

# **Observations on the The Space of Four Dimensional String and M theory Vacua.**

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In Honor of David Olive.

The work of David Olive has had a major impact on the development of string theory.

I think this impact has been greatest in two areas:

- a) world-sheet CFT (eg GKO,GSO, etc...)
- b) strong-weak coupling duality.

The latter will play an important role in what I have to say here.

David Olive's work with Montonen, suggesting that non-Abelian gauge theories have a dual description at strong coupling in terms of magnetic degrees of freedom, found its home in string theory where it lead to string dualities and M theory.

We learned that some vacua of different string theories are in fact equivalent.

It is important to try and understand if these dualities can have a bearing on ``realistic vacua of string and M theory".

We will discuss this question today.

There are *many* different vacua of string and *M* theory.

Some examples are:

Eleven dimensional flat spacetime.

Type IIB on  $AdS_5 \times S^5$  with N units of flux

Heterotic Strings on  $K3 \times R^{5,1}$

***None*** of these vacua are realistic.

There are too many large dimensions.

The gauge groups are wrong.

Etc. etc.

However, there is very strong evidence that these are *exact*, non-perturbative vacua.

This is due to strong-weak coupling dualities.

So, I think, we have to live with the fact that M theory predicts many vacuum states which are ***not realistic***.

This is not a problem as long as the theory also predicts the existence of a vacuum state which is realistic.

But, in order to proceed, we need to have some idea about the number of ``realistic vacua" in the theory.

This is not really a well posed question, because one can (and many people have) conceive of many different UV completions of the standard models of particle physics and cosmology.

Also, we don't actually know any *realistic* vacua. So, in defining ``realistic vacua" we need to make some choices.

My choice is *unbroken supersymmetry* at some energy scale above the electroweak scale.

This is because of hints from unification and hierarchy stabilisation as well as the *idea* that string theory *predicts* supersymmetry.

Since particle physics also requires non-Abelian gauge symmetry and chiral fermions, we will be interested in

The Space of

String and M theory vacua with four large dimensions, non-Abelian gauge symmetry, chiral fermions and unbroken supersymmetry at some energy scale

$$E > E_{ew}$$

Comments:

I have absolutely no idea how big this space is!!

One could obtain a rough lower bound by simply enumerating the number of known vacua with these properties.

(Douglas has initiated a study of this bound, hep-th/0303194)

Even though we have defined the space, its not really *the* space!

This is because I did not add the criterion that supersymmetry is *broken* at low energy. So we get an upper bound.

Here is a (more or less complete) list of the known vacua in the space:

- a  $E_8 \times E_8$  heterotic strings on Calabi-Yau 3-folds
- b  $SO(32)$  heterotic strings on Calabi-Yau 3-folds
- c Type IIA orientifolds of C-Y 3-folds with D6-branes
- d Type IIB orientifolds of C-Y 3-folds with D(odd) branes.
- e F-theory on Calabi-Yau 4-folds
- f  $M$  theory on G2-holonomy manifolds with singularities
- g Non-geometric CFT's, where the extra dimensions are represented by a SCFT without a geometric limit.
- h Freund-Rubin compactifications of  $M$  theory on singular Einstein manifolds with  $R > 0$

Comments:

Vacua from g behave similarly to those in a – f, which can also be perturbatively defined by CFT.

One can also add fluxes to some of these vacua. We will discuss this below.

Vacua from a – f can be related by dualities. For example: all of the c-vacua are also f-vacua; all of the d-vacua are limits of e-vacua; when the G2-manifolds in f are K3-

fibred, they are dual to a- or b-vacua.

More importantly, there are vacua in this list which are **apparently not** dual to other vacua in this list!

Precisely: if we **assume** that all dualities between string and/or  $M$  theory vacua originate from the known, **fundamental** dualities between vacua with 16 or more supercharges, then there **are** vacua which are **not** dual to other vacua.

Eg, if the G2-holonomy manifolds in f-vacua are not K3-fibered then they **cannot** be dual to the heterotic string vacua. If they are also not locally foliated by circles, then they are **not** dual to Type IIA vacua.

Similarly, if the F-theory d-vacua are defined with non K3-fibered C-Y 4-folds, then they are **not** dual to the heterotic string vacua.

The a,b,c,d,e,f,g-vacua all classically have  $\Lambda = 0$  whereas the Freund-Rubin vacua in h have classically  $\Lambda < 0$ , suggesting that h-vacua are **distinct** from the rest.

