2-d CHARGED BLACK HOLE

COSMOLOGY

SINGULARITIES ALGEBRA

CO-SETS

$$G/H \qquad C_{G/H} = C_G - C_H$$

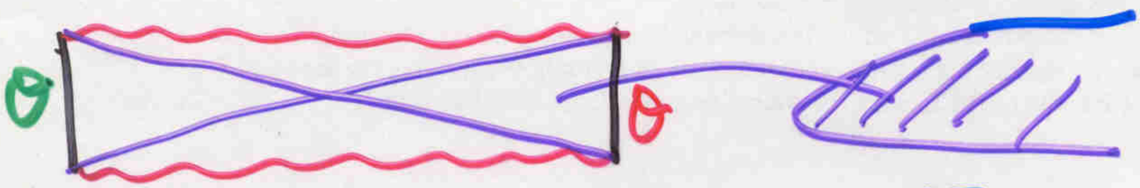
DAVID OLIVE

A. GIVEON, A. KONECHNY, A. SEVER

VICTIM?

OVER-ACHIEVER?

L E



T

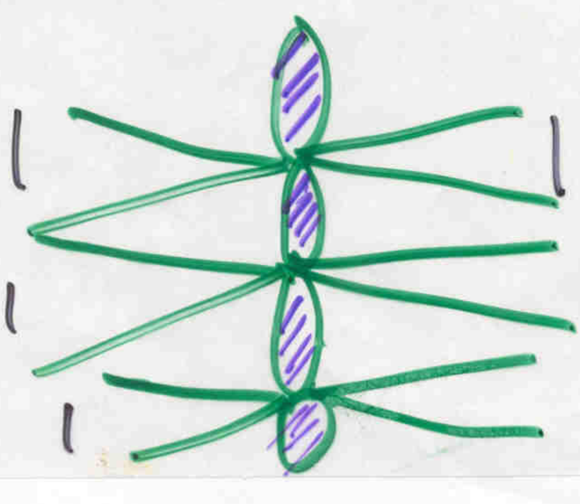
(β_Q)

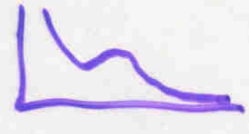
$H_1 \times H_2$

θθ θθ ~ THERMAL

$\theta\theta \sim \theta(t_1)\theta(t_1 + \frac{i\beta}{2})$

$$\sum \exp\left(-\frac{\beta E_n}{2}\right)|n\rangle_1 |n\rangle_2$$





COMPACT

?

T ?

2-d DILATONIC CHARGED BLACK HOLES

COMPACT

$$\frac{SL(2,R) \times U(1)}{U(1)} \xrightarrow[3]{} \mathcal{L}(G', B', \phi', A)$$

NON COMPACT

$\xrightarrow[KK]{}$ 2-d CBHD

TECHNOLOGY!

OLIVE SL(2,R) × U(1) MANIFOLD

GAUGE BY A U(1) SUCH THAT:

$(g, x) \in SL(2,R) \times U(1) \quad X \equiv X + 2\pi L$

$$(g, x_L, x_R) \xrightarrow[\text{GAUGE}]{} \left(\exp\left(\frac{\rho \sigma_3}{\sqrt{k}}\right) g \exp\left(\frac{\tau \sigma_3}{\sqrt{k}}\right), \right.$$

$$\left. x_L + \rho', \; x_R + \tau' \right)$$

k IS LEVEL OF SL(2,R) ALA

FOR A SINGLE AND NON-ANOMALOUS

U(1); $\underline{\tau} \equiv (\tau, \tau') \overset{\text{i.e.}}{=} |\tau| \, (1,0)$ $\underline{u}$

$(\rho, \rho') \equiv \underline{\rho} = R \underline{\tau} \qquad R \in SO(2)$

$$R = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix}$$

$$S = \overset{SL(2,\mathbb{R})}{S}\left[e^{x/\sqrt{k}\,\hat{\underline{\rho}}\,\sigma_3/\sqrt{k}}\, g\, e^{x/\sqrt{k}\,\hat{\underline{\tau}}\,\sigma_3/\sqrt{k}} \right]$$

$$+ \overset{U(1)}{S'}\left[x + \hat{\rho}' + \hat{\tau}' \right] - \frac{1}{2\pi}\int d^2z \left(\partial \hat{\underline{\rho}} - R\,\partial \hat{\underline{\tau}}\right)^T \left(\bar{\partial} \hat{\underline{\rho}} - R\,\bar{\partial} \hat{\underline{\tau}}\right)$$

$$S[g] = \frac{k}{4\pi}\left[\mathrm{Tr}\int_{\Sigma} g^{-1}\partial g\, g^{-1}\bar{\partial} g - \frac{1}{3}\int_{B} \mathrm{Tr}\left(g^{-1} dg\right)^3 \right]$$

$$S'[x] = \frac{1}{2\pi}\int_{\Sigma} \partial x\, \bar{\partial} x$$

CONSTRAINT ON $\hat{\tau}, \hat{\rho}$

$\underset{\text{i.e.}}{u =} (1,0)$ $\quad \hat{\underline{\tau}} = \left(\hat{\underline{\tau}}^T \underline{u}\right) \underline{u} \qquad \hat{\underline{\rho}} = \left(\hat{\underline{\rho}}^T R \underline{u}\right) R \underline{u}$

ACTION IS INVARIANT UNDER

$$\hat{\underline{\rho}} \to \hat{\underline{\rho}} - \underline{\rho}$$

$$\hat{\underline{\tau}} \to \hat{\underline{\tau}} - \underline{\tau}$$

WITH RELATED $\underline{\tau}, \underline{\rho}$

THE ACTION DEPENDS ON $\hat{\underline{\rho}}, \hat{\underline{\tau}}$

ONLY THROUGH

$$A = \underline{u}^T \partial \hat{\underline{\tau}}$$

$$\bar{A} = (R\underline{u})^T \bar{\partial} \hat{\underline{\rho}}$$

$$S = S[g] + S'[x] +$$

$$\frac{1}{2\pi} \int d^2 z \quad A \bar{J}^T \underline{u} + \bar{A} J^T R\underline{u} + 2A\bar{A}(R\underline{u})^T M \underline{u}$$

