ASPECTS OF ADS/CFT DUALITY

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- string theory – gauge theory duality : large N
- $\mathcal{N}=4$ SYM $=$ $AdS_5 \times S^5$ string theory

Precise map of states/operators?

How string action "emerges" from gauge theory?

Lessons for less susy / non-conform. cases?

Frolov, A.T. (2003)

Kruczenski, Ryzhov, A.T. hep-th/0403120

$\mathcal{N}=4$ Super Yang Mills

Gliozzi, Scherk, Olive (1977)
Brink, Schwarz, Scherk (1977)

very special $d=4$ QFT:
(super) conformal theory

Sohnius, West (1981)
Brink, Lindgren, Nilsson (1983)

$d=2$ CFT's: related to integrable lattice models or quantum spin chains
(+ integrable massive deformations)

Integrability in $d=4$? "Solve" SYM?

Hidden 2-d structure in large N ($\lambda = g^2_{YM} N$) $SU(N)$ SYM?

Dilatation operator

(previous relations: one-loop anomalous dim's $N \to \infty$ QCD: Lipatov et al)

Indication: AdS/CFT

$\rightarrow$ relation to integrable 2-d σ-model

Results on gauge theory side:

anomalous dim's for composite operators

$\rightarrow$ relation to spin chains in 1+1

High energy scattering in planar QCD $\leftrightarrow$

XXX_0 $sl(2)$ spin chain

Lipatov 93

Faddeev

Korchemski 94

$Tr(\bar{\Psi} \underbrace{D \ldots D}_{S} \Psi)$, etc.

One-loop anom. dim. of quasi-partonic operators in QCD

(Belitsky, Braun, Derkachov, Korchemski, Manashov 98-99)

$\mathcal{N}=4$ SYM: $Tr(\Phi^* D^S \Phi)$ + similar

$\Delta = S + 2 + f(\lambda) \ln S$. $S \to \infty$

$f = \underline{a_1 \lambda} + \underline{a_2 \lambda^2} + \ldots$

Kotikov

Lipatov 2000-2003

Dolan, Osborn 2001

Arbitrary high twist:

$Tr(D^n \Phi \ldots D^k \Phi \ldots D^m \Phi)$ $S = n+k+m$

Belitsky

Gorsky, Korchemski

$XXX_{-1/2}$ apply integrability

Other sectors of $\mathcal{N}=4$ SYM?

<u>Pure scalar sector</u>:

$$\mathrm{tr}\,(\Phi_{I_1} \dots \Phi_{I_n}) \qquad I = 1, \dots, 6$$

One loop anom. dim. operator $(N \to \infty)$
($\equiv$ dilatation operator) $\equiv$ H of "Heisenberg"
SO(6) integrable spin chain

Minahan
Zarembo 2002

integrability $\to$ ∞ set of charges
$[H, Q_i] = 0$ $\to$ Bethe ansatz

Subsector: $\mathrm{tr}\,(\Phi_1^{J_1} \Phi_2^{J_2})$: $SU(2) \subset SO(6)$

$XXX_{1/2}$ SU(2) Heisenberg spin chain

<u>General operators</u> : $(N \to \infty$, one-loop$)$

symmetry of SYM : psu (2,2|4)
superconf. algebra

SO(6) and sl (2) sectors are parts of it

$\to$ psu (2,2|4) superspin chain

Beisert 2003
Staudacher
....

<u>All loops:</u> long-range interactions

2d structure ?

Relation to $AdS_5 \times S^5$ σ-model?

$\mathcal{N}=4$ SYM : $SU(N)$

$N \to \infty$, $\lambda = g_{YM}^2 N$ = fixed

$$\mathcal{L} = \mathrm{Tr}\left[F_{\mu\nu}^2 + (\partial\phi_I)^2 + [\phi,\phi]^2 + \text{ferm.}\right]$$

$\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6$: $\Phi_1 = \phi_1 + i\phi_2$

adjoint rep.