This is one point that we will try to develop

evidence for here: Freund-Rubin vacua are not dual to the "special holonomy vacua" a – f.

If this is indeed true, then it means that the space of realistic vacua is disconnected, even after accounting for dualities.

This makes the task of making predictions for physics beyond the standard models more difficult.

There are presumably more classes of realistic vacua than just those we know about already! For example, someone may eventually find some new classes of flux compactifications of  $M$  theory. My bet is that new vacua do exist.

I now want to move on and give some more details about what I have said already.

First of all, I want to review a modern approach to particle physics model building via  $M$  theory on a manifold,  $X$ , with  $G_2$ -holonomy.

The classical vacuum is flat four dimensional spacetime.



When  $X$  is smooth, the low energy description of the physics is 4d supergravity with an Abelian gauge group and massless, neutral, moduli.

In order to obtain non-Abelian gauge symmetry  $X$  must contain a 3-manifold  $Q$ , along which there is an orbifold singularity (BA hep-th/9812205, 0011089) These singularities are ADE in type ie

locally 
$$X \sim \frac{R^4}{\Gamma_{ADE}} \times Q$$

Thus, along  $Q$  we have a non-Abelian ADE gauge field.

In order to obtain chiral fermions  $X$  contains additional singular points  $p_i$  on  $Q$  at which there is a conical singularity (Atiyah-Witten hep-th/0107177 and BA-Witten hep-th/0109152)

Near these singularities the metric is

$$ds^2 = dr^2 + r^2 g(Y)$$

with  $Y$  a compact 6-manifold whose topology determines which representation of the gauge

group the chiral fermions reside in.

For both kinds of singularity, the light charged particles are actually wrapped membranes.

So the picture of the seven extra dimensions in these vacua is roughly

Notice that all the charged matter particles are localised at points in the extra dimensions.

This means that there can be no local interactions between them.

Instead, interactions are generated by non-local *instanton* effects. Every interaction between the charged particles is generated by a Euclidean membrane instanton which wraps a (supersymmetric) 3-cycle  $N$  which passes through the points  $p_i$  supporting the particles involved.

The coupling constant  $\lambda$  for this interaction term is of order

$$\lambda \sim e^{-T_2 \text{Vol}(N)}$$

with  $T_2$  the membrane tension.

Because the couplings are exponentially suppressed it is extremely *natural* to obtain hierarchies of Yukawa couplings.

In fact it is also natural for some couplings to be extremely small....

In GUT models built this way the couplings have a minimal value given by

$$\lambda_0 \sim e^{-2\frac{\pi}{\alpha_{gut}}} \text{ which is roughly } 10^{-70} \text{ for}$$

$$\alpha_{gut} = \frac{1}{25} \text{ !!!!!!!}$$

So in principle one can suppress "unwanted operators" using this mechanism. In practice this turns out to be difficult if one wants to preserve Grand Unification.

Luckily, these vacua can also have *natural* discrete symmetries which can also suppress such operators, (Witten hep-ph/0201018).

Notice also that the theory automatically *predicts* localised chiral fermions.

Another prediction of such vacua is that supersymmetry breaking, if it occurs, will generically be *gravity mediated*.

This is because the 3-manifolds which support the gauge fields generically do not intersect in X

Even though this is a generic prediction, ***some*** models will exist in which supersymmetry breaking is ***gauge mediated***. Consider the models which are dual to Type IIA orientifolds of C-Y 3-folds with partially wrapped intersecting D6-branes.

In this dual picture,  $Q$  is a 3-manifold in the Calabi-Yau,  $Z$  and is a surface which the D6-branes wrap. In a six dimensional space such as  $Z$ , a collection of 3-manifolds will generically intersect in points, and hence there will be massless particles charged under both of the gauge groups which are involved. So, it is natural in such vacua that there are gauge interactions between *all* gauge groups.

Existence:

Unlike Calabi-Yau's, G2-manifolds are much harder to construct explicitly. However, there are many more G2-manifolds than there are Calabi-Yau 3-folds!!! This is again because of dualities.

So, even though we don't know that many G2-manifolds explicitly, we do know that they exist.

The problems of G2-compactification:

The main problem, which is not specific to these vacua but all string/M theory vacua is a proper understanding of supersymmetry breaking and the cosmological constant problem.

A problem more specific to G2 compactification is explaining why the couplings of charged particles take the values they take.