WITH

$$J^T = \left(\sqrt{k}\, \mathrm{Tr}\left[\partial g g^{-1} \sigma_3\right], 2\partial X\right]$$

$$\bar{J}^T = \left(\sqrt{k}\, \mathrm{Tr}\left(g^{-1}\bar{\partial} g\, \sigma_3\right], 2\bar{\partial} X\right]$$

$$\underset{2\times 2}{M} = \begin{pmatrix} \frac{1}{2}\mathrm{Tr}\left(g^{-1}\sigma_3 g \sigma_3\right) & 0 \\ 0 & 1 \end{pmatrix}_{2\times 2} + R_{2\times 2}$$

$$\Rightarrow S = S[g] + S'[X]$$

$$+ \frac{1}{2\pi}\int d^2 z \left\{ \left(\bar{A} + \frac{\bar{J}^T \underline{u}}{2(R\underline{u})^T M \underline{u}}\right) 2(R\underline{u})^T M \underline{u} \left(A + \frac{J^T R \underline{u}}{2(R\underline{u})^T M \underline{u}}\right) \right.$$

$$\left. - \frac{(\bar{J}^T \underline{u})(J^T R \underline{u})}{2(R\underline{u})^T M \underline{u}} \right\}$$

INTEGRATE OUT $A\ \bar{A}$ AND *

$$S = S[g] + S'[x] - \frac{1}{4\pi}\int d^2 z \, \frac{\bar{J}^T \underline{u}\ J^T R \underline{u}}{(R\underline{u})^T M \underline{u}}$$

$$\phi = \phi_0 - \frac{1}{2}\log\left((R\underline{u})^T M \underline{u}\right)$$

$$SL(2,\mathbb{R}) \ni g(\alpha,\beta,\gamma;\varepsilon_1,\varepsilon_2,\delta) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad ad-bc=1$$

$$\| $$

$$\exp(\alpha\sigma_3)(-1)^{\varepsilon_1}(i\sigma_2)^{\varepsilon_2}\, g_\delta(\theta)\exp(\beta\sigma_3)$$

$$\varepsilon_{1,2} = 0,1 \qquad \delta = I, 1, 1'$$

NON COMPACT

$$g_I = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$g_1 = g_{1'}^{-1} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$$

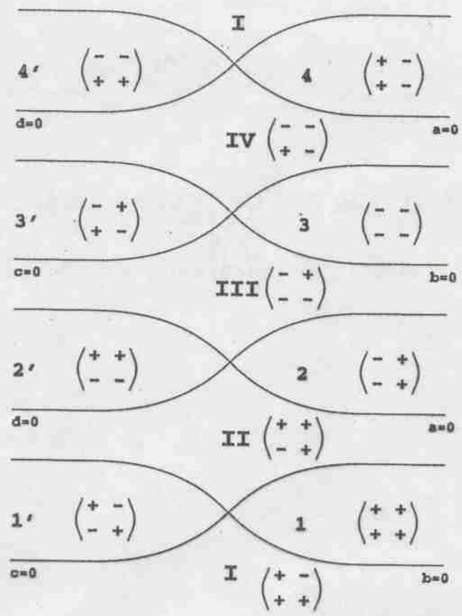


Figure 3: A two dimensional slice of $SL(2,\mathbb{R})$.

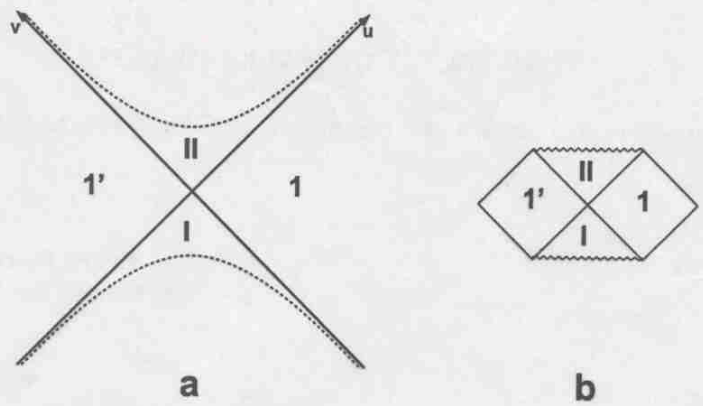
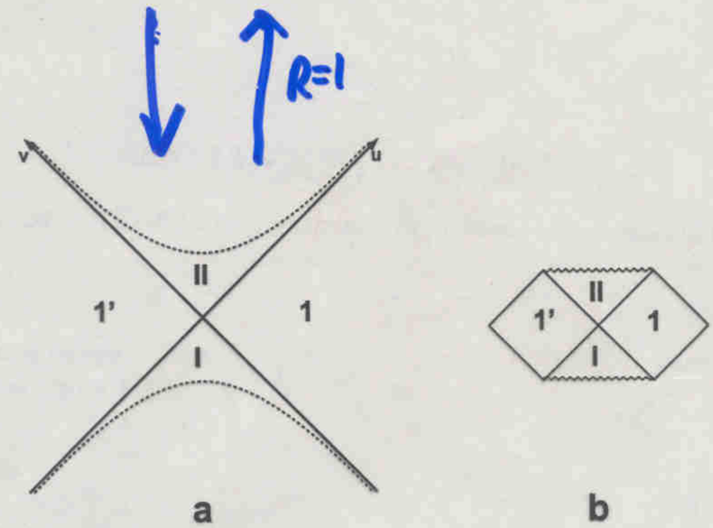


Figure 1: **a**: Kruskal diagram and **b**: Penrose diagram of the Schwarzshild or the 2-d black hole.

gauge invariant vertex operators in the $SL(2,\mathbb{R})$ CFT. As we shall describe later, one



