DUALITY AND GEOMETRY

DIOFEST 2004

MONTONEN-OLIVE DUALITY

$$g_{YM} \longrightarrow 1/g_{YM}$$

$$\underset{\sim}{E} \longrightarrow \underset{\sim}{B} \qquad \underset{\sim}{B} \longrightarrow -\underset{\sim}{E}$$

"WE HAVE DARED TO PRESENT OUR SPECULATIONS, AS YET UNPROVED, BECAUSE WE FEEL THEY DO SUCCEED IN RELATING UNCORRELATED FACTS AND WOULD BE, IF TRUE, OF SOME IMPORTANCE FOR THE FURTHER UNVEILING OF ~~SE~~ THE SECRETS OF QUANTUM GAUGE FIELD THEORIES"

DUALITY CAN TAKE GEOMETRIC STRING SOLUTIONS TO "NON-GEOMETRIC" ONES.

- GENERALISED GEOMETRY?

CMH hepth/9811021, 0203146

CMH + DABHOLKAR 0210209

CMH + OZER 0308133

S-DUALITY OF N=4 SYM

BREAK TO U(1): $SL(2,\mathbb{Z})$

$dF = 0$ BIANCHI

$dG = 0$ FIELD EQUN $G = \frac{1}{g^2} * F + \theta F$

$\begin{pmatrix} F \\ G \end{pmatrix}$ $SL(2)$ DOUBLET

$$\tau = \theta + \frac{i}{g^2} \to \frac{a\tau + b}{c\tau + d}$$

g SMALL: $F = dA$, FORMULATE IN TERMS OF A, ELECTRIC CHARGES "FUNDAMENTAL", MAGNETIC CHARGES "SOLITONIC"

g LARGE PERT THEORY IN $\tilde{g} = \frac{1}{g}$,

$G = d\tilde{A}$, MAGNETIC CHARGES FUNDAMENTAL ETC

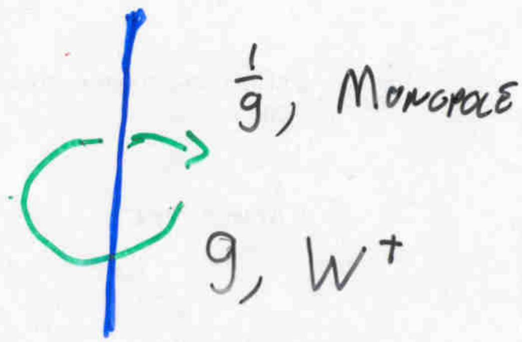
$g \sim 1$? NO PERTURBATION THEORY.

COMPARE $g \ll 1$ AND $g \gg 1$ REGIMES?

S-Duality Twists

1-Cycles with $SL(2;\mathbb{Z})$ Monodromy

1) Cosmic String



2) $M = S^1 \times \mathbb{R}^3$

Twisted Reduction on S^1

$$\Phi(x, y) = g(y)\, \phi(x)$$

$$g : S^1 \longrightarrow SL(2,\mathbb{R})$$

$$g(y) = \exp\left[M \frac{y}{2\pi R} \right]$$

Monodromy $\quad \mathcal{M} = e^{M} \in SL(2,\mathbb{Z})$

[Generalise to any Global Duality Symmetry G,

$g : S^1 \longrightarrow G \,, \quad \mathcal{M} \in G(\mathbb{Z})$]

Classified by Conjugacy Classes of $\mathcal{M}$

- $\tau(y)$ "NATURAL" IF EMBED IN SUGRA / STRING

  $\tau(y)$ IS SCALAR FIELD

- SYMMETRY OF FIELD EQUNS, SO REDUCE FIELD EQUNS
- EXPECT MASSES FROM ∂_y

TWO MASSIVE VECTORS A_μ^i IN $D=3$, + SELF-DUALITY

$$\boxed{* dA^i \sim \tilde{M}^i{}_j A^j}$$

$$\tilde{M}^i{}_k = J^i{}_j(\tau) M^j{}_k \,, \quad J^2 = -1$$

CHERN-SIMONS GAUGED SUGRAS IN $D=3$

[CM4 + OZER]