$\Phi_2 = \phi_3 + i\phi_4$

$\Phi_3 = \phi_5 + i\phi_6$

$SO(2,4) \times SO(6)$

conformal × R-symmetry

Local ops : $\mathcal{O} = \mathrm{Tr}\left(F_{\mu\nu}^2 \ldots DF \ldots \Phi \ldots \psi \ldots\right)$

Dimensions : $\Delta = n + \gamma(\lambda) = ?$

"Protected" sector : $\gamma(\lambda) = 0$

BPS or CPO $\mathrm{Tr}\left(\phi_{\{I_1} \ldots \phi_{I_n\}}\right)$

e.g. $\mathrm{Tr}\, \Phi_1^J$: $\Delta = J$

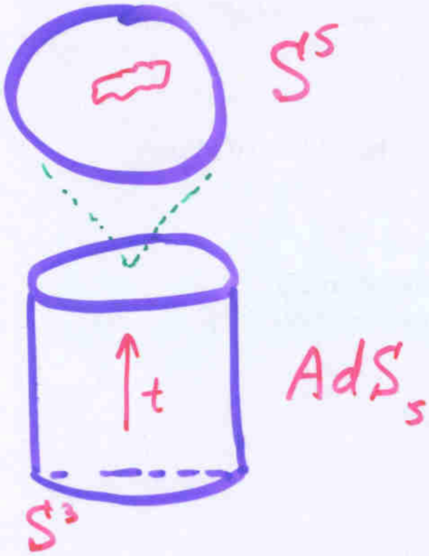
"Near-BPS" : $\mathrm{Tr}\left(\Phi_1^J \Phi_2^k\right) + \ldots$

$J \gg k$

Non-BPS : $\mathrm{Tr}\left(\Phi_1^{J_1} \Phi_2^{J_2}\right) + \ldots$

$J_1 \sim J_2$

$AdS_5 \times S^5$ string theory:



$$X_I X_I = 1$$
$$I = 1, \dots, 6$$

$$\eta^{MN} Y_M Y_N = -1$$
$$\eta^{MN} = (-++++-)$$

$$SO(6) \times SO(2,4) \quad + \text{ SUSY}$$

$$I = T \int_0^{2\pi} d\sigma \int d\tau \left(\partial Y^M \partial Y^N \eta_{MN} + \partial X^I \partial X^I + \text{fermions} \right)$$

$$T = \frac{R^2}{2\pi\alpha'} \equiv \frac{\sqrt{\lambda}}{2\pi}$$

2d CFT
integrable

Spectrum of string states ?

AdS_5 energy $\quad E = E(Q_i, n; T)$

Q_i : $\quad 2 + 3$ Cartan charges of $SO(2,4) \times SO(6)$

$$(\underbrace{S_1, S_2}_{AdS_5 \text{ spins}} ; \underbrace{J_1, J_2, J_3}_{S^5 \text{ spins}}) + \text{higher charges}$$

Duality :

string states (on $R \times S^1$) $\simeq$ SYM states (in $R \times S^3$)

or string vertex ops $\simeq$ or SYM operators

$\simeq \mathrm{Tr}(\Phi \dots \Phi \dots)$

$$E_{AdS} = \Delta(\lambda)$$

energy dimension

How to check ?

string side : $T \sim \sqrt{\lambda}$: $\frac{1}{\sqrt{\lambda}}$ expansion

gauge side : $\lambda = g^2_{YM} N$: λ - expansion

- BPS sector : protected 1998
  sugra states = CPO's
  symmetries $\rightarrow$ matching
- near - BPS non-trivial 2001
- non - BPS very non-trivial 2003

$$E = \sum_n \frac{c_n}{(\sqrt{\lambda})^n}$$ " α' - " expansion

$$\Delta = \sum_n a_n \lambda^n$$ perturb. gauge theory

Recent progress :

Look at sector of states with **large** quantum numbers J (BMN, GKP)

New limits like

$$J \to \infty\ , \quad \boxed{\tilde{\lambda} \equiv \frac{\lambda}{J^2}} = \text{fixed} < 1$$

Generic string states: $E \sim \frac{1}{\sqrt{\alpha'}} \sim \sqrt[4]{\lambda}$

Semiclassical states : + excitations

$E \sim T = \sqrt{\lambda}\ , \quad J \sim \sqrt{\lambda}$

$E \sim J + \ldots$

"Long" SYM operators $Tr(\underbrace{\Phi \ldots \Phi}_{J})$

$$\Delta = E = J + f(\lambda, J)$$

duality map becomes more explicit

- **Near BPS** states : BMN

S^5

ground state $E = J$ BPS

excitations :

