S-MATRIX WITHOUT LORENTZ

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S-MATRIX THEORY NOT ALL THAT DAFT...

A FRAMEWORK ANY THEORY WITH A MASS GAP HAS TO SATISFY
(ONLY HOPE FOR UNIQUENESS OF NONTRIVIAL S TOO MUCH ?)

APOGEE:
FINITE ENERGY SUM RULES →
DUAL RESONANCE MODELS →
STRING THEORY

FROM A HYPERBOREAN PERSPECTIVE CAMBRIDGE WAS A NATURAL CHOICE IN 1970

TO GET DAVID AS A SUPERVISOR WAS A JACKPOT

1960's :

DAVID TRIED CAREFULLY TO ESTABLISH THE ANALYTICITY STRUCTURE OF THE S-MATRIX WITHOUT USING THE CRUTCHES OF QFT

PHYS. REV. 135 (1964) B 745 :

"RECENTLY SOME DEGREE OF UNDERSTANDING OF THE WORKING OF UNITARITY IN S-MATRIX THEORY HAS BEEN DEVELOPED... IN THIS SORT OF WORK A LARGE NUMBER OF PROPERTIES OR INGREDIENTS HAVE BEEN USED. APART FROM THE QUANTUM AND LORENTZ ASSUMPTIONS THESE ARE : (1) UNITARITY ... (13) CONNECTION BETWEEN SPIN AND STATISTICS."

WHAT IF NO LORENTZ ?

WILL USE CRUTCHES PROVIDED BY
NON-COMMUTATIVE FIELD THEORIES
(OR KOSTELECKÝ -TYPE THEORIES)

- "SPIN" AN INESSENTIAL COMPLICATION

DO ASSUME TRANSLATIONAL
INVARIANCE (E, $\vec{p}$ CONSERVED)

"ACTIVE" $\neq$ "PASSIVE"

OBSERVER LORENTZ INVARIANCE OK (WE ARE FREE TO CHOOSE REFERENCE FRAME AT WILL)

"PARTICLE LORENTZ SYMMETRY" (INVARIANCE UNDER L-TRANSFORMATIONS OF SYSTEM IN A FIXED OBSERVER REFERENCE FRAME) BROKEN

N.C.F.T :

$$\theta^{\mu\nu} \leftrightarrow \quad e^i \sim \theta^{0i}$$

$$b^i \sim \epsilon^{ijk}\theta_{jk}$$

OBSERVER L.I. :

CAN CHOOSE E.G. $\vec{e}, \vec{b}$ IN XZ-PLANE

ASYMPTOTIC STATES

$$|\vec{p}\,\rangle$$

DISPERSION RELATION

$$E(\vec{p}\,) \neq \sqrt{\vec{p}^{\,2}+m^2}$$

N.C.F.T. w. $\theta^{0i} = 0$

$$E = E\big(\vec{p}^{\,2}, \underbrace{\vec{p}\circ\vec{p}}_{\theta^{ij}\,\theta_{ik}\,p_j\,p^k}\big)$$

N.B. EVEN IF IN FREE N.C.F.T

$$S_{n.c.} = S_c \,, \quad \text{i.e.} \quad E_{free} = \sqrt{\vec{p}^{\,2}+m^2}$$

ASYMPTOTIC PARTICLES FEEL SELF-INTERACTION AND $E \neq \sqrt{\vec{p}^{\,2}+m^2}$

2 → 2 SCATTERING $a+b \to c+d$

(CHAICHIAN, TUREANU, C.M. hep-th 0305243)

USE OBSERVER L.I. TO GO TO A FRAME

$$\vec{p}_a + \vec{p}_b = \vec{0} = \vec{p}_c + \vec{p}_d \qquad \begin{array}{l} \vec{p}_a = \vec{p} \\ \vec{p}_c = \vec{p}\,' \end{array}$$

ENERGY CONSERVATION

$$E = E(\vec{p}) + E(-\vec{p}) = E(\vec{p}\,') + E(-\vec{p}\,')$$

CONSTRAINS $\vec{p}\,'$!