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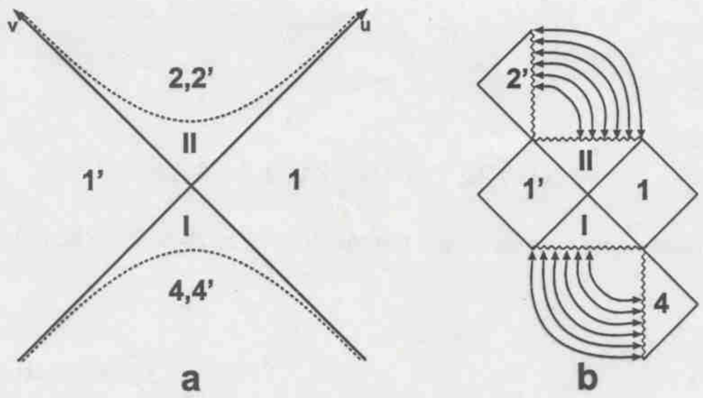


Figure 4: a: The 2 dimensional black hole ($\psi = 0$): the solid lines are horizons and the dashed lines

$uv = 1$

SINGULARITIES $\xrightarrow{\psi \neq 0}$ INNER HORIZON

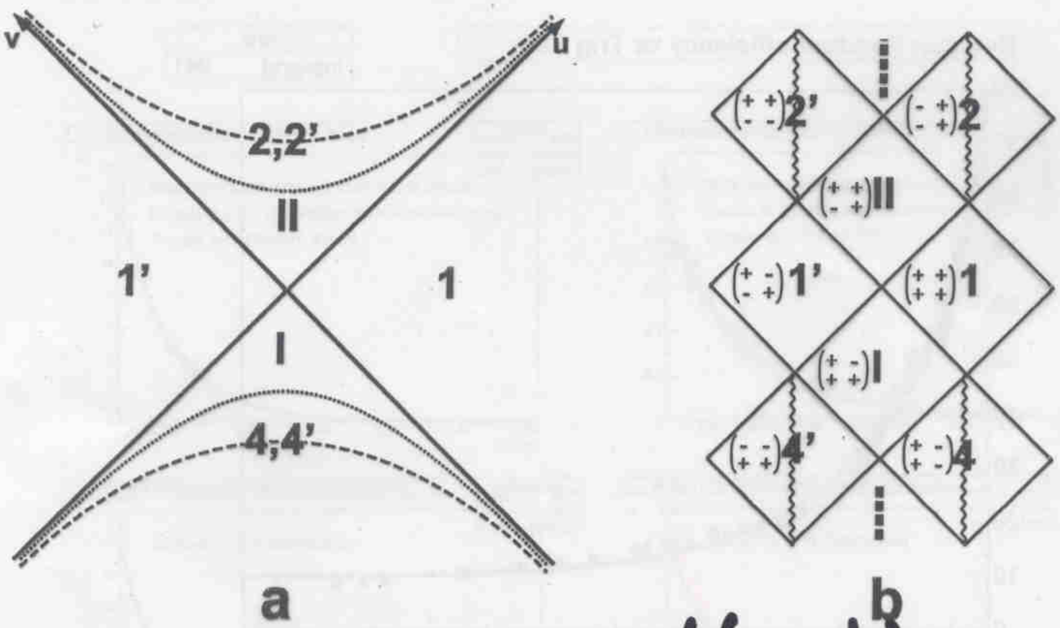
$\psi = 0$

FIXED LINE OF G. T

$\psi \neq 0$

NULL GAUGE ORBIT

$\lambda \equiv \tan \frac{\psi}{2}$

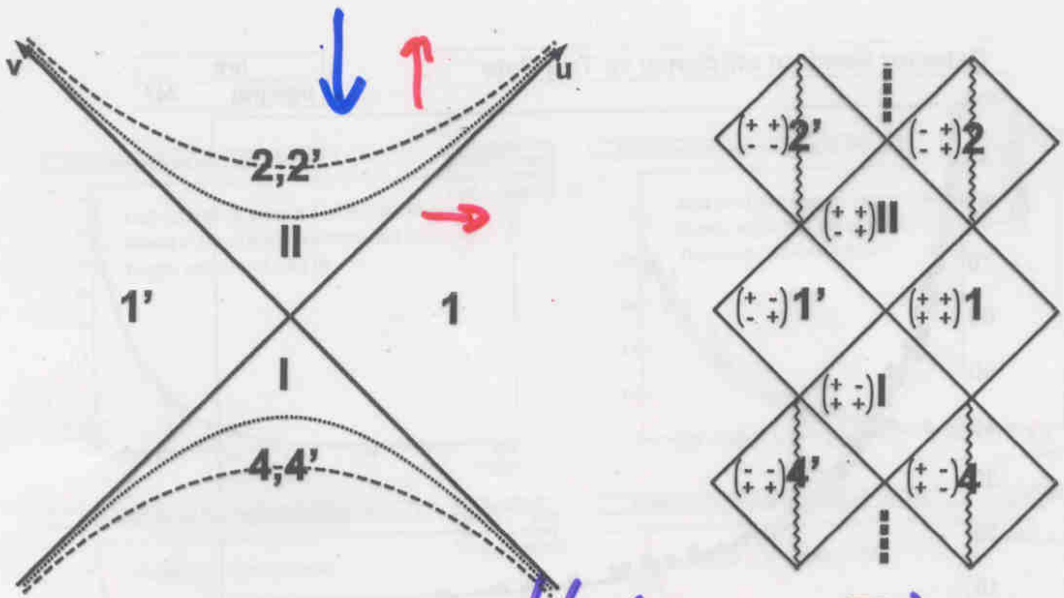


$$M_{ADM} = 4m\exp(-2\phi_0)$$

$$Q_{el} = 2\sqrt{2}\, q \exp(-2\phi_0)$$

$$2m = \frac{1+\mu^2}{1-\mu^2} \qquad q = \frac{\mu}{1-\mu^2}$$

μ CONTROLS



$$1 \geq \left|R(j, m, \bar{m})\right|^2 = \frac{\cosh(\pi(2s - m - \bar{m})) + \cosh(\pi(m-\bar{m}))}{\cosh(\pi(2s + m + \bar{m})) + \cosh(\pi(m-\bar{m}))}$$

so ?