$$E_n = J + N_n \sqrt{1 + \tilde{\lambda} n^2} + O\left(\frac{1}{J}\right)$$

$$\mathcal{O} \sim Tr\left(\Phi_1^J\, \Phi_2\, \Phi_2\right) + \ldots$$

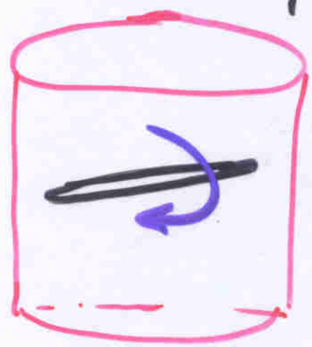
- Non-BPS single-spin states: GKP

interpolating functions of λ

(quantum string corrections not suppressed at $J \to \infty$)

Qualitative agreement: J-dependence matches!

Example: Folded string in AdS_5



$$E = J + f(\lambda) \ln J + \ldots$$

$$J \to \infty$$

$$\mathcal{O} = \mathrm{Tr}\left(\Phi^* D^J \Phi\right)$$

string side:

$$f(\lambda) = a_0 \sqrt{\lambda} + a_1 + \frac{a_2}{\sqrt{\lambda}} + \ldots$$

classical energy non-analytic in λ

quantum corrections not suppressed

FT

SYM:

$$\Delta = J + f(\lambda) \ln J + \ldots$$

$$f(\lambda) = b_1 \lambda + b_2 \lambda + \ldots$$

Same J-dependence (!)

but $f(\lambda)$ hard to match

FT 2003

New development:

sector of non-BPS states with several large spins

$$J_1 \sim J_2 \sim \sqrt{\lambda} \,, \quad \boxed{J = J_1 + J_2} \to \infty$$

$$\boxed{\tilde{\lambda} = \frac{\lambda}{J^2}} \lesssim 1$$

- Classical string energy has analytic expansion in $\tilde{\lambda}$

$$E = J \, F\left(\frac{J_1}{J_2}, \lambda\right) = J\left[1 + c_1 \tilde{\lambda} + c_2 \tilde{\lambda}^2 + \dots\right]$$

$$c_n = c_n\left(\frac{J_1}{J_2}\right)$$

- Quantum string (α') corrections suppressed at $J \to \infty$ $\longmapsto$

classical E is exact in $J \to \infty$ limit

Should be possible to compare directly to SYM perturbative results (if $J \to \infty$, $\tilde{\lambda} < 1$ limit is well-defn on SYM side too)

"SU(2) sector":

strings with (J_1, J_2) in S^5

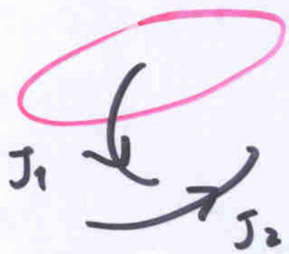
$$\mathcal{O} = \mathrm{Tr}\left(\Phi_1^{J_1} \Phi_2^{J_2}\right) + \text{perm.}$$

Simplest example:

circular string rotating in $S^3 \subset S^5$
with $J_1 = J_2$

Flat space $R^{1,4}$:

$$ds^2 = -dt^2 + dX_1^2 + \dots dX_4^2$$



$$X_1 = x_1 + i x_2 = \cos n\sigma \; e^{in\tau}$$

$$X_2 = x_3 + i x_4 = \sin n\sigma \; e^{in\tau}$$

$$\left(|X_1|^2 + |X_2|^2 = 1 \right)$$

$J_1 = J_2 \sim n$

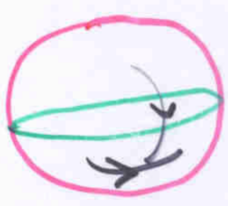
$$E = \sqrt{\frac{2}{\alpha'} n J} \sim \sqrt{T J}$$

$R_t \times S^3 \subset AdS_5 \times S^5$:

$$ds^2 = -dt^2 + \underbrace{|dX_1|^2 + |dX_2|^2}_{dS_3}$$

$$X_1 = \cos\psi \, e^{i\varphi_1}$$
$$X_2 = \sin\psi \, e^{i\varphi_2}$$

$$|X_1|^2 + |X_2|^2 = 1$$



S^3

$$\psi = n\sigma \,, \quad \varphi_1 = \omega_1 \tau \,, \quad \varphi_2 = \omega_2 \tau$$

$$\omega_1 = \omega_2 \quad \rightarrow \quad J_1 = J_2 = \sqrt{\lambda}\, \omega$$

$$\boxed{E = \sqrt{J^2 + n^2 \lambda}} \sim \sqrt{J^2 + T^2}$$

(regular $T \to 0$ limit)

$$E = J \sqrt{1 + n^2 \tilde{\lambda}} = J \left(1 + a_1 \tilde{\lambda} + a_2 \tilde{\lambda}^2 + \dots \right)$$