$$L \sim \Omega_{ij} A^i \wedge dA^j + \tilde{M}_{ij} A^i \wedge * A^j$$

D=4 DOUBLED FORMALISM

CREMMER
JULIA LIU POPE

$$A^i = \begin{pmatrix} A \\ \tilde{A} \end{pmatrix}$$

$$L = \frac{1}{2} K_{ij}(t)\, F^i \wedge * F^j$$

$SL(2,\mathbb{R})$ INVARIANT

HALVE D.O.F.'s : "SELF-DUALITY"

$$F^i = J^i{}_j(t) * F^j$$

COVARIANT

$$(J*)^2 = +1$$

"SOLVE" FOR A, OR $\tilde{A}$

TO GET CONVENTIONAL

FORMULATION.

TWIST $\Rightarrow$ CAN'T DO THIS GLOBALLY

$$J^i{}_j = \Omega^{ik} K_{kj}$$

$$\Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

GEOMETRIC ORIGIN OF SL(2) :

M5 - BRANE $\rightarrow$ (2,0) SUSY THEORY

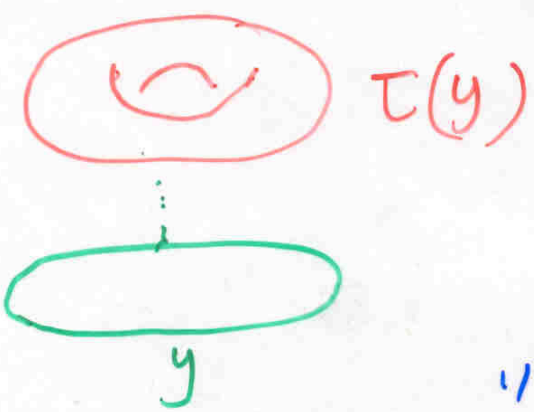
COMPACTIFY ON T^2 :

$\rightarrow$ $N=4$ SYM

$SL(2;\mathbb{Z})$ GEOMETRICAL

τ : MODULUS OF T^2

TWISTED REDUCTION TO $D=3$



T^2 BUNDLE OVER S^1

TOTAL SPACE : B_3

"TWISTED T^3"

$D=3$ THEORY FROM (2,0) THEORY ON B

STRING THEORY COMPACTIFICATION ON B_3

e.g. $\mathcal{M} = \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix} \quad m \in \mathbb{Z}$

$$ds^2_B = (2\pi R)^2 dy^2 + \frac{a}{\tau_2} \left| dx_1 + \tau(y)\, dx_2 \right|^2$$

$$\tau(y) = \tau_0 + m y \qquad B(R, a)$$

T - DUALIZE ON x_1 : [CMH]

UNTWISTS $B \longrightarrow T^3$

H - FLUX $\qquad H \sim m\, dy\, dx^1 dx^2$

"TWIST" DUAL TO "H - FLUX"

HOWEVER, FURTHER T - DUALITIES "PROBLEMATIC"

$CY \sim T^3$ FIBRATION [SYZ]

$\xrightarrow{T^3 \text{ T-DUAL}}$ MIRROR CY

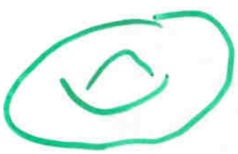
$CY + \text{FLUX ON } T^3 \longrightarrow \; ?$

DUALS OF FLUX COMPACTIFICATIONS?

Massive IIA from D=11?

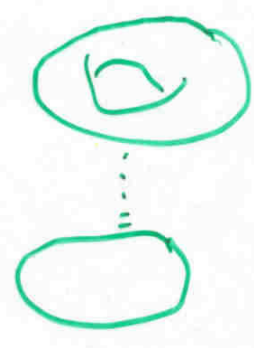
Can't get IIB or Romans IIAm Sugra from D=11 Sugra

But M-Theory on T^2



$\Rightarrow$ IIB in limit Area $\rightarrow 0$ [Schwarz]

M-Theory on B



$\Rightarrow$ IIAm string in limit Volume $\rightarrow 0$ [CMH]

"Smeared KK-Monopole"

$\longrightarrow$ D8 brane

T^n Reduction: String Theory

Geometrical $SL(n;\mathbb{Z}) \subset O(n,n;\mathbb{Z})$
T-Duality

Further Reduction on S^1,
Twisted by $M \in O(n,n;\mathbb{Z})$

If $M \in SL(n;\mathbb{Z})$ [CMH + Dabholkar]
Lifts to Reduction on
T^n Bundle over S^1.