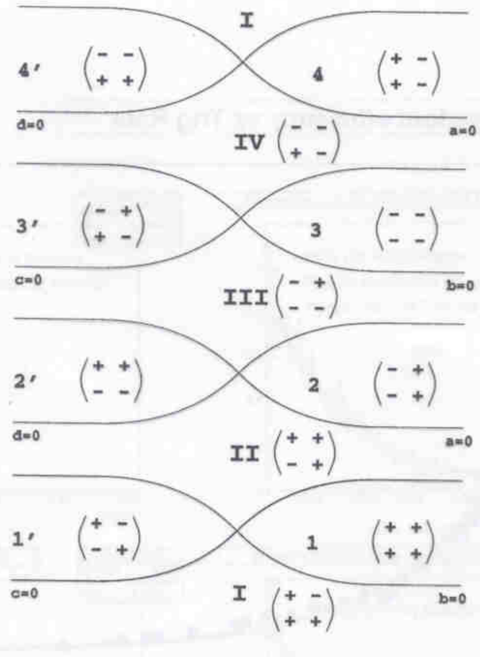


Figure 3: A two dimensional slice of $SL(2,\mathbb{R})$.

$$\Phi = \Phi_0 - \frac{1}{2}\log\left((R\underline{u})^T M\underline{u}\right) , \tag{25}$$

where $|W| \leq 2$. In the regions where $W > 2$, θ_B in (24), (25) should be replaced by $i\theta_A$. In the regions with $W < -2$, substitute $i\theta_C$ for $\theta_B - \frac{\pi}{2}$:

$$\begin{array}{cccc} B & |W| \leq 2\ , & I, II, III, IV\ : & \theta_B \\ A & W > 2\ , & 1, 1', 3, 3'\ : & \theta_B \rightarrow i\theta_A \\ C & W < -2\ , & 2, 2', 4, 4'\ : & \theta_B \rightarrow i\theta_C + \frac{\pi}{2}\ . \end{array} \tag{26}$$

If we take the vector

$$\underline{u}^T = (1, 0) , \tag{27}$$

then G_{xx} is constant [4] and after rescaling $x \rightarrow \sqrt{k}x$ the action and the dilaton become:

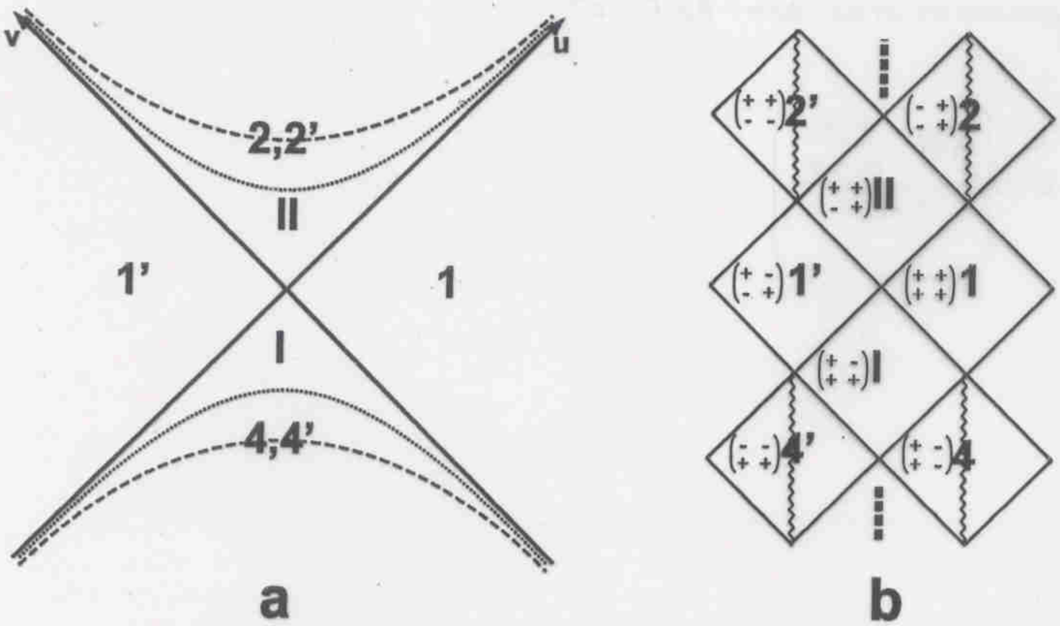
$$\begin{aligned} S =& \frac{k}{2\pi}\int_\Sigma \partial x\bar{\partial}x + \frac{k}{2\pi}\int d^2z \left[-\partial\theta_B\bar{\partial}\theta_B + \sin^2(\theta_B)\partial y\bar{\partial}y\right] + \\ &+ \frac{k}{\pi}\int d^2z \frac{\sin^2(\theta_B)\bar{\partial}y\left(\sin^2(\theta_B)\cos(\psi)\partial y - \sin(\psi)\partial x\right)}{1+\cos(\psi)\cos(2\theta_B)} = \\ =& \frac{k}{2\pi}\int d^2z\left[-\partial\theta_B\bar{\partial}\theta_B + \frac{\partial y\bar{\partial}y - 2p\bar{\partial}y\partial x}{\cot^2(\theta_B)+p^2} + \partial x\bar{\partial}x\right] \end{aligned} \tag{28}$$

$$\Phi = \tilde{\Phi}_0 - \frac{1}{2}\log\left(\cos^2(\theta_B) + p^2\sin^2(\theta_B)\right) , \quad \tilde{\Phi}_0 \equiv \Phi_0 + \frac{1}{2}\log\left(\frac{1+p^2}{2}\right) , \tag{29}$$