Other similar solitonic string solutions (integrable σ-model)

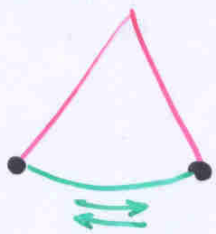
Strings with rigid shape (at center of AdS_5)

$$\psi = \psi(\sigma), \quad \varphi_1 = w_1 \tau, \quad \varphi_2 = w_2 \tau, \quad t \sim \tau$$

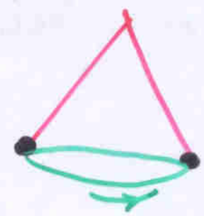
$$\psi'' = -(w_1^2 - w_2^2) \sin 2\psi$$

"Elliptic problem"

cf. Pendulum in gravit. field



folded string



circular string

$$E = E(J_1, J_2, \lambda) = J\, F\left(\frac{J_1}{J_2}, \tilde{\lambda}\right)$$

$$= J\left[1 + c_1 \tilde{\lambda} + c_2 \tilde{\lambda}^2 + \ldots\right]$$

$$\left.\begin{aligned} \left(\frac{E}{K(z)}\right)^2 - \left(\frac{J_1}{\mathcal{E}(z)}\right)^2 &= \frac{4}{\pi^2} z \lambda \\ \left(\frac{J_2}{K(z) - \mathcal{E}(z)}\right)^2 - \left(\frac{J_1}{\mathcal{E}(z)}\right)^2 &= \frac{4}{\pi^2} \lambda \end{aligned}\right\} \; E = E(J_1, J_2, \lambda)$$

K, $\mathcal{E}$ - elliptic integrals

$$c_1 = c_1\left(\frac{J_2}{J}\right) = \frac{2}{\pi^2} K(z_0)\left[E(z_0) - (1-z_0)K(z_0)\right]$$

$$z_0 = z_0\left(\frac{J_2}{J}\right): \quad \frac{E(z_0)}{K(z_0)} = 1 - \frac{J_2}{J}$$

Reproduced on SYM side: $\mathrm{Tr}\left(\Phi_1^{J_1}\Phi_2^{J_2}\right)$

$$\mathcal{O}_{J_1,J_2} = \mathrm{Tr}\left(\Phi_1 \ldots \Phi_2 \ldots \Phi_1 \ldots\right) + \ldots \qquad J \to \infty$$

$$\Delta = J + \frac{\lambda}{J} d_1\left(\frac{J_2}{J}\right) + \frac{\lambda^2}{J^3} d_2\left(\frac{J_2}{J}\right) + \ldots$$

$$= J\left[1 + \tilde{\lambda}\, d_1 + \tilde{\lambda}^2 d_2 + \ldots\right]$$

Problem: find eigenvalues of anom. dim. matrix (= dilatation operator D of SYM) for "long" ($J \gg 1$) operators

$$D = J + \lambda D_1 + \lambda^2 D_2 + \ldots$$

"one-loop" "two-loop"

Hamiltonian of Heisenberg-type spin chain

Use Bethe ansatz to find spectrum of eigenvalues

"one-loop" and "two-loop": matches string theory predictions!

Minahan Zarembo 2002

Beisert Minahan Staudacher Zarembo 2003

(Φ_1, Φ_2) sector : closed under renorm.