But T-Dualizing This $\longrightarrow$

$$M \to U M U^{-1} = M'$$

in general $M' \in O(n,n;\mathbb{Z})$

General Monodromies
in T- or U-Duality Groups
Should Be Allowed

$\Rightarrow$ Consistent
String Backgrounds

Winding

Momentum

Twisted Reductions ≡ Orbifolds

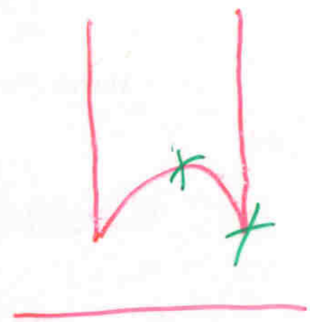
CMH + Dabholkar

— If $\mathcal{M}^m = \mathbb{1}$

— Moduli $\tau = \tau_0$ $\mathbb{Z}_m$ Fixed Point

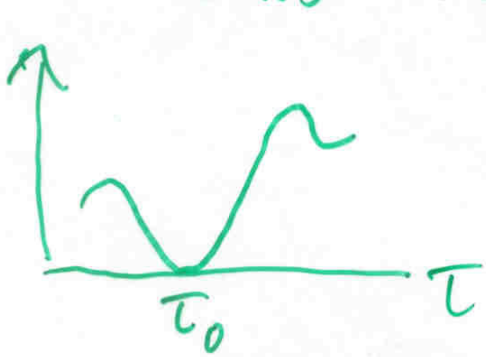
$\Rightarrow$ Orbifold of $S^1 \times T^n(\tau)$

By $\mathcal{M}$ and $\frac{2\pi}{m}$ shift on S^1



$\Rightarrow$ Relation between Classification of Conj Classes of $G(\mathbb{Z})$ & Fixed Points

$\Rightarrow$



τ_0 Minimum of Potential $V(\tau)$, $V(\tau_0) = 0$

— Can continue away from $\tau = \tau_0$

"Climb" $V(\tau)$, No Minkowski Vacuum

— For T-Duality Twists Asymmetric Orbifold

Modular Invariance Gives Extra Constraint

TWISTS AND GEOMETRY

T^n : $E_{ij} = G_{ij} + B_{ij}$ $\quad i = 1, \ldots, n$

$\mathcal{M} \in SL(n, \mathbb{Z})$: GEOMETRIC :

$$E_{ij}(x, y+2\pi R) = \mathcal{M}_i{}^k \, E(x,0)_{kl} \, (\mathcal{M}^t)^l{}_j$$

$$E' = \mathcal{M}^t E \mathcal{M}^t$$

$$\mathcal{M} \in O(n, n; \mathbb{Z})$$

NON-GEOMETRIC

$$E' = \frac{aE + b}{cE + d}$$

$$\mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(n, n; \mathbb{Z})$$

CMH + DABHOLKAR
LOWE NASTASE RAMGOOLAM
HELLERMAN MCGREEVY WILLIAMS
KACHRU, SCHULZ, TRIPATHY, TRIVEDI

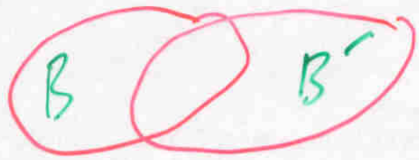
STRING GEOMETRY?