where

$$p \equiv \tan(\frac{\psi}{2}) . \tag{30}$$

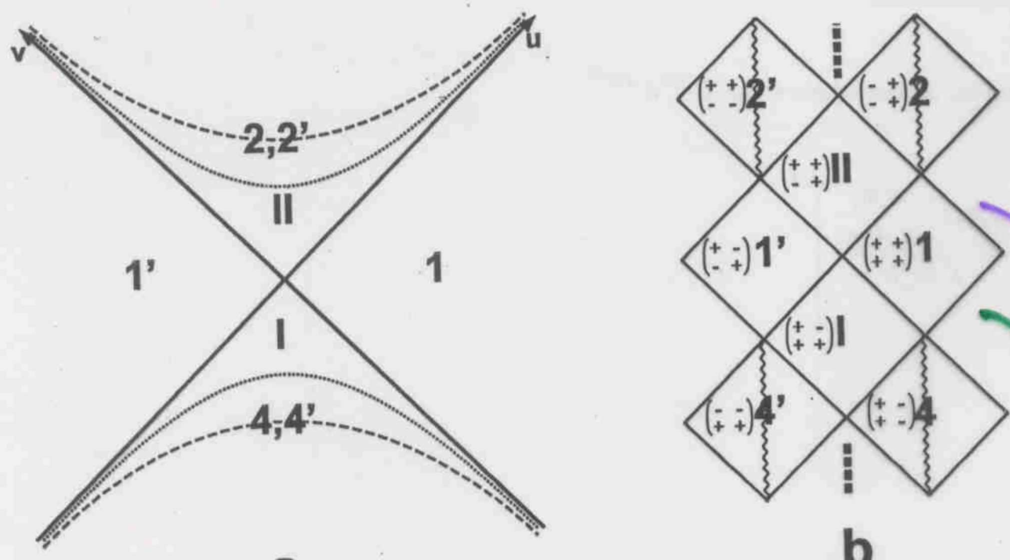
[4] Actually, $G_{xx} = const$ iff $(G+B)_{yx} = 0$ and, therefore, in this case the $\frac{SL(2)\times U(1)}{U(1)}$ background can be used in the heterotic string.



T DUALITY

$\psi_E \leftrightarrow \pi i - \psi_E$

$\rho_E^2 \to \frac{1}{\rho_E^2}$



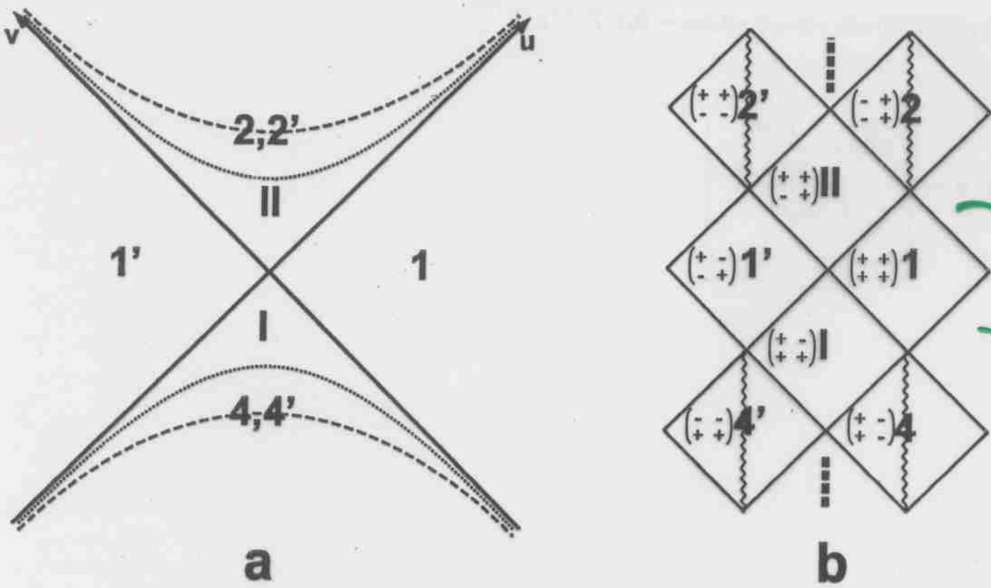
a   b

$$\frac{1}{k}\, ds^2 = d\theta^2 - \frac{\coth^2\theta}{(\coth^2\theta - p^2)^2}\, dy^2$$

$$\frac{1}{k}\, ds^2 = d\theta^2 - \frac{\tanh^2\theta}{(\tanh^2\theta - p^2)^2}\, dy^2$$

$$p \equiv \tan\left(\frac{\psi}{2}\right)$$

i.e $0 \leq \psi \leq \frac{\pi}{2}$ $\quad p^2 < 1$ SINGULARITY SHORT



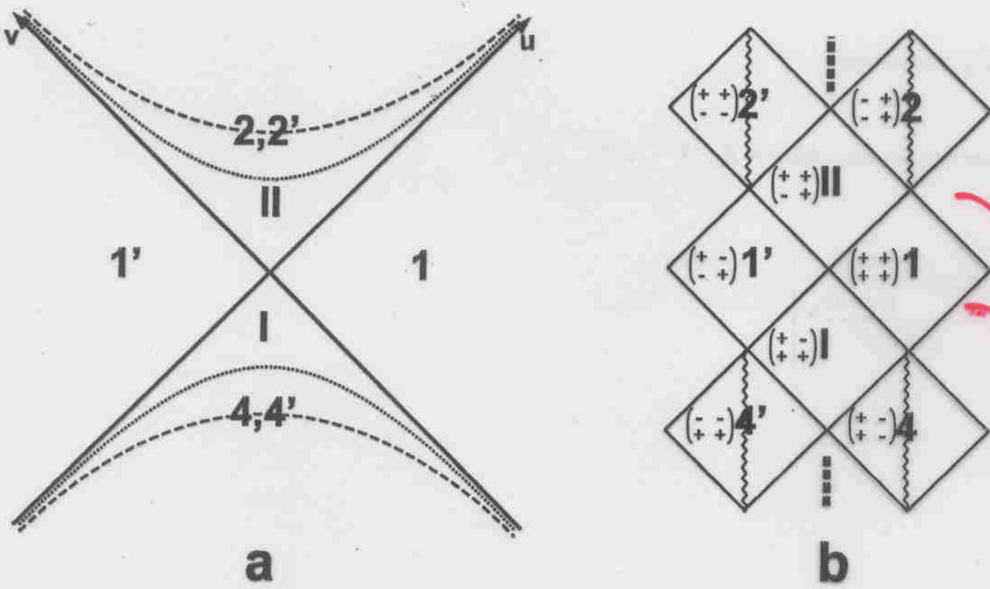
$$\phi = \tilde{\phi}_0 - \frac{1}{2}\log\left(\cosh^2\theta - \ell^2 \sinh^2\theta\right)$$