Spin chain interpretation : $\Phi_1 : |\uparrow\rangle$
$\Phi_2 : |\downarrow\rangle$

$$\mathcal{O} \sim \mathrm{Tr}\left(\Phi_1 \Phi_1 \Phi_1 \Phi_2 \Phi_2 \Phi_1 \ldots \right)$$

$$\underbrace{\uparrow \quad \uparrow \quad \uparrow \quad \downarrow \quad \downarrow \quad \uparrow \ \ldots}_{J \text{ sites}}$$

Large N : planar graphs in

$$\langle \mathrm{tr}(\Phi_1^* \Phi_2^* \ldots)_x \ \mathrm{tr}(\Phi_1 \Phi_2 \ldots)_y \rangle \sim \frac{1}{|x-y|^{2\Delta}}$$

with $[\Phi, \Phi]^2$ interaction $\rightarrow$

local interactions in spin chain

MZ
Beisert
Kristiansen
Staudacher

$$D_n = \sum_{a=1}^{J} \mathcal{D}_n^{(a)} \, , \qquad \boxed{D = \sum_{n=0}^{\infty} \lambda^n D_n}$$

$$\mathcal{D}_0 = 1 \, , \qquad \mathcal{D}_1 = (1 - \vec{\sigma}_a \vec{\sigma}_{a+1})$$

$$\mathcal{D}_2 = -3 + 4 \vec{\sigma}_a \cdot \vec{\sigma}_{a+1} - \vec{\sigma}_a \cdot \vec{\sigma}_{a+2}$$

$$\mathcal{D}_3 = 20 - 29 \vec{\sigma}_a \vec{\sigma}_{a+1} + 10 \vec{\sigma}_a \cdot \vec{\sigma}_{a+2} - \vec{\sigma}_a \vec{\sigma}_{a+3}$$
$$+ \vec{\sigma}_a \cdot \vec{\sigma}_{a+3} \, \vec{\sigma}_{a+1} \vec{\sigma}_{a+2} - \vec{\sigma}_a \cdot \vec{\sigma}_{a+2} \, \vec{\sigma}_{a+1} \vec{\sigma}_{a+3}$$

$$\vec{S} = \tfrac{1}{2} \vec{\sigma} \, , \qquad [S^i, S^j] = i \epsilon^{ijk} S^k$$

Ferromagnetic long-range chain

Expected to be integrable

$$H = - \sum_{a=1}^{J} \sum_{k} \lambda_k \vec{S}_a \vec{S}_{a+k} + O(S^4)$$

ground state $(E=0)$: $\uparrow\uparrow \ldots \uparrow$

$\mathcal{O} = \mathrm{tr}\, \Phi_1^J$ BPS

$J \to \infty$: expect effective low-energy description

long-wave-length modes relevant

$$H = -\lambda \sum_a \vec{S}_a \cdot \vec{S}_{a+1}$$

$$i \dot{\vec{S}}_a = [H, \vec{S}_a]$$

$$\dot{S}^i_a = \lambda\, \varepsilon^{ijk} S^j_a \left(S^k_{a+1} + S^k_{a-1} \right)$$

Semiclassical + continuum limit :

$$\langle n | \vec{S} | n \rangle = \tfrac{1}{2} \vec{n} \,, \qquad \vec{n}^2 = 1$$

$$n(\sigma_a) = n\left(\tfrac{2\pi}{J} a\right), \qquad a = 1, \ldots, J$$

$$n_{a+1} + n_{a-1} - 2n_a \longrightarrow n'' + O\left(\tfrac{1}{J}\right)$$

$$\boxed{\dot{n}_i = \tfrac{1}{2} \tilde{\lambda}\, \varepsilon_{ijk} n_j n''_k} \qquad \tilde{\lambda} = \frac{\lambda}{J^2}$$

Landau-Lifshits eq. for classical ferromagnetic

Effective action:

$$S = J \int dt \int_0^{2\pi} \frac{d\sigma}{2\pi} \left(\mathcal{L}_{WZ} - \langle n | H | n \rangle \right)$$

$$\mathcal{L}_{WZ} \equiv C_t(n) = \text{"} C \dot{n} \text{"} = \int_0^1 d\xi \, \epsilon_{ijk} n_i \dot{n}_j \partial_\xi n_k$$

vector potential of magnetic monopole at center of S^2

$$\mathcal{L} = C_t(n) - \tilde{\lambda} n'^2 + O\left(\frac{1}{J^2} n''\right)$$

Coherent state: $\boxed{\langle n | \vec{S} | n \rangle = \frac{1}{2} \vec{n}}$

$$|n\rangle = e^{-i\phi S_3} e^{-i\theta S_1} |\uparrow\rangle$$

$$\langle \psi_f | e^{-iHt} | \psi_i \rangle \sim \int \mathcal{D}\vec{n} \, e^{i S(\vec{n})}$$

$$S = \int dt \left(\sum_a C_t(n_a) - \underbrace{\langle n | H | n \rangle}_{\lambda \sum_a (n_{a+1} - n_a)^2 + \dots} \right)$$

Compute low-energy eff. action:

generalization of LL action $J \to \infty$, $\tilde{\lambda} = \text{fixed}$

$$\mathcal{L} = C_t(n) - \left(\tilde{\lambda} n'^2 + \tilde{\lambda}^2 (n''^2 + n'^4) + \dots \right)$$

Match to string theory?