Recall that these are generated by membrane instantons and are exponentially suppressed by the volume of the instanton. These couplings are actually functions of the parameters  $s_i$  of the Einstein metric on  $X$ . So, to explain why, say

$$\frac{\lambda_t}{\lambda_e} \sim 10^6 \text{ is equivalent to explaining why}$$

$$s_t \sim s_e - 14$$

This is a mild fine tuning, but needs to be understood.

We can offer an explanation prior to supersymmetry breaking, with the hope that the reasoning is still valid in models without bose-fermi mass degeneracy.....

### Moduli Stabilisation by Fluxes:

Low energy M theory is well approximated by 11d supergravity in smooth regions of spacetime.

This theory has two bose fields: a metric  $g$  and a 3-form potential  $C$ , with field strength  $G$ .

The classical solution corresponding to a G2-holonomy manifold  $X$  times Minkowski spacetime has  $G=0$ .

This classical theory has zero potential. Hence the parameters of the metric  $s_i$  are massless fields whose vevs are undetermined.

However, at volumes large compared to  $L_P$ , the energy density of  $C$  is small and hence the theory with non-zero  $G$  can be regarded as a perturbation of the  $G = 0$  case.

Since the Lagrangian for  $C$ ,  $G^*G$  depends on the metric on  $X$ , it depends on the moduli fields  $s_i$  and therefore induces a potential for the moduli. If this potential has isolated stable critical points, the vev's of the moduli are determined!

Unfortunately, this does not happen  
(BA-Spence hep-th/0007213; Beasley-Witten hep-th/0203061)

Instead, the potential is positive, with a runaway behaviour.

As we saw above, at special kinds of singular subspaces, the theory has additional localised light degrees of freedom.

In [BA hep-th/0212294] we proved that, under certain topological conditions, a combination of fluxes for both  $G$  and the gauge fields at a singularity is enough to guarantee the existence of a supersymmetric vacuum with negative  $\Lambda$  in which the  $s_i$  are *uniquely* determined.

In [BA hep-th/0303234] we went on to show that in such vacua, choices for  $G$  exist for which the Yukawa couplings between quarks, leptons and



Higgses are correct.

The bad news is that choices of flux exist which give virtually any set of Yukawa couplings!

Other problems which need to be addressed are:

Calculating the phases of the mixing matrix.

Showing that the Yukawa couplings are unchanged by supersymmetry breaking effects.

Explaining how, after supersymmetry breaking, the cosmological constant is very slightly above zero.

I now want to go on to discuss the so-called Freund-Rubin vacua, and explain why I think that these vacua cannot be dual to the vacua I discussed above.

## Freund-Rubin Vacua:

In 1980, Freund and Rubin found a solution of 11d supergravity (a.k.a. low energy M theory) in which spacetime is of the form

$$S^7 \times AdS_4$$

This describes a compactification on a round 7-sphere to anti de Sitter space.

The cosmological constants of the two spaces are proportional to each other, so the Kaluza-Klein excitations have a mass of the same order as the fluctuations of the AdS.

By replacing  $S^7$  by another Einstein,  $R > 0$  manifold,  $Y$ , we obtain other solutions.

This led to an industry in the early 80's in which one attempts to find a suitable  $Y$  for which the low energy physics is that of the standard models.

Unfortunately Witten proved that, when  $Y$  is smooth one can never obtain chiral fermions from such vacua. The discovery of the heterotic string then put an end to this program..

Maldacena revived interest in these vacua when he proposed that they are all ***holographically dual*** to 3d conformal field theories (which reside on the world-volumes of membranes at the boundary of AdS4).

This was strongly motivated by the fact that the metric of  $N$  membranes in flat spacetime becomes the Freund-Rubin vacuum at large  $N$ .

In the context of  $G_2$ -holonomy compactifications we saw that special kinds of singularities of the extra dimensions lead to chiral fermions in four dimensions, so it is natural to ask if something similar happens in Freund-Rubin compactifications?

This is the question posed in [BA, F. Denef, C. Hofman, N. Lambert hep-th/0308046]

Well, the chiral fermion singularities are a very local concept, so if  $R > 0$  Einstein 7-manifolds exist with locally the same singularities as we discussed above, we will have Freund-Rubin vacua with chiral fermions.

Here is an example which illustrates many of the important features.