USUAL PICTURE: MANIFOLD

$$(M;\ G_{\mu\nu},\ B_{\mu\nu},\ \Phi)$$

B_2: GERB CONNECTION

$B' = B + d\Lambda$ IN OVERLAPS



FOR T-DUALITY:

M HAS T^n FIBRATION

T^n BUNDLE WITH $SL(n;\mathbb{Z})$ TRANSITION FUNCTIONS

$U(1)^n$ ISOMETRY

KILLING VECTORS k^i_μ $\quad i = 1,\dots n$

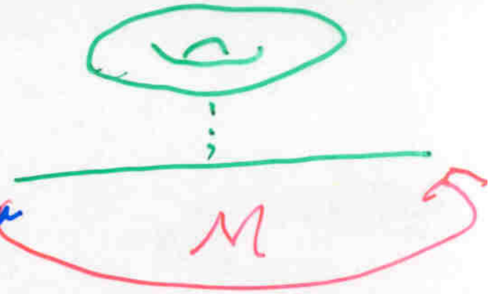
$\mathcal{L}_k G = 0 \qquad \mathcal{L}_k H = 0 \qquad (\mathcal{L}_k B = d\Lambda)$

$\Rightarrow \quad d(\iota_k H) = 0$

REQUIRE $\quad \iota_k H = dv \quad$ EXACT

$T: \quad k \longleftrightarrow v \qquad G_{mi} \longleftrightarrow B_{mi}$

BUNDLE OVER S^1:



SINGLE TRANSITION FUNCTION $\mathcal{M}$.

T-DUALITY $\longrightarrow$ NEED TO ALLOW TRANSITION FUNCTIONS IN $O(n,n;\mathbb{Z})$

$\rightsquigarrow$ T^{2n} BUNDLE OVER S^1, ... ?

COORDS $\quad x^\mu = x^\mu_L + x^\mu_R$

$$\tilde{x}^\mu = x^\mu_L - x^\mu_R$$

$$X^\mu(\sigma,\tau) = X^\mu_L(\sigma-\tau) + X^\mu_R(\sigma+\tau)$$

$$X^\mu_L = x^\mu_L + p^\mu_L(\sigma-\tau) + \sum_n \alpha^\mu_n e^{in(\sigma}$$

STRING FIELD THEORY

$$\Phi[X(\sigma)] \longrightarrow \Phi(x, \tilde{x}, \alpha, \bar{\alpha})$$

$$\rightarrow \sum \phi_A(x)\, f_A(\tilde{x}, \alpha, \bar{\alpha})$$

OR ? $\quad \sum \phi_A(x,\tilde{x})\, f_A(\alpha,\bar{\alpha})$

HITCHIN: GENERALIZED COMPLEX STRUCTURES ON T^*M, $O(n,n)$ STRUCTURE

IF TRANSITION FUNCTIONS MIX $x, \tilde{x}$
CAN'T USE $\phi(x)$, NEED $\phi(x, \tilde{x})$

DOUBLED FORMALISM

$$X^I = \begin{pmatrix} x^i \\ \tilde{x}^i \end{pmatrix}$$ COORDS ON T^{2n} $\quad y^\alpha$ COORDS ON BASE

$$L_{IJ} = \begin{pmatrix} 0 & \mathbb{1}_n \\ \mathbb{1}_n & 0 \end{pmatrix}$$

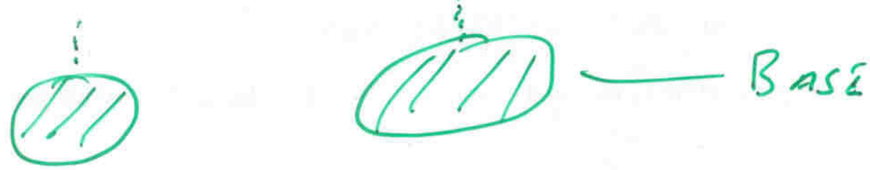
$$\mathcal{H}_{IJ}(G, B) = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

$$= \mathcal{V}\mathcal{V}^t$$

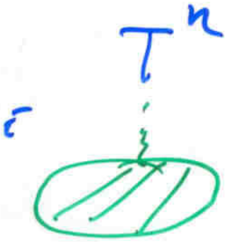
$\mathcal{V}$ PARAMETERIZES $\dfrac{O(n,n)}{O(n) \times O(n)}$

LOCALLY CAN PICK

$$T^n \subset T^{2n}$$



GET USUAL PICTURE ON SPACETIME



T-DUALITY: CHANGES SUBSPACE $T^n \subset T^{2n}$

GIVES DUAL PICTURE

e.g. $n=1$: $\quad T^2 = S^1_R \times S^1_{1/R}$

GLOBALLY

NEED CONSISTENT CHOICE $T^n \subset T^{2n}$

FOR EACH POINT IN BASE:-

NEED GLOBAL SECTION

NOT POSSIBLE IN GENERAL e.g. IF

MONODROMY IN $O(n,n;\mathbb{Z})$

NO "REAL" SPACETIME !