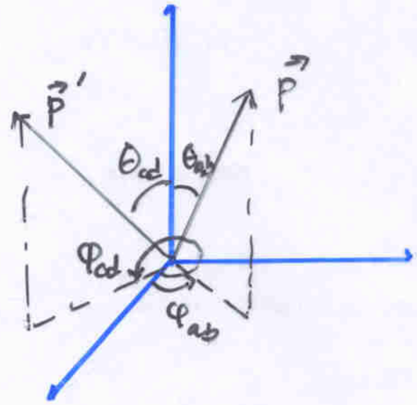
ASSUME IT FIXES $|\vec{p}\,'|$ REL. TO $|\vec{p}|$

(DOES NOT FIX)

⇒ AMPLITUDE DEPENDS ON 5 (6) VARS.

E (OR $|\vec{p}|$), θ_{ab}, φ_{ab}, θ_{cd}, φ_{cd}

$(+\ |\vec{p}\,'|)$



(IF $\vec{c} \parallel \vec{b}$ HAVE EXTRA SO(2) SYMMETRY, ROTATE AWAY ONE φ

⇒ 4 (5) VARIABLES)

ELASTIC UNITARITY

$$-i\left[a_{lm,l'm'}(E)-(-1)^{m+m'}a^*_{l',-m',\,l-m}(E)\right]$$

$$=\frac{|\vec{p}|}{4\pi E}\sum_{l_1,m_1}(-1)^{m'}a^*_{l'-m'\,l_1m_1}(E)\,a_{lm\,l_1m_1}(E)$$

(HERMITIAN ANALYTICITY ?

$M_{ab}(s)^* = M_{ba}(s^*)$ D.I.O. 1962)

PARTIAL WAVE ANALYSIS :

$$A(\vec{p}_a, \vec{p}_b ; \vec{p}_c \vec{p}_d) =$$

$$= 4\pi \sum_{\substack{l,l' \\ m,m'}} a_{lm,l'm'}(E,\dots)\, Y_{lm}(\theta_{ab}, \varphi_{ab})\, Y_{l'm'}(\theta_{cd}, \varphi_{cd})$$

$$\left(\vec{c} \parallel \vec{b} \; : \; a_{lm\, l'm'} = \delta_{m'0}\, a_{lm\, l'} \right)$$

UNITARITY (DEMAND!) BOUND
FOR $E(\vec{p}) = \sqrt{\vec{p}^2 + m^2}$ (WRONG!)

$$\sigma_{el} \leq \sigma_{tot}$$

$$\Rightarrow \; -i \sum_{\substack{l\, l' \\ m\, m'}} \left(a_{lm\, l'm'}(E) - (-1)^{m+m'} a^*_{l-m,\, l'-m'}(E) \right) Y_{lm}(\Omega_{ab}) Y_{l'm'}(\Omega_{ab})$$

$$\geq \frac{|\vec{p}|}{4\pi E} \sum_{\substack{l\, l'\, l_1 \\ m\, m'\, m_1}} a_{lm\, l_1 m_1}(E)\, a^*_{l'-m'\, l_1 m_1}(E) (-1)^{m'}\, Y_{lm}(\Omega_{ab}) Y_{l'm'}(\Omega_{ab})$$

$\Rightarrow$ e.g.

$$(-i) \sum_{l,m} (-1)^m \left(a_{lm,\, l-m}(E) - a^*_{l-m,\, lm}(E) \right)$$

$$\geq \frac{|\vec{p}|}{4\pi E} \sum_{\substack{l\, l_1 \\ m\, m_1}} (-1)^m \left| a_{lm\, l_1 m_1}(E) \right|^2$$

NUMBER OF VARIABLES FOR n-LEG ON-SHELL AMPLITUDE: (NO RESIDUAL CONTINUOUS SYMMETRY)

$$N = \underset{\substack{\uparrow \\ \#\text{ of momentum} \\ \text{components}}}{3n} - \overset{\substack{\text{Energy-mom. conserv.} \\ \downarrow}}{4} - \underset{\substack{\uparrow \\ \text{One } \vec{p} \text{ can be boosted to } 0 \\ \text{(Observer Lorentz invariance)}}}{3} = 3n - 7$$

BUT: If $\sum E_{in} = \sum E_{out}$ only determines a boundary for allowed values of momentum components

$$N = 3n - 6$$

(Happens e.g. for $E^2(\vec{p}) = \vec{p}^{\,2} + \vec{p} \circ \vec{p} + m^2$)