Aims:

- String – SYM (spin chain) matching beyond comparing particular states? (Avoid Bethe)
- Derive string 2-d action from SYM side?
- Direct relation between string profiles and SYM operators?
- exact $E(J,\tilde{\lambda}) \rightarrow$ hints about exact SYM dil. operator?

Key ideas:

Kruczenski 2003

KRT 2004

- new expansion on string side: gauge away coll. coord. for $J = J_1 + J_2$ and expand in $J \to \infty$, $\tilde{\lambda} < 1$:

  $R \times S^3$ σ-model $\rightarrow$ effective non-rel. action for $\vec{n}(\sigma,\tau)$ ("string profile")

$$\boxed{\tilde{\mathcal{L}}_{class} = C \cdot \dot{n} - H(n', n'', n''', \ldots; \tilde{\lambda})}$$

- low-energy eff. action for $\langle \vec{S} \rangle \sim \vec{n}$ on ferrom. spin chain side:

$$\boxed{\mathcal{L}_{eff} = C \cdot \dot{n} - H(n', n'', n''', \ldots, \tilde{\lambda})}$$

Actions agree order by order in $\tilde{\lambda}$

String side : (J_1, J_2) sector

$$X_1 = U_1 e^{i\alpha} \,, \quad X_2 = U_2 e^{i\alpha}$$

$$|U_1|^2 + |U_2|^2 = 1 \qquad CP^1$$

$$U_1 = \cos\psi\, e^{i\beta} \,, \quad U_2 = \sin\psi\, e^{-i\beta}$$

$\alpha =$ coll. coord. for $J = J_1 + J_2$
(orbital $\oplus$ internal, cf. x^+)

Hopf fibration parametrization of S^3

$$ds^2_{S^3} = (d\alpha + C)^2 + (d\psi^2 + \sin^2 2\psi\, d\beta^2)$$

$$C = \cos 2\psi\, d\beta$$

"Shape" vector $\boxed{\vec{n} \equiv U^+ \vec{\sigma}\, U} \,, \quad U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$

$$\boxed{ds^2_{S^3} = (D\alpha)^2 + \frac{1}{4}\, d\vec{n}\, d\vec{n}}$$

$$\vec{n} = (\sin 2\psi \cos 2\beta ; \sin 2\psi \sin 2\beta ; \cos 2\psi)$$

$$D\alpha = d\alpha + C$$

WZ $\quad C = -\frac{1}{2}\int_0^1 d\xi \; \varepsilon_{ijk}\, n_i\, \partial_\xi n_j\, dn_k$

$$dC = +\frac{1}{2}\, \varepsilon_{ijk}\, n_i\, dn_j \wedge dn_k$$

cf. on spin chain side:

$$\langle n | \vec{\sigma} | n \rangle = \vec{n}$$

$|n\rangle = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$ coherent state $\rightarrow \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$

t, α – "longitudinal" coordinates: gauge away

$\vec{n}$ – physical or $\perp$ $\rightarrow$ "seen" on SYM side

$$E = \sqrt{\lambda} \int \frac{d\sigma}{2\pi}\, \partial_0 t$$

$$J = \sqrt{\lambda} \int \frac{d\sigma}{2\pi}\, D_0 \alpha \qquad U(1)$$

$$Q_i = \sqrt{\lambda} \int \frac{d\sigma}{2\pi} \left(D_0 \alpha\, n_i + \frac{1}{2} \epsilon_{ijk} n_j \partial_0 n_k \right) \qquad SO(3)$$

$J = J_1 + J_2 \quad , \quad Q_3 \sim J_1 - J_2$

Match charges to spin chain side

J = length of chain $\rightarrow$ choose gauge where J is distrib. homogen.:

$$t = \tau \quad , \quad D_0 \alpha = \text{const}$$

Solve constraints $\rightarrow$ eliminate α

$V \equiv \alpha - t$: WZ term C from $D\alpha$

$$\mathcal{L} = -\partial t\, DV - \frac{1}{2}(DV)^2 - \frac{1}{8}(\partial \vec{n})^2$$