$$A_y = \frac{\sqrt{k}\,\ell}{\ell^2 - \coth^2\theta}$$

$$\phi = \check{\phi}_0 - \frac{1}{2}\log\left(\ell^2\cosh^2\theta - \sinh^2\theta\right)$$

$$A_y = \frac{\sqrt{k}\,\ell\,\tanh^2(\theta)}{\ell^2 - \tanh^2\theta}$$

$$\tilde{\phi}_0 \equiv \phi_0 + \frac{1}{2}\log\left(\frac{1+\ell^2}{2}\right)$$



$$\frac{1}{k}\, ds^2 = dr^2 + \frac{\coth^2(r)}{\coth^2 r + \hbar_E^2}\, d\phi^2 \qquad \begin{aligned} y &\to i\phi \\ \psi &\to i\psi_E \\ \hbar^2 &\to -\hbar_E^2 \end{aligned}$$

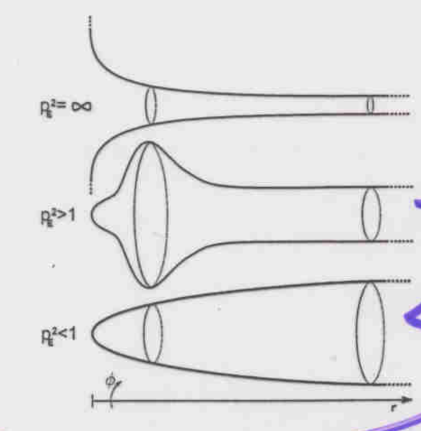
$$\frac{1}{k}\, ds^2 = dr^2 + \frac{\tanh^2 r\,(\hbar_E^2)^2}{(\tanh^2 r + \hbar_E^2)^2}\, d\phi^2 \qquad \begin{aligned} y &\to i\hbar_E^2 \\ \psi &\to i\psi_E \\ \hbar^2 &\to -\hbar \end{aligned}$$

$$\frac{1}{k}\, ds^2 = \cancel{f} - f(r)\, dt^2 + f^{-1}(r)\, dr^2$$

$$f(r) = 1 - 2m \exp(-2r) + q^2 \exp(-4r)$$

$$f(r_\pm) = 0 \qquad r_\pm = \frac{m \pm \sqrt{m^2 - q^2}}{q^2}$$

$$T_A = \frac{r_+ - r_-}{r_+}\, \frac{1}{2\pi\sqrt{\alpha' k}} \qquad T_B = \frac{r_+ - r_-}{r_-}\, \frac{1}{2\pi\sqrt{\alpha' k}}$$



$$\rightarrow T_C = \frac{1-h^2}{h^2} \cdot \frac{1}{2\pi\sqrt{\alpha' k}}$$

$$T_A = \frac{1-h^2}{2\pi\sqrt{\alpha' k}}$$

FOR A GIVEN h ONE KNOWS WHERE ONE IS.

$h \to 0$

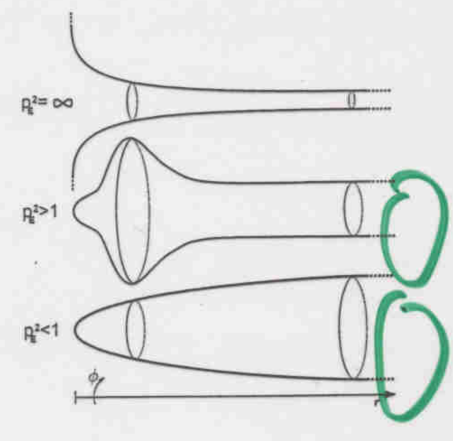
Rescale Euc. time by h_E^2

WILSON LOOP $\int d\phi\, A_\phi(r\to\infty) = \beta\mu_{el}$

$$A_\phi^{(A)}(r\to\infty) = A_\phi^{(B)}(r\to\infty) = \frac{\sqrt{k}\,\lambda_E}{\lambda_E^2+1}$$

$$r_A = \frac{\lambda}{\sqrt{\alpha}} = \frac{q}{\sqrt{\alpha}\, r_+} \;;\quad r_c = \frac{1}{\sqrt{\alpha}\,\lambda} = \frac{q}{\sqrt{\alpha}\, r_-}$$

$d \geq 4$
KERR



WILSON LOOP

STRING FIXED

F.T. $A_\phi(\theta=0)$ FIXED

REGULARITY, METRIC, A_μ

* GAUGE → ANALYTICAL CONT:

** ANALYTICAL CONT OF GAUGING

$$(g, X_L, X_R) \longrightarrow \left(\exp\left(\frac{\rho\, i\sigma_2}{\sqrt{2}}\right) g \exp\left(\frac{\tau\, i\sigma_2}{\sqrt{2}}\eta\right), X_L + \rho', X_R + \tau' \right)$$

TIME LIKE COMPACT

$\eta = 1$ AXIAL 1 1'

$\eta = -1$ VECTOR 2 2'

ANOMALY FREE ⇔

$$\underline{\rho} = R\, \underline{\tau}$$

$$R = \begin{pmatrix} \cosh \psi_E & \sinh \psi_E \\ \sinh \psi_E & \cosh \psi_E \end{pmatrix}$$

COSMOLOGY

$$\frac{SL(2,R) \times SU(2)}{U(1) \times U(1)}$$

OLIVE

N.C

4 Cosmology. Nappi-Witten solution.

A close coset CFT relative of the charged black hole is the cosmological Nappi-Witten background [6]. Recently this background was studied in more detail in [1], [2]. This background is obtained from a coset CFT

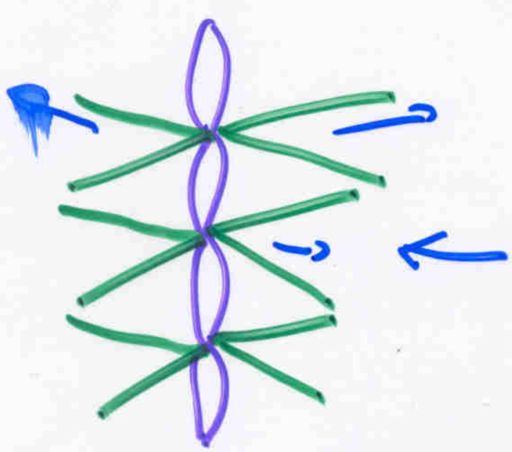
$$SL(2,\mathbb{R}) \times SU(2)/U(1) \times U(1)$$

Let $(g_1, g_2) \in SL(2,\mathbb{R}) \times SU(2)$. The nonanomalous $U(1) \times U(1)$ group action is chosen as

$$\begin{aligned} \delta g_1 &= \epsilon\sigma_3 g_1 + (\tilde{\epsilon}\cos(\alpha) - \epsilon\sin(\alpha)) g_1 \sigma_3 \\ paper\delta g_2 &= i\tilde{\epsilon}\sigma_3 g_2 + (\tilde{\epsilon}\sin(\alpha) + \epsilon\cos(\alpha)) g_2 i\sigma_3 \end{aligned}$$

where α is an angular variable analogous to the parameter ψ that labels the charged black hole backgrounds.

The four-dimensional space-time manifold described by the corresponding gauged WZW contains 12 regions (see [1]) for details) cyclically repeated in the maximally extended solution. Four out of 12 of these regions are time-dependent and decribe a universe evolving from big bang to a big crunch singularity. The remaining 8 regions are static, contain closed



me-like curves and are usually referred to as "whiskers" [1]. In addition to the presence of closed time-like curves the whiskers contain a time-like singularity surface called a domain wall in [1].

Explicitly the background in a whisker is described as follows. Let us choose a parameterization of the corresponding submanifold of $SL(2,\mathbb{R}) \times SU(2)$ as (this corresponds to the $\epsilon = \epsilon' = 0$, $\delta = I$ region in the notation of [1])

$$g_1 = e^{\gamma\sigma_3} e^{\theta\sigma_1} e^{\beta\sigma_3}\,, \qquad g_2 = e^{i\gamma'\sigma_3} e^{i\theta'\sigma_2} e^{i\beta'\sigma_3}$$

Here $g_1 \in SL(2,\mathbb{R})$, $g_2 \in SU(2)$. We choose the gauge fixing condition $\gamma = \beta = 0$. There are no residual gauge transformations in this case as γ and β are noncompact. The metric in such a whisker can be derived to be

$$\frac{ds^2}{k} = (d\theta)^2 + (d\theta')^2 + g_{\lambda_+\lambda_+}(d\lambda_+)^2 + g_{\lambda_-\lambda_-}(d\lambda_-)^2 \tag{28}$$ L_metr

$$\begin{aligned} g_{\lambda_-\lambda_-} &= -\frac{\tanh^2(\theta)}{b^2 - \tanh^2(\theta)\cot^2(\theta')} \\ g_{\lambda_+\lambda_+} &= \frac{b^2 cot^2(\theta')}{b^2 - \tanh^2(\theta)\cot^2(\theta')} \\ b^2 &= \frac{1-\sin(\alpha)}{1+\sin(\alpha)} \end{aligned} \tag{29}$$ L_metr

Here $\lambda_\pm = \gamma' \pm \beta'$ have periodicity of 2π. As evident from the form of the metric shifts of the coordinates $\lambda_\pm$ generate two commuting isometries.

In addition there are nontrivial B-field and dilaton backgrounds

$$B_{\lambda_+\lambda_-} = \frac{kb^2}{b^2 - \tanh^2(\theta)\cot^2(\theta')}\,, \tag{30}$$ B

$$\Phi = \Phi_0 - \frac{1}{2}\log(\cosh^2(\theta)\sin^2(\theta') - b^2\sinh^2(\theta)\cos^2(\theta'))\,. \tag{31}$$ D

The surface specified by equation

$$b^2 - \tanh^2(\theta)\cot^2(\theta') = 0$$

is a curvature singularity to which we refer to as a singular domain wall.

In parallel with our discussion of Euclidean charged black hole background one may try to define a Euclidean space by Wick rotating the Killing coordinates $\lambda_\pm$ and the parameter α:

$$\lambda_+ \to i\tilde{\lambda}_+\,, \quad \lambda_- \to i\tilde{\lambda}_-\,, \quad \alpha \to \frac{\pi}{2} + i\alpha_E \tag{32}$$ Wick

where we included a shift by $\pi/2$ in the last transformation solely for the sake of later convenience. After such a rotation we obtain a Euclidean signature space with metric

$$\begin{aligned} \frac{ds^2}{k} &= (d\theta)^2 + (d\theta')^2 + \frac{\tanh^2(\theta)(d\tilde{\lambda}_-)^2}{b_E^2 + \tanh^2(\theta)\cot^2(\theta')} + \frac{b_E^2 cot^2(\theta')(d\tilde{\lambda}_+)^2}{b_E^2 + \tanh^2(\theta)\cot^2(\theta')} \\ b_E^2 &= \frac{\cosh(\alpha_E)+1}{\cosh(\alpha_E)-1} = \tanh^2\left(\frac{\alpha_E}{2}\right)\,. \end{aligned} \tag{33}$$ E_metr

We see that the domain wall has disappeared. The subspace $\theta = \theta' = 0$ is singular, it has a trumpet-like curvature singularity. There are also potential conical singularities at $\theta' = 0$ and $\theta' = \pi/2$ subspaces when the coordinates $\tilde{\lambda}_-$ or $\tilde{\lambda}_+$ respectively shrink to zero size. The pereodicities of coordinates $\tilde{\lambda}_\pm$ are not yet fixed though. We may fix them by requiring the absence of conical singularities. That means that the periods of both $\tilde{\lambda}_\pm$ should be 2π.

In the asymptotic region $\theta \to \infty$ the metric (33) takes the form

$$\frac{ds^2}{k} \approx (d\theta)^2 + (d\theta')^2 + \frac{(d\tilde{\lambda}_-)^2}{b_E^2 + \cot^2(\theta')} + \frac{b_E^2 cot^2(\theta')(d\tilde{\lambda}_+)^2}{b_E^2 + \cot^2(\theta')}$$

NON ISOTROPIC T !

that is uniformly bounded in both $\tilde{\lambda}_\pm$ directions. This suggests that this Euclidean solution defines a vacuum state in the original Minkowski signature space characterized by a canonical distribution in the eigenvalues of Killing vectors $\frac{\partial}{\partial\tilde{\lambda}_\pm}$. With one of the Killing vectors being time-like the corresponding distribution should be thermal. The dependence of the asymptotic sizes on θ' presumably can be interpreted in terms of the high anisotropy of the outgoing thermal radiation.

5 Euclidean $SL(2,\mathbb{R}) \times SU(2)/(U(1) \times U(1))$ CFT.

In the case of charged black holes the Euclidean background obtained after Wick rotation was also obtainable directly from the corresponding gauged WZW model. We can try the same strategy for finding the Euclidean solution for Nappi-Witten background. Namely let us start with $SL(2,\mathbb{R}) \times SU(2)$ WZW model and gauge away one time-like and one space-like $U(1)$ so that the remaining space is a four-dimensional Euclidean one. A general nonanomalous $U(1) \times U(1)$ action of this kind has the form

$$\begin{aligned} \delta g_1 &= \epsilon i\sigma_2 g_1 + (\tilde{\epsilon}\eta_1\eta_2 \sinh(\alpha_E) + \epsilon\eta_1 \cosh(\alpha_E)) g_1 i\sigma_2 \\ \delta g_2 &= i\tilde{\epsilon}\sigma_3 g_2 + (\tilde{\epsilon}\eta_2 \cosh(\alpha_E) + \epsilon \sinh(\alpha_E)) g_2 i\sigma_3 \end{aligned} \qquad (34)$$ Eg_tr

Here $\eta_1, \eta_2 = \pm 1$ are discrete parameters corresponding to the axial/vector choice of gaugings on $SL(2,\mathbb{R})$ and $SU(2)$. As we hope to obtain a Nappi-Witten coset model after Wick rotation that includes rotation of the mixing angle $\alpha_E \to i\alpha$ we should be choosing $\eta_1 = 1$, $\eta_2 = -1$. We will restrict our considerations to this case below. We choose parameterizations

$$\begin{aligned} g_1 &= e^{i\alpha\sigma_2} e^{\theta\sigma_3} e^{\beta i\sigma_2} \\ g_2 &= e^{i\alpha'\sigma_3} e^{i\theta'\sigma_2} e^{i\beta'\sigma_3} . \end{aligned}$$

We can perform initial gauge fixing by imposing the condition $\alpha = \beta = 0$. Now however, in contrast with Minkowski signature case, the coordinates $\alpha, \beta, \alpha', \beta'$ are all noncompact. It is easy to derive from (34) that the residual gauge transformations preserving $\alpha = \beta = 0$ are generated by the shifts

$$\begin{aligned} \tilde{\lambda}_- &\to \tilde{\lambda}_- + 2\pi b_E , \\ \tilde{\lambda}_+ &\to \tilde{\lambda}_+ + \frac{2\pi}{b_E} \end{aligned} \qquad (35)$$ res_g

where

$$\tilde{\lambda}_\pm = \alpha' \pm \beta'$$

$SU(2,R) \times SU(2)$ G.I.

$m, \bar{m}$ $m' \bar{m}'$

j j'

$m - \bar{m} \in \mathbb{Z}$

$$m + \bar{m}\cosh(\alpha_E) + \bar{m}'\sinh(\alpha_E) = 0$$

$$m' - \bar{m}\sinh(\alpha_E) + \bar{m}'\cosh(\alpha_E) = 0$$

$$\underset{\text{INTEGER}}{m - \bar{m}} = \underbrace{\frac{1 + \cosh\alpha_E}{\sinh\alpha_E}}_{\text{"}\Rightarrow\text{" INTEGER RATIONAL}} \underset{\text{INTEGER}}{(\bar{m}' - m')}$$

$\alpha \to i\alpha_E + \frac{\pi}{2}$

$\Rightarrow$ * $m = \bar{m} = 0$ $\bar{m}' = m' = 0$

** $\coth\left(\frac{\alpha_E}{2}\right) \left(= \frac{1}{b_E}\right) =$ INTEGER/RATIONAL

EVEN SO ASSUME $b_E = P/Q$ RELATIVELY PRIME

$$\tilde{\lambda}_- = \tilde{\lambda}_- + \frac{2\pi}{Q} \qquad \tilde{\lambda}_+ = \tilde{\lambda}_+ + \frac{2\pi}{P}$$

THE METRIC (41)

HAS CONICAL SINGULARITIES

AT $\theta' = 0$ AND $\theta = 0$

DEFICIT $\frac{2\pi}{Q}$ $\frac{2\pi}{P}$

SO NEED $b_E = \frac{1}{1} = 1$ $(\alpha_E \to \infty)$

"NON CONVENTIONAL" EUCLIDEAN

SOLUTIONS.

TEMPERATURE FOR COSMOLOGY