We begin with the metric of N membranes:

$$ds^2 = H^{-\frac{2}{3}}(r) g_{2+1} + H^{\frac{1}{3}}(r) \{ dr^2 + r^2 g_7(Y) \}$$
$$H = 1 + \frac{a^6}{r^6}; \quad a^6 \sim \frac{N}{Vol(Y)}$$

This describes N membranes in the background

$$ds^2 = g_{2+1} + dr^2 + r^2 g_7(Y)$$

In the large N limit this metric becomes

$$ds^2 = g(AdS_4) + a^2 g_7(W)$$

where AdS4 has  $\Lambda \sim a^{-2}$ . This is the Freund-Rubin solution.

Now, let

$$ds^2 = dy^2 + y^2 g_6(W) \text{ be a G2-holonomy cone.}$$

(there are several **explicit** examples)

Now,

$ds^2 = g_{2+1} + dx^2 + dy^2 + y^2 g_6(W)$  is a background

for M theory. So we can consider N membranes in this background. To do this we need to identify a good "radial" coordinate. This is provided by the following equality

$$dx^2 + dy^2 + y^2 g_6(W) = dr^2 + r^2 \{ d\alpha^2 + \sin^2 \alpha g_6(W) \}$$

where  $0 \leq \alpha \leq \pi$

So, we can use r as the variable in the membrane metric. This gives a Freund-Rubin metric at large N for which the Einstein metric on the extra dimensions is

$$g(Y) = d\alpha^2 + \sin^2 \alpha g_6(W)$$

At  $\alpha = 0, \pi$  there are conical singularities of the form

$d\alpha^2 + \alpha^2 g_6(W)$  exactly as in the G2-holonomy case. So, if W is such that we get chiral fermions in that case, they will also be present here.

However,  
in these, simplest examples, the chiral fermion at 0  
is CPT conjugate to that at  $\pi$ , so the full  
spectrum is non-chiral.

By some fancier arguments, however, we were able  
to demonstrate the existence of genuinely chiral  
examples.

Instead of getting bogged down in those details,  
I want to emphasise some general points.

In all the examples we constructed, the rank of  
the four-dimensional gauge group is a *free*  
parameter.

Classically, the cosmological constant (or  $N$ , the  
membrane number) is also a free parameter.

All of these Freund-Rubin examples have a  
holographic dual – a 3d SCFT.

In simple cases one can explicitly write the metric  
for the supergravity solution.

All of these features are in *contrast* to the G2-  
holonomy and Calabi-Yau vacua. There one

cannot explicitly write the metric. The rank of the gauge group is fixed.

Since the manifolds (although singular) involved in the Freund-Rubin vacua are so simple compared to the special holonomy vacua, the examples do not admit K3-fibrations (although perhaps some do).

For these and probably other reasons I think that the generic Freund-Rubin vacua in our space are totally disconnected from the G2-holonomy or Calabi-Yau vacua.

My bet is that there are also other classes of vacua in our space which are not dual to either the Freund-Rubin vacua or the special holonomy vacua.

This means that we have to carefully study the physics of Freund-Rubin (and would be other classes of) vacua to address whether or not they can fail to be realistic at a finer level.

In some sense it would be more satisfying if one could show that Freund-Rubin vacua cannot actually describe the real world.

Failing that....

Since there seem to be completely disconnected components of the space of vacua and, moreover, since there appear to be many vacua in each disconnected component, we ought to be worried about what we should do to make predictions.

Instead of running around like headless chickens searching for a single realistic vacuum, perhaps we should ask questions about the generic, model independent predictions of vacua of the various types.

For instance, we already argued that generic G2-holonomy compactifications will have gravity mediated supersymmetry breaking, if at all.

A more abstract, statistical approach might also be used to demonstrate the existence of vacua with certain properties (Douglas).



An important outstanding problem.

The Freund-Rubin vacua classically have a negative cosmological constant.

Can quantum effects generate corrections to the potential which allow a de Sitter vacuum?

Actually this is not just a problem for Freund and Rubin.

Generically *all* of the vacua in our space will suffer from this problem.

The reason is that N=1 supersymmetry in four dimensions allows very non-trivial non-perturbative effects and generically such a system will have a non-trivial potential which arises via a superpotential,  $W$ .

The conditions for unbroken supersymmetry with non-zero (always  $< 0$ ) cosmological constant are  $p$  equations for  $p$  unknowns and will typically have solutions.

This is unlike the case of zero c.c. which is an overdetermined system.

I also want to emphasise that KKLT gave a proposal for the existence of de Sitter vacua in Type IIB string theory.

Attempting to implement this proposal leads to difficulties. So, we don't have a single example of a de Sitter vacuum.

Maybe string theory predicts that the acceleration of the universe is not due to a cosmological constant.