Re-organize action: $J \to \infty$, $\tilde{\lambda} < 1$

$$\mathcal{L} = C(n) - \mathcal{H}, \qquad C = \text{"}c_i \dot{n}_i\text{"}$$

$$\mathcal{H} = \sqrt{\left(1 + \frac{1}{4}\tilde{\lambda} n'^2\right)\left(1 - \frac{1}{4}\tilde{\lambda}^2 \dot{n}^2\right) + \frac{1}{16}\tilde{\lambda}^3 (\dot{n} n')^2}$$

$$\mathcal{H} = \sqrt{-\det \gamma_{ab}}, \qquad \gamma_{ab} = \tilde{\eta}_{ab} + \frac{1}{4}\partial_a \vec{n}\, \partial_b \vec{n}$$

$$\mathcal{L} = C(n) - \frac{1}{8}\tilde{\lambda} n'^2 + \frac{1}{8}\tilde{\lambda}^2\left(\dot{n}^2 + \frac{1}{16} n'^4\right) + \dots$$

Leading-order eqs: Landau-Lifshits

$\dot{n} \sim n'' + \dots$ linear in $\dot{n}$

(cf. friction: $\tilde{\lambda}\ddot{\varphi} + \dot{\varphi} - \varphi'' + \dots = 0$)

Can do local field redefns to eliminate higher powers of $\dot{n}$:

$$\mathcal{L} = C(n) - \frac{1}{8}\tilde{\lambda} n'^2 + \frac{1}{32}\tilde{\lambda}^2\left(n''^2 - \frac{3}{4} n'^4\right) - \frac{1}{64}\tilde{\lambda}^3\left(n'''^2 - \frac{7}{4} n'^2 n''^2 - \frac{25}{2}(n' n'')^2 + \frac{13}{16} n'^6\right) + O(\tilde{\lambda}^4)$$

Same energy/solutions as in "exact" σ-model

All quadratic terms ("BMN" limit)

$$\mathcal{L} = C - \frac{1}{4} n_i \sqrt{1 - \tilde{\lambda}\partial_1^2}\, n_i + O(n^4)$$

Spin chain side:

$$J \to \infty, \quad \tilde{\lambda} = \frac{\lambda}{J^2} \quad : \quad \langle n | \mathcal{D}_1 | n \rangle \to n'^2$$

$$\langle n | \mathcal{D}_2 | n \rangle = \frac{1}{2}(n_a - n_{a+2})^2 - 2(n_a - n_{a+1})^2$$

$$\to -\frac{1}{2}(\partial^2 n)^2 + O\left(\frac{1}{J}\right)$$

$$\langle n | \mathcal{D}_3 | n \rangle = (\partial^3 n)^2 + \dots$$

Include quantum corrections: $\tilde{\lambda}^2 n'^4 + \dots$

cf: σ-model 

$$\mathcal{L} = \mathcal{L}_{WZ} - H(n, n', n'', \dots)$$

$$H = \frac{1}{8}\tilde{\lambda}\, n'^2 - \frac{1}{32}\tilde{\lambda}^2\left(n''^2 - \frac{3}{4} n'^4\right) + \frac{1}{64}\tilde{\lambda}^3\left(n'''^2 + \dots\right)$$

Matches string σ-model!

Explains all previous $\tilde{\lambda}, \tilde{\lambda}^2$ results

Conclusions:

- Beginning of understanding of precise workings of AdS / CFT
  space-time / w-sheet picture, locality
  New "BMN-type" limit on string side
  (but <u>not</u> just c.o.m. motion isolated)
- Matching of integrable structures follows from eff. lagrangian matching
- implies relation between
  "real space" strings and SYM operators
  (spin chain states)

Open questions and extensions:

- Matching at $\tilde{\lambda}^3$ in all detail?
  puzzles on spin chain side
- extension to (J_1, J_2, J_3) SU(3) sector:
  obvious
  extension to (J, S) sector: $XXX_{-1/2}$ chain
- similar ideas for $N=2$ susy examples?
- new tools for computing anom. dim in gauge theories (high en. QCD)?