_{\tau^-} &= \vec{p}_{\rho^-} + \vec{p}_-^{\,\nu' s}, \quad \vec{p}_-^{\,\nu' s} \approx c_- \vec{p}_{\rho^-}. \end{aligned}$$

where $c_\pm$ are proportionality constants, to be determined.

• cluster transverse mass (III):

$$H^0 \to \tau^+\tau^- \to \mu^+ \ \bar{\nu}_\tau \ \nu_\mu, \quad \rho^- \ \nu_\tau$$

A lot more complicated with (many) more $\nu's$?



Not really!

$\tau^+\tau^-$ ultra-relativistic, the final states from a τ decay highly collimated:

$$\theta \approx \gamma_\tau^{-1} = m_\tau/E_\tau = 2m_\tau/m_H \approx 1.5^\circ \quad (m_H = 120 \text{ GeV}).$$

We can thus take

$$\vec{p}_{\tau^+} = \vec{p}_{\mu^+} + \vec{p}_+^{\,\nu's}, \quad \vec{p}_+^{\,\nu's} \approx c_+\vec{p}_{\mu^+}.$$
$$\vec{p}_{\tau^-} = \vec{p}_{\rho^-} + \vec{p}_-^{\,\nu's}, \quad \vec{p}_-^{\,\nu's} \approx c_-\vec{p}_{\rho^-}.$$

where $c_\pm$ are proportionality constants, to be determined.

This is applicable to any decays of fast-moving particles, like

$$T \to Wb \to \ell\nu, \ b.$$

Experimental measurements: p_{ρ^-}, p_{μ^+}, $\not{p}_T$:

$$c_+(p_{\mu^+})_x + c_-(p_{\rho^-})_x = (\not{p}_T)_x,$$
$$c_+(p_{\mu^+})_y + c_-(p_{\rho^-})_y = (\not{p}_T)_y.$$

Unique solutions for $c_\pm$ exist if

$$(p_{\mu^+})_x/(p_{\mu^+})_y \neq (p_{\rho^-})_x/(p_{\rho^-})_y.$$

Physically, the τ^+ and τ^- should form a finite angle, or the Higgs should have a non-zero transverse momentum.

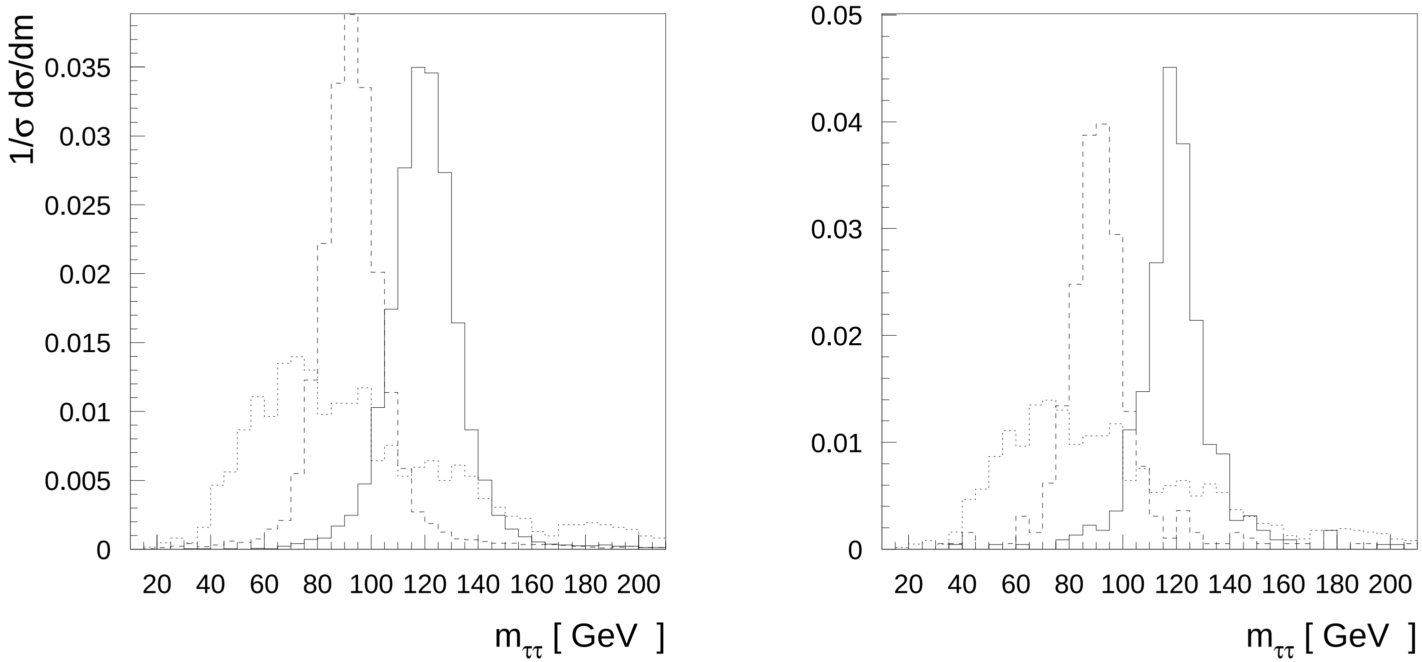
Experimental measurements: p_{ρ^-}, p_{μ^+}, $\not{p}_T$:

$$c_+(p_{\mu^+})_x + c_-(p_{\rho^-})_x = (\not{p}_T)_x,$$
$$c_+(p_{\mu^+})_y + c_-(p_{\rho^-})_y = (\not{p}_T)_y.$$

Unique solutions for $c_\pm$ exist if

$$(p_{\mu^+})_x/(p_{\mu^+})_y \neq (p_{\rho^-})_x/(p_{\rho^-})_y.$$

Physically, the τ^+ and τ^- should form a finite angle, or the Higgs should have a non-zero transverse momentum.



(b). Two-body versus three-body kinematics

- Energy end-point and mass edges:

utilizing the "two-body kinematics"

Consider a simple case:

$$e^+e^- \to \tilde{\mu}_R^+ \ \tilde{\mu}_R^-$$
$$\text{with two} - \text{body decays}: \ \ \tilde{\mu}_R^+ \to \mu^+\tilde{\chi}_0, \quad \tilde{\mu}_R^- \to \mu^-\tilde{\chi}_0.$$

In the $\tilde{\mu}_R^+$-rest frame: $E_\mu^0 = \frac{M_{\tilde{\mu}_R}^2 - m_\chi^2}{2M_{\tilde{\mu}_R}}$.

In the Lab-frame:

$$(1-\beta)\gamma E_\mu^0 \le E_\mu^{lab} \le (1+\beta)\gamma E_\mu^0$$
$$\text{with} \ \ \beta = \left(1 - 4M_{\tilde{\mu}_R}^2/s\right)^{1/2}, \quad \gamma = (1-\beta)^{-1/2}.$$

Energy end-point: $E_\mu^{lab} \Rightarrow M_{\tilde{\mu}_R}^2 - m_\chi^2$.

Mass edge: $m_{\mu^+\mu^-}^{max} = \sqrt{s} - 2m_\chi$.

(b). Two-body versus three-body kinematics

- Energy end-point and mass edges:

utilizing the "two-body kinematics"

Consider a simple case:

$$e^+e^- \to \tilde{\mu}_R^+ \; \tilde{\mu}_R^-$$

$$\text{with two} - \text{body decays}: \quad \tilde{\mu}_R^+ \to \mu^+ \tilde{\chi}_0, \quad \tilde{\mu}_R^- \to \mu^- \tilde{\chi}_0.$$

In the $\tilde{\mu}_R^+$-rest frame: $E_\mu^0 = \frac{M_{\tilde{\mu}_R}^2 - m_\chi^2}{2M_{\tilde{\mu}_R}}$.

In the Lab-frame:

$$(1-\beta)\gamma E_\mu^0 \le E_\mu^{lab} \le (1+\beta)\gamma E_\mu^0$$

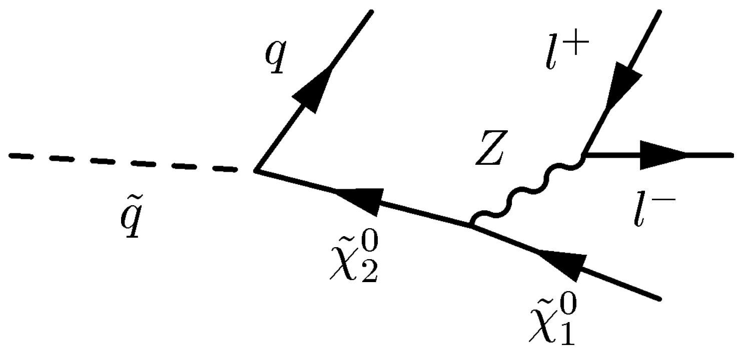
$$\text{with} \quad \beta = \left(1 - 4M_{\tilde{\mu}_R}^2/s\right)^{1/2}, \quad \gamma = (1-\beta)^{-1/2}.$$

Energy end-point: $E_\mu^{lab} \Rightarrow M_{\tilde{\mu}_R}^2 - m_\chi^2$.

Mass edge: $m_{\mu^+\mu^-}^{max} = \sqrt{s} - 2m_\chi$.

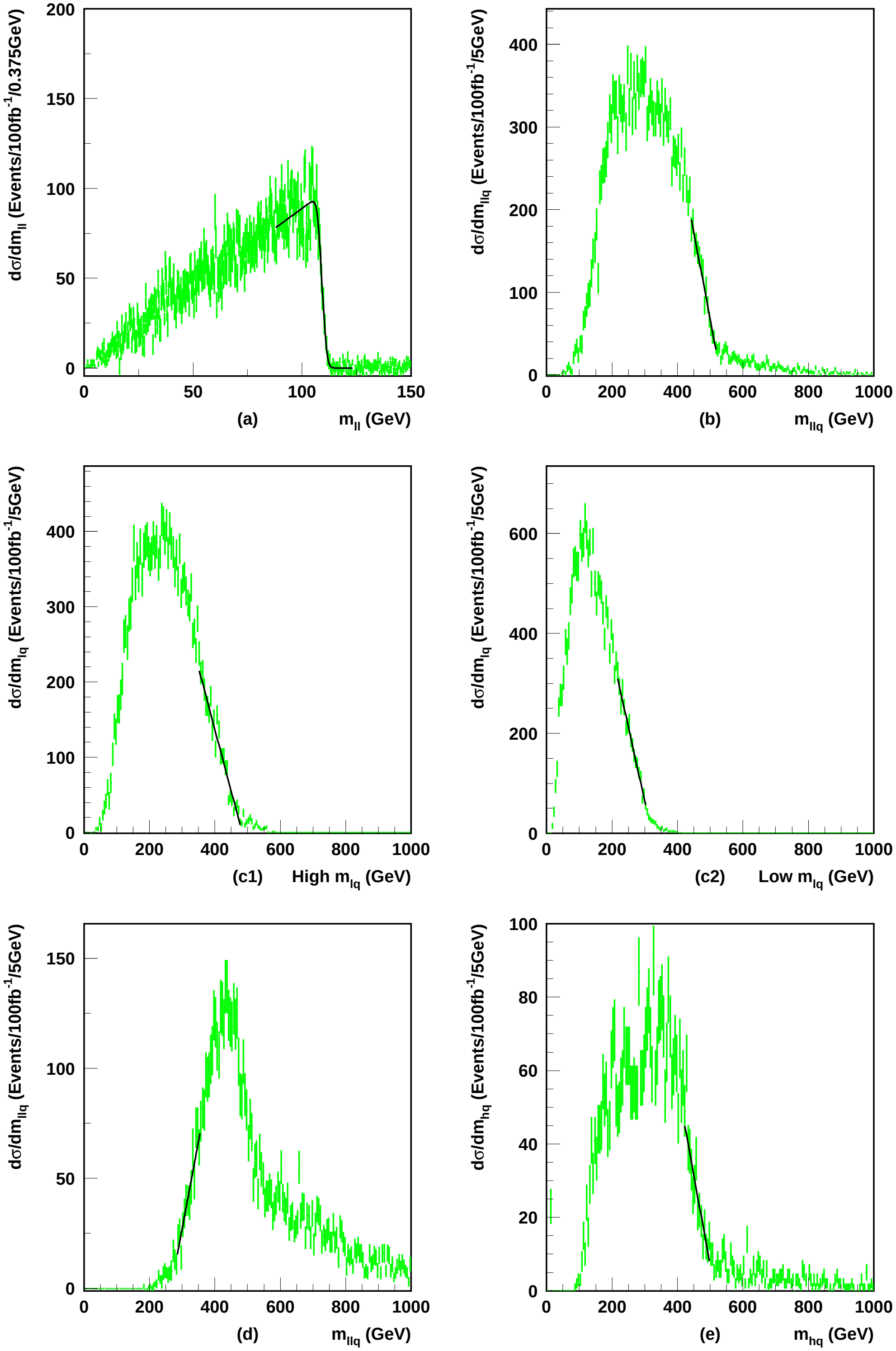
Same idea can be applied to hadron colliders ...

Consider a squark cascade decay:



1[st] edge : $M^{max}(\ell\ell) \approx M_{\chi_2^0} - M_{\chi_1^0}$;

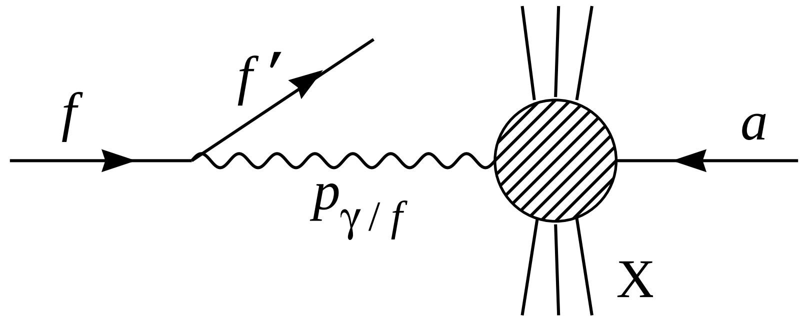
2[nd] edge : $M^{max}(\ell\ell j) \approx M_{\tilde{q}} - M_{\chi_1^0}$.


dσ/dm_ll (Events/100fb^-1/0.375GeV)
(a)
m_ll (GeV)
dσ/dm_llq (Events/100fb^-1/5GeV)
(b)
m_llq (GeV)
dσ/dm_lq (Events/100fb^-1/5GeV)
(c1)
High m_lq (GeV)
dσ/dm_lq (Events/100fb^-1/5GeV)
(c2)
Low m_lq (GeV)
dσ/dm_llq (Events/100fb^-1/5GeV)
(d)
m_llq (GeV)
dσ/dm_hq (Events/100fb^-1/5GeV)
(e)
m_hq (GeV)

(c). t-channel singularity: splitting.

- Gauge boson radiation off a fermion:

The familiar Weizsäcker-Williams approximation



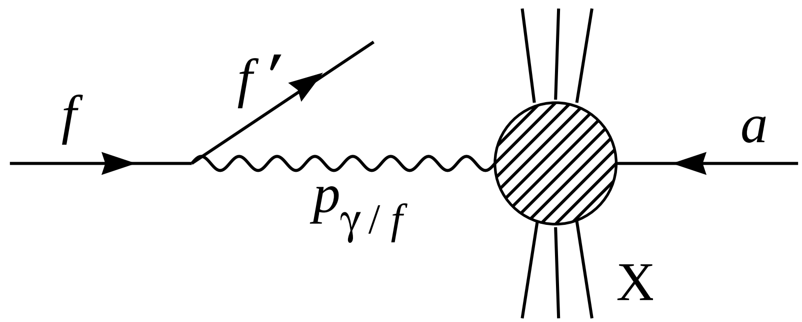
$$\sigma(fa \to f'X) \approx \int dx\ dp_T^2\ P_{\gamma/f}(x, p_T^2)\ \sigma(\gamma a \to X),$$

$$P_{\gamma/e}(x, p_T^2) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}(\frac{1}{p_T^2})|_{m_e}^{E}.$$

(c). t-channel singularity: splitting.

- Gauge boson radiation off a fermion:

The familiar Weizsäcker-Williams approximation



$$\sigma(fa \to f'X) \approx \int dx\, dp_T^2\, P_{\gamma/f}(x, p_T^2)\, \sigma(\gamma a \to X),$$

$$P_{\gamma/e}(x, p_T^2) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}(\frac{1}{p_T^2})|_{m_e}^{E}.$$

† The kernel is the same as $q \to qg^*$ $\Rightarrow$ generic for parton splitting;

† The high energy enhancement $dp_T^2/p_T^2 \to \ln(E/m_e)$ reflects the collinear behavior.

- Generalize to massive gauge bosons:

$$P^T_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2},$$

$$P^L_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1-x}{x} \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.$$

• Generalize to massive gauge bosons:

$$P^T_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2},$$

$$P^L_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1-x}{x} \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.$$

Special kinematics for massive gauge boson fusion processes:

For the accompanying jets,

At low-p_{jT},

$$\left.\begin{array}{l} p_{jT}^2 \approx (1-x)M_V^2 \\ E_j \sim (1-x)E_q \end{array}\right\} \textit{forward jet tagging}$$

At high-p_{jT},

$$\left.\begin{array}{l} \frac{d\sigma(V_T)}{dp_{jT}^2} \propto 1/p_{jT}^2 \\ \frac{d\sigma(V_L)}{dp_{jT}^2} \propto 1/p_{jT}^4 \end{array}\right\} \textit{central jet vetoing}$$

has become important tools for Higgs searches, single-top signal etc.

(C). Charge forward-backward asymmetry A_{FB}:

The coupling vertex of a vector boson V_μ to an arbitrary fermion pair f

$$i\sum_{\tau}^{L,R} g_\tau^f \, \gamma^\mu \, P_\tau \qquad \rightarrow \qquad \text{crucial to probe chiral structures.}$$

The parton-level forward-backward asymmetry is defined as

$$A_{FB}^{i,f} \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4}\mathcal{A}_i\mathcal{A}_f,$$

$$\mathcal{A}_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}.$$

where N_F (N_B) is the number of events in the forward (backward) direction defined in the parton c.m. frame relative to the initial-state fermion $\vec{p}_i$.

At hadronic level:

$$A_{FB}^{\mathsf{LHC}} = \frac{\int dx_1 \sum_q A_{FB}^{q,f} \Big(P_q(x_1)P_{\bar{q}}(x_2) - P_{\bar{q}}(x_1)P_q(x_2)\Big)\,\mathsf{sign}(x_1 - x_2)}{\int dx_1 \sum_q \Big(P_q(x_1)P_{\bar{q}}(x_2) + P_{\bar{q}}(x_1)P_q(x_2)\Big)}.$$

At hadronic level:

$$A_{FB}^{\mathsf{LHC}} = \frac{\int dx_1 \sum_q A_{FB}^{q,f} \Big(P_q(x_1)P_{\bar{q}}(x_2) - P_{\bar{q}}(x_1)P_q(x_2)\Big)\,\mathsf{sign}(x_1 - x_2)}{\int dx_1 \sum_q \Big(P_q(x_1)P_{\bar{q}}(x_2) + P_{\bar{q}}(x_1)P_q(x_2)\Big)}.$$

Perfectly fine for Z/Z'-type:

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$.

In pp collisions, however, what is the direction of $\vec{p}_{quark}$?

At hadronic level:

$$A_{FB}^{\mathsf{LHC}} = \frac{\int dx_1 \sum_q A_{FB}^{q,f} \Big(P_q(x_1)P_{\bar{q}}(x_2) - P_{\bar{q}}(x_1)P_q(x_2)\Big)\, \mathsf{sign}(x_1 - x_2)}{\int dx_1 \sum_q \Big(P_q(x_1)P_{\bar{q}}(x_2) + P_{\bar{q}}(x_1)P_q(x_2)\Big)}.$$

Perfectly fine for Z/Z'-type:

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$.

In pp collisions, however, what is the direction of $\vec{p}_{quark}$?

It is the boost-direction of $\ell^+\ell^-$.

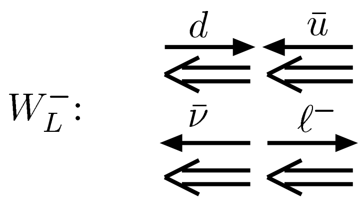
How about $W^{\pm}/W'^{\pm}(\ell^{\pm}\nu)$-type?

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$,
AND ℓ^{+} (ℓ^{-}) along the direction with $\bar{q}$ (q) $\Rightarrow$ OK at the Tevatron,

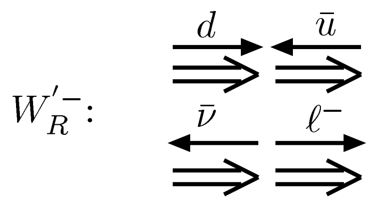
How about $W^{\pm}/W'^{\pm}(\ell^{\pm}\nu)$-type?

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$,
AND ℓ^{+} (ℓ^{-}) along the direction with $\bar{q}$ (q) $\Rightarrow$ OK at the Tevatron,

But: (1). cann't get the boost-direction of $\ell^{\pm}\nu$ system;
(2). Looking at $\ell^{\pm}$ alone, no insight for W_L or W_R!



(a)

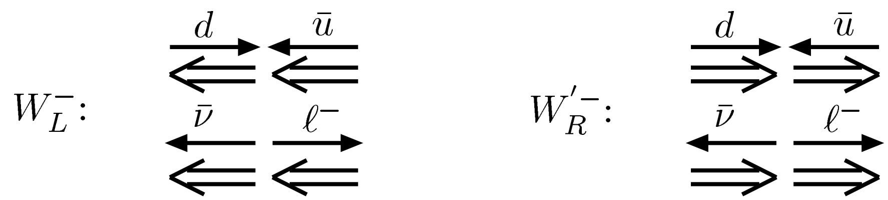


(b)

How about $W^{\pm}/W'^{\pm}(\ell^{\pm}\nu)$-type?

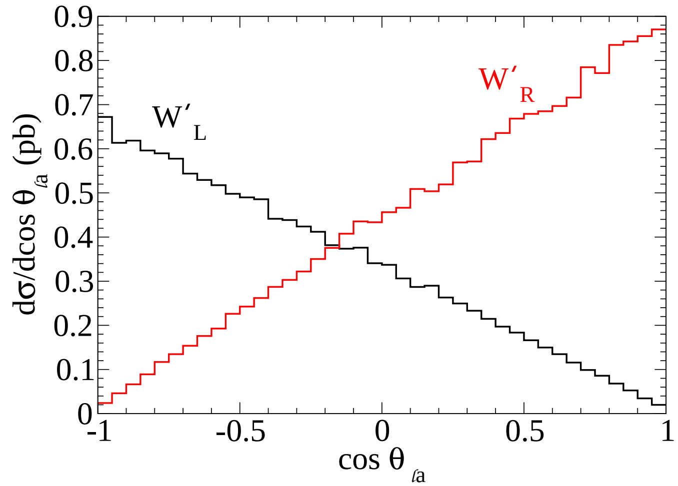
In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$,
AND ℓ^{+} (ℓ^{-}) along the direction with $\bar{q}$ (q) $\Rightarrow$ OK at the Tevatron,

But: (1). cann't get the boost-direction of $\ell^{\pm}\nu$ system;
(2). Looking at $\ell^{\pm}$ alone, no insight for W_L or W_R!



(a) (b)

In $p\bar{p}$ collisions: (1). a reconstructable system; (2). with spin correlation:
Only tops: $W' \to t\bar{b} \to \ell^{\pm}\nu\ \bar{b}$:



(D). CP asymmetries A_{CP}:

To non-ambiguously identify CP-violation effects, one must rely on CP-odd variables.

(D). CP asymmetries A_{CP}:

To non-ambiguously identify CP-violation effects,
one must rely on CP-odd variables.

Definition: A_{CP} vanishes if CP-violation interactions do not exist
(for the relevant particles involved).

This is meant to be in contrast to an observable:
that'd be *modified* by the presence of CP-violation,
but is *not zero* when CP-violation is absent.

e.g. $M_{(\chi^{\pm}\ \chi^{0})}, \ \ \sigma(H^{0},\ A^{0}), ...$

(D). CP asymmetries A_{CP}:

To non-ambiguously identify CP-violation effects,
one must rely on CP-odd variables.

Definition: A_{CP} vanishes if CP-violation interactions do not exist (for the relevant particles involved).

This is meant to be in contrast to an observable:
that'd be *modified* by the presence of CP-violation,
but is *not zero* when CP-violation is absent.

$$\text{e.g. } M_{(\chi^{\pm}\ \chi^{0})}, \quad \sigma(H^0,\ A^0), \ldots$$

Two ways:

a). Compare the rates between a process and its CP-conjugate process:

$$\frac{R(i \to f) - R(\bar{i} \to \bar{f})}{R(i \to f) + R(\bar{i} \to \bar{f})}, \quad \text{e.g.} \quad \frac{\Gamma(t \to W^+ q) - \Gamma(\bar{t} \to W^- \bar{q})}{\Gamma(t \to W^+ q) + \Gamma(\bar{t} \to W^- \bar{q})}.$$

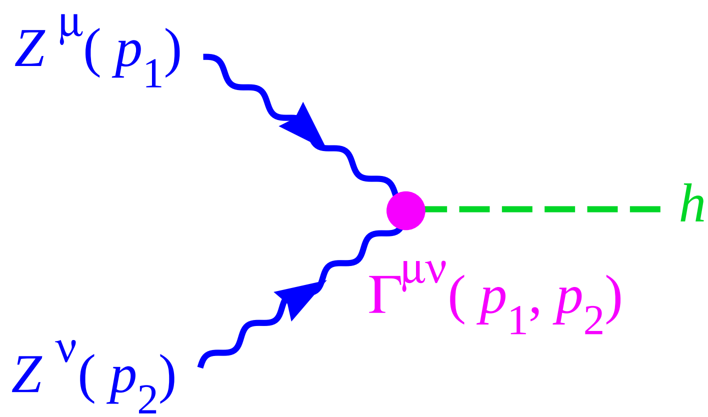
b). Construct a CP-odd kinematical variable for an initially CP-eigenstate:

$$\mathcal{M} \sim M_1 + M_2 \sin\theta,$$
$$A_{CP} = \sigma^F - \sigma^B = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

b). Construct a CP-odd kinematical variable for an initially CP-eigenstate:

$$\mathcal{M} \sim M_1 + M_2 \sin\theta,$$
$$A_{CP} = \sigma^F - \sigma^B = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

E.g. 1: $H \to Z(p_1)Z^*(p_2) \to e^+(q_1)e^-(q_2),\ \mu^+\mu^-$



$$\Gamma^{\mu\nu}(p_1, p_2) = i\frac{2}{v}\ h[a\ M_Z^2 g^{\mu\nu} + b\ (p_1^\mu p_2^\nu - p_1 \cdot p_2 g^{\mu\nu}) + \tilde{b}\ \epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}]$$

$a = 1,\ b = \tilde{b} = 0$ for SM.

In general, $a,\ b,\ \tilde{b}$ complex form factors,

describing new physics at a higher scale.

For $H \to Z(p_1)Z^*(p_2) \to e^+(q_1)e^-(q_2),\ \mu^+\mu^-$, define:

$$O_{CP} \sim (\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2),$$

$$\text{or}\ \ \cos\theta = \frac{(\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2)}{|\vec{p}_1 - \vec{p}_2||\vec{q}_1 \times \vec{q}_2)|}.$$

For $H \to Z(p_1)Z^*(p_2) \to e^+(q_1)e^-(q_2),\ \mu^+\mu^-$, define:

$$O_{CP} \sim (\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2),$$
$$\text{or}\ \ \cos\theta = \frac{(\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2)}{|\vec{p}_1 - \vec{p}_2||\vec{q}_1 \times \vec{q}_2)|}.$$

E.g. 2: $H \to t(p_t)\bar{t}(p_{\bar{t}}) \to e^+(q_1)\nu_1 b_1,\ e^-(q_2)\nu_2 b_2.$

$$-\frac{m_t}{v}\bar{t}(a + b\gamma^5)t\ H$$
$$O_{CP} \sim (\vec{p}_t - \vec{p}_{\bar{t}}) \cdot (\vec{p}_{e^+} \times \vec{p}_{e^-}).$$

thus define an asymmetry angle.