T^{2n} BUNDLE OVER BASE

INVARIANT L_{IJ} $\quad SL(2n;\mathbb{Z}) \to O(n,n;\mathbb{Z})$

$$(X^I, y^\alpha) = (x^i, \tilde{x}^i, y^\alpha)$$

$$V(y) \, , \quad C_I(y) = k_{I\alpha}\, dz^\alpha + v_{I\alpha} * dz^\alpha$$

$$L = \frac{1}{2} K_{IJ}\, \partial_a X^I \partial^a X^J + C_I \wedge * dX^I + \cdots$$

$O(n,n)$ INVARIANT IF C_I TRANSFORMS AS $\underset{\smile}{2n}$

CONSTRAINT

$$dX^I = L^{IJ} * \left(K_{JK}\, dX^K + C_J \right)$$

HALVES DEGREES OF FREEDOM

[cf TSEYTLIN; SCHWARZ, SEN, ...]

CONSTRAINT: SELF-DUALITY

$$P = \mathcal{V}^t dX$$

$O(n,n)$ SINGLET, TRANSFORMS UNDER $O(n) \times O(n)$

BASIS: $L = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $P = \begin{pmatrix} P_L \\ P_R \end{pmatrix}$

$O(n) \times O(n)$: $\begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix}$

$\Lambda_1 \quad \Lambda_2$

$\rightarrow$

$$P_L \simeq * P_L$$
$$P_R \simeq - * P_R$$

LEFT, RIGHT MOVERS

SPLIT X INTO L, R MOVERS

DEPENDS ON

$$\mathcal{V}[G(y), B(y), C(y)]$$

$O(n,n)$ ACTION ON

$K(G,B)$, $C(y) \rightarrow$ STANDARD BUSHER RULES FOR G, B, k, V

$$P^I = dX^I \, ;$$

$$dP^I = 0$$

$$P^I = \Pi^I{}_J * P^J$$

$\Rightarrow$ FIELD EQUATION

$$d(\Pi * P) = 0$$

USUAL DESCRIPTION:

"CHOOSE" HALF OF $P^I = \begin{pmatrix} p^i \\ \tilde{p}^i \end{pmatrix}$,

- SOLVE $dp^i = 0 \rightarrow p^i = dx^i$

$\rightarrow$ GET FIELD EQUATION FOR x^i

EQUIVALENT TO STANDARD σ-MODEL

$O(n,n)$ MIXES $(p^i, \tilde{p}^i) \Longrightarrow$

IF MONODROMY MIXES $p, \tilde{p}$,

NO WAY OF CHOOSING p

GLOBALLY, ALTHOUGH $\tilde{p}$

CAN ALWAYS FIND x LOCALLY p

– SOME STRING BACKGROUNDS HAVE NO SPACETIME FORMULATION, BUT CAN FORMULATE IN "DOUBLED" SPACE WITH T^{2n} FIBRATION

– LOCALLY T^n FIBRATION EMERGES, SPACETIME, BUT CAN'T PATCH

T-MONODROMY MIXES MOMENTUM + WINDING MODES.

U-MONODROMY MIXES IN BRANE WRAPPING MODES.

– FURTHER DIMENSIONS

$$\begin{array}{ccc} T^4 & \subset & T^{10} \\ \vdots & & \vdots \\ M_7 & & M_7 \end{array}$$

4 MOMENTA
6 WRAPPED MEMBRANES

– GENERALIZATION TO SPACES THAT ARE NOT TORUS FIBRATIONS?