KÄLLÉN - LEHMANN REPRESENTATION

(LIAO & SIBOLD hep-th/0209221)

$$E = E(\vec{p}^{\,2}, \vec{p} \circ \vec{p})$$

$$i D_F(x) = \langle 0 | T(\varphi(x)\varphi(0)) | 0 \rangle$$

$$= \int \frac{d^4p}{(2\pi)^4} \int_0^\infty dm^2 \, \rho(m^2, \vec{p} \circ \vec{p}) \, \frac{i}{p^2 - m^2 + i\varepsilon} \, e^{-ip\cdot x}$$

$\Rightarrow$ FOR FIXED $\vec{p} \circ \vec{p}$ THE PROPAGATOR IS ANALYTIC IN $s = p^2$ EXCEPT FOR POLES AND A BRANCH CUT ON THE POSITIVE AXIS

FORWARD DISPERSION RELATIONS

LSZ - REDUCTION FORMULAE STILL VALID IF EXTERNAL LEG AMPUTATION OPERATORS $\Box + m^2$ REPLACED BY

$$K_x = \frac{\partial^2}{\partial t^2} + E^2(-i\vec{\nabla}_{\vec{x}})$$

E.G. $ab \to cd$

$$S_{fi} = \langle \vec{p}_c \, \vec{p}_d \text{ in} | \vec{p}_a \, \vec{p}_b \text{ in} \rangle +$$

$$+ i^2 \int d^4x \, d^4y \; f^*_{\vec{p}_d}(x) \, f_{\vec{p}_b}(y) \, \theta(x^0 - y^0) \langle \vec{p}_c | [j(x), j(y)] | \vec{p}_a \rangle$$

(+ polynomial terms)

where $\quad K_x \varphi(x) = j(x)$

$$f_{\vec{p}}(x) = \frac{1}{(2\pi)^{3/2}\sqrt{2E(\vec{p})}} \, e^{-i(E(\vec{p})t - \vec{p}\cdot\vec{x})}$$

TO PROCEED, NEED SOMETHING TO REPLACE MICROCAUSALITY:

$$[j(x), j(y)] = 0 \qquad (x-y)^2 < 0$$

FOR $\vec{e} \parallel \vec{b}$ ALVAREZ-GAUMÉ, BARBON & ZWICKY (hep-th/0103069) PROPOSE ($\theta^{12} \neq 0$, $\theta^{03} \neq 0$)

$$[j(x), j(y)] = 0 \quad \text{IF} \quad (x^0-y^0)^2 - (x^3-y^3)^2 < 0$$

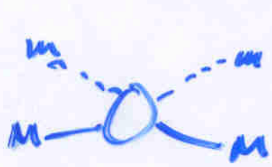
ASSUMING THIS ($\&$ $\theta^{03} = 0$) CHAICHIAN ET AL. (hep-th 0306158, 0403032) SHOW FOR

$$F(E,\vec{q}) = \int d^4x\, e^{i(Ex^0 - \vec{q}\cdot\vec{x})}\, \theta(x^0) \langle M | [j(\tfrac{x}{2}), j(-\tfrac{x}{2})] | M \rangle$$

$$F(E) = \tfrac{1}{2}\left(F(E,\vec{q}) + F(E,-\vec{q})\right)$$

* $\left(E^2 - \frac{m^4}{4M^2}\right) F(E)$ ANALYTIC IN E-PLANE EXCLUDING CUTS $(-\infty, -m)$; (m, ∞)

* $$F(E) = \frac{2E^n}{\pi} \int_m^\infty dE' \frac{\operatorname{Im} F(E')}{(E')^{n-1}(E'^2 - E^2)} + \sum_{k\ \text{even}}^{n-2} C_k E^k + \text{pole terms}$$



* NO CONCLUSIONS ON ANALYTICITY IN $\cos\theta$ (SCATTERING ANGLE)

* HOWEVER, IF ASSUME

$$[j(x), j(0)] = 0$$

FOR $x_0^2 - x_3^2 - (x_1^2 + x_2^2 - \ell^2) < 0$

$(\ell^2 \propto \theta)$

AND A SPECTRAL CONDITION

$$p_0^2 - p_3^2 - (p_1^2 + p_2^2) \geq 0 \,, \quad p_0 > 0$$

GET ANALYTICITY IN LEHMANN ELLIPSE

* NO EXTENSION TO MARTIN ELLIPSE SEEMS POSSIBLE