LARGE N DUALITY

HIROSI OOGURI (CALTECH)

WITH VAFA / 0205297

WITH BERKOVITS AND VAFA / 0310118

WITH OKUDA . TO APPEAR

GAUGE THEORY (OPEN STRING) $\longleftrightarrow$ CLOSED STRING

LARGE N DUALITY

- AdS/CFT CORRESPONDENCE (MALDACENA)

$\mathcal{N}=4$ GAUGE THEORY $\longleftrightarrow$ TYPE II STRING ON $AdS_5 \times S^5$

$\Downarrow$ DEFORMATION

$\mathcal{N}=1$ GAUGE THEORIES (KLEBANOV + STRASSLER, MALDACENA + NUNEZ, ...)

- TOPOLOGICAL STRING DUALITY (GOPAKUMAR + VAFA)

$\Downarrow$ BCOV

$\mathcal{N}=1$ GAUGE THEORIES $\longleftrightarrow$ TYPE II STRING ON CY_3 WITH RR FLUXES

(VAFA)

IT IS DESIRABLE TO HAVE MICROSCOPIC UNDERSTANDING OF THESE DUALITIES.

IN THIS TALK, I WILL PRESENT
A MICROSCOPIC EXPLANATION (WORLDSHEET DERIVATION)
OF THE TOPOLOGICAL STRING DUALITY.

PLAN OF THE TALK

1. WORLDSHEET DERIVATION OF TOPOLOGICAL STRING DUALITY

   CONIFOLD CASE (WITH VAFA)

2. GENERALIZATION

   DIFFERENT DBRANES (OPEN STRING)

   $\updownarrow$

   DIFFERENT COULOMB BRANCES (CLOSED STRIING)

   (WITH OKUDA)

3. LIFTING TO SUPERSTRING

   (WITH BERKOVITS, VAFA)

1. TOPOLOGICAL OPEN / CLOSED STRING DUALITY

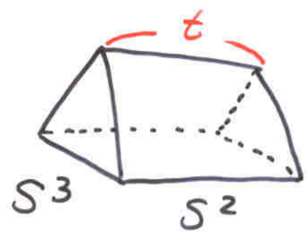
STATEMENT :

TOPOLOGICAL CLOSED STRING (A-MODEL)

ON THE RESOLVED CONIFOLD

λ : STRING COUPLING

t : SIZE OF S^2



$\Updownarrow$ EQUIVALENT

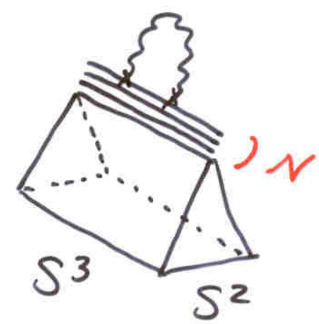
TOPOLOGICAL OPEN STRING

ON THE DEFORMED CONIFOLD

WITH N D BRANES ON S^3.

λ : STRING COUPLING

$t = N\lambda$: 't HOOFT COUPLING



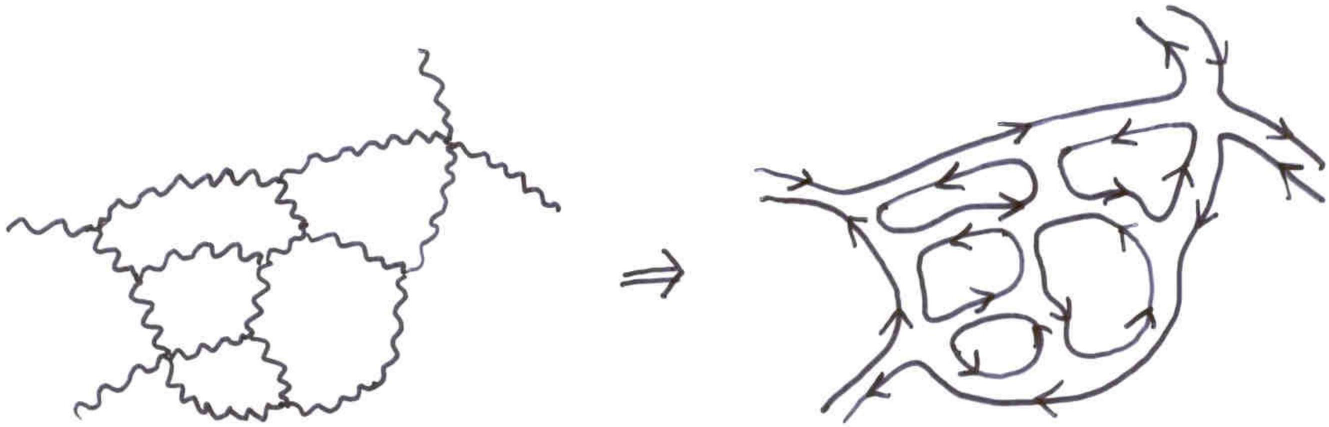
IDEA OF PROOF :

- START WITH THE CLOSED STRING SIDE
- EXPAND IN POWERS OF t

  * $t \to 0$ IS A SINGULAR LIMIT OF THE CLOSED STRING

- REPRODUCE THE OPEN STRING COMPUTATION

't HOOFT EXPANSION



EACH INDEX LOOP COMES
WITH THE 't HOOFT COUPLING $t = N\lambda = N g_{YM}^2$

GAUGE THEORY DIAGRAM

$\Downarrow$ FILL THE INDEX LOOP 

BY A DISK  (WHY NOT  ?

CLOSED STRING WORLD SHEET

$t = N g_{YM}^2$ IS A PARAMETER
OF THE WORLDSHEET THEORY
(SIZE OF S^2 IN THE PRESENT CASE)

WE WILL GO IN THE OPPOSITE DIRECTION.

CLOSED STRING WORLDSHEET $\Rightarrow$ GAUGE THEORY

THE CLOSED STRING DUAL IS ON THE RESOLVED CONIFOLD WITH THE KÄHLER MODULI t = SIZE OF S^2.

$t \to 0$: CONIFOLD SINGULARITY

TO UNDERSTAND THE EXPANSION AROUND $t=0$, WE USE THE LINEAR SIGMA-MODEL.

- CHIRAL MULTIPLETS $(\phi_1, \phi_2, \phi_3, \phi_4)$ WITH CHARGES $Q = (+1, +1, -1, -1)$
- $U(1)$ GAUGE MULTIPLET COUPLED TO Q

t COUPLES TO THE THETA TERM OF THE GAUGE FIELD.

$t \neq 0 \Rightarrow$
- HIGGS BRANCH THEORY FLOWS IN THE IR TO THE SIGMA-MODEL ON THE CONIFOLD.
- NO COULOMB BRANCH

$t = 0 \Rightarrow$ COULOMB BRANCH EMERGES.

IT TURNS OUT TO BE MORE CONVENIENT TO USE THE MIRROR DESCRIPTION (WITH OKUDA)

ACCORDING TO HORI-VAFA,

$$(\phi_1, \phi_2, \phi_3, \phi_4) \quad \text{WITH} \quad Q = (+1, +1, -1, -1)$$

$\downarrow$ T-DUALITY ALONG THE PHASE ROTATIONS $\phi_i \to e^{i\theta}\phi_i$

LANDAU-GINZBURG MODEL WITH SUPERPOTENTIAL

$$W = \sigma(x_1 + x_2 - x_3 - x_4 - t) + \sum_{i=1}^{4} e^{-x_i}$$

- (x_1, x_2, x_3, x_4) : T-DUAL OF $\log \phi_i$
- σ : U(1) GAUGE MULTIPLET
  EXPRESSED AS A TWISTED CHIRAL MULTIPLET
- e^{-x_i}, GENERATED BY INSTANTON EFFECTS

THE CLOSED STRING DUAL IS

THE LANDAU - GINZBURG MODEL WITH SUPERPOTENTIAL

$$W = \sigma(x_1 + x_2 - x_3 - x_4 - t) + \sum_i e^{-x_i}$$

$t \neq 0$: NO FLAT DIRECTION FOR W

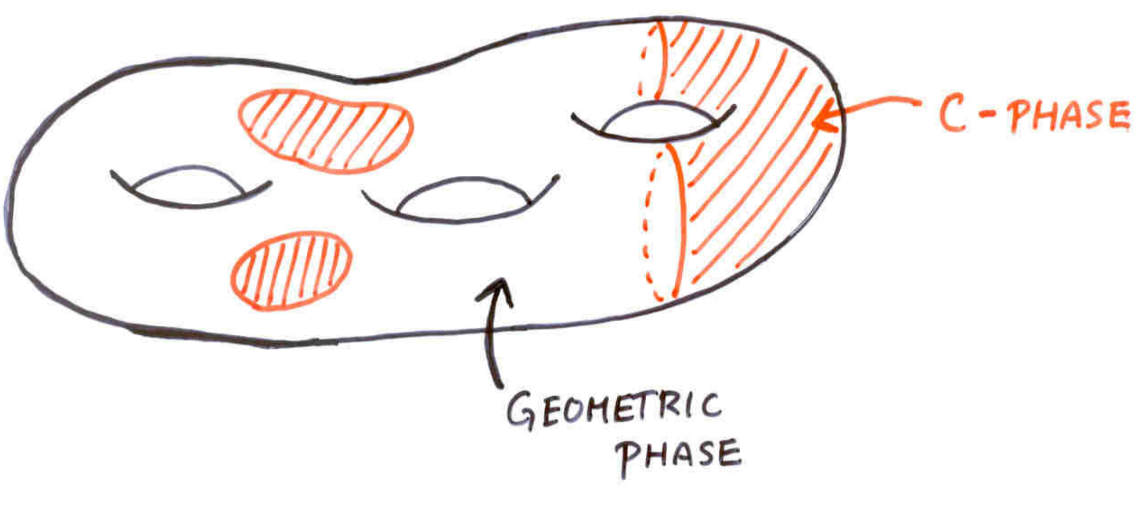
$\Rightarrow$ FLOWS TO THE SIGMA-MODEL

ON THE MIRROR OF THE RESOLVED CONIFOLD

$t = 0$: $\exists$ FLAT DIRECTION

$$\begin{cases} \sigma : \text{ARBITRARY} \\ x_1 = x_2 = -\log\sigma \\ x_3 = x_4 = -\log\sigma + \pi i \end{cases}$$

"C - PHASE"



TO COMPARE WITH OPEN STRING COMPUTATION,
WE INTERPRET

C-PHASE = HOLE ON THE WORLDSHEET

FOR THIS TO WORK, WE NEED :

(1) EVERY C-DOMAIN HAS THE TOPOLOGY OF THE DISK.

CONTRIBUTIONS FROM ALL OTHER TOPOLOGIES
MUST VANISH.

= 0

(2) EACH DISK IN C-PHASE GIVES THE FACTOR OF t

$= t = N\lambda$

(1) EACH C-DOMAIN CONTRIBUTES AS

$$\oint d\sigma_0 \frac{\partial}{\partial\sigma_0} \mathcal{F}^{(C)}(\sigma_0)$$

- $\mathcal{F}^{(C)}(\sigma_0)$: PARTITION FUNCTION FOR THE C-DOMAIN WITH THE BOUNDARY CONDITION $\sigma = \sigma_0$.
- $\oint d\sigma_0 \frac{\partial}{\partial\sigma_0}$ IS DUE TO A JACOBIAN FACTOR.

BY THE TOPOLOGICAL BRST SYMMETRY,

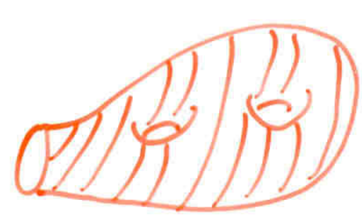
$\mathcal{F}^{(C)}(\sigma_0)$ IS HOLOMORPHIC IN σ_0

$$\Rightarrow \quad \oint d\sigma_0 \frac{\partial}{\partial\sigma_0} \mathcal{F}^{(C)}(\sigma_0) = 0$$

IF $\mathcal{F}^{(C)}$ IS SINGLE-VALUED.

THIS IS THE CASE FOR

,



, ...

THE ONLY EXCEPTION IS THE DISK.

(2) THE DISK AMPLITUDE FOR THE LG MODEL IS

$$\mathcal{F}^{(C)}(\sigma_0) = \int^{\sigma_0} d\sigma \, dx \; e^{-W(\sigma, x)}$$

$$\oint d\sigma_0 \frac{\partial}{\partial\sigma_0} \mathcal{F}^{(C)}(\sigma_0)$$

$$= \oint d\sigma_0 \, dx \, \exp\left[-\sigma_0(x_1 + x_2 - x_3 - x_4 - t) - \sum_i e^{-x_i}\right]$$

$$= \oint d\sigma_0 \frac{1}{\sigma_0^2} e^{t\sigma_0}$$

$$\sim t = N\lambda$$

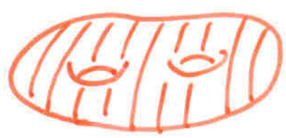
THE 't HOOFT COUPLING.

THIS IS WHAT WE WANTED TO SHOW.

WHAT ABOUT PURE C-PHASE?

THE ENTIRE WORLDSHEET IS A HOLE.

→ THIS DOES NOT CORRESPOND TO ANY TERM IN THE 't HOOFT EXPANSION.

$$\int_{\mathcal{M}_g}$$  $$= \underbrace{\chi(\mathcal{M}_g)}_{\text{EULER CHARACTERISTIC OF } \mathcal{M}_g} \cdot \frac{1}{t^{2g-2}}$$

THIS IS SINGULAR AS $t \longrightarrow 0$,

REFLECTING THE SINGULAR NATURE OF CONIFOLD.

$$\sum_g \lambda^{2g-2} \cdot \chi(\mathcal{M}_g) \cdot \frac{1}{(N\lambda)^{2g-2}} \qquad t = N\lambda$$

$$= \sum_g \frac{B_{2g}}{2g(2g-2)} N^{2-2g} = -\log \mathrm{vol}\, U(N)$$

$$\left(\mathrm{vol}\, U(N) = \frac{(2\pi)^{\frac{1}{2}N(N+1)}}{(N-1)!(N-2)! \cdots 3!2!1!} \right)$$

THIS REPRODUCES THE CORRECT MEASURE FACTOR FOR THE GAUGE THEORY.

THE PURE C-PHASE IS NEEDED TO REPRODUCE THE GAUGE THEORY RESULT BEYOND THE 't HOOFT EXPANSION.

2. GENERALIZATION

CAN WE IDENTIFY D-BRANE BOUNDARY CONDITIONS FROM THE POINT OF VIEW OF THE CLOSED STRING DUAL?

CHERN-SIMONS GAUGE THEORY ON THE LENSE SPACE S^3/Z_p

$\Updownarrow$

CLOSED STRING ON THE Z_p ORBIFOLD OF THE RESOLVED CONIFOLD.

✡ SINCE $\pi_1(S^3/Z_p) = Z_p$,

THERE ARE p DIFFERENT D-BRANES.

$$U(N) \to U(N_0) \times U(N_1) \times \cdots \times U(N_{p-1})$$

WHERE $U(N_k)$ SECTOR HAS HOLONOMY $e^{2\pi i k/p}$.

✡ IT TURNS OUT THAT THE CLOSED STRING DUAL HAS p DIFFERENT C-PHASES.

D-BRANE $\longleftrightarrow$ C-PHASE.

P = 2 CASE

THE Z_2 ORBIFOLD OF THE RESOLVED CONIFOLD IS DESCRIBED BY THE LINEAR SIGMA-MODEL WITH

- CHIRAL MULTIPLETS $(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5)$

  WITH CHARGES
  $$Q_1 = (-2, 1, 1, 0, 0)$$
  $$Q_2 = (-2, 0, 0, 1, 1)$$

- GAUGE MULTIPLETS COUPLED TO Q_1 AND Q_2.

$\Updownarrow$ MIRROR

LANDAU - GINZBURG MODEL WITH

$$W = \sigma_1 (-2x_0 + x_1 + x_2 - t_1) + \sigma_2 (-2x_0 + x_3 + x_4 - t_2) + \sum_{i=0}^{4} e^{-x_i}$$

FOR GENERIC VALUES OF t_1, t_2 THERE IS NO FLAT DIRECTION FOR W.

THERE IS A CO-DIMENSION ONE SUBSPACE WHERE C-PHASES EMERGE.

$$\frac{\partial W}{\partial x_i} = 0 \quad \Rightarrow \quad \begin{cases} x_0 = -\log(-2\sigma_1 - 2\sigma_2) \\ x_1 = x_2 = -\log \sigma_1 \\ x_3 = x_4 = -\log \sigma_2 \end{cases}$$

$$\frac{\partial W}{\partial \sigma_a} = 0 \quad \Rightarrow \quad \begin{cases} e^{t_1} = 4\left(1 + \frac{\sigma_1}{\sigma_2}\right)^2 \\ e^{t_2} = 4\left(1 + \frac{\sigma_2}{\sigma_1}\right)^2 \end{cases}$$

THESE ARE CONSISTENT ONLY WHEN

$$\Delta(t_1, t_2) = 16\left(e^{-t_1} - e^{-t_2}\right)^2 - 8\left(e^{-t_1} + e^{-t_2}\right) + 1 = 0$$

- IF THIS IS SATISFIED,

  THERE IS A FLAT DIRECTION TO RESCALE (σ_1, σ_2).

- IN SPECIAL COORDINATES

$$\hat{t}_1 = \hat{t}_1(t_1, t_2), \quad \hat{t}_2 = \hat{t}_2(t_1, t_2)$$

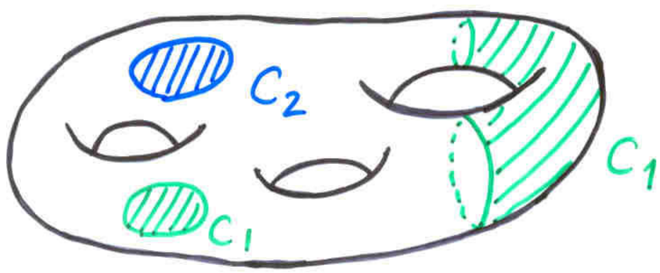
$$\Delta(t_1, t_2) = 0$$

$$\Leftrightarrow \begin{cases} \hat{t}_1 = 0 \quad : \quad C_1 \text{ PHASE} \\ \qquad \text{OR} \\ \hat{t}_2 = 0 \quad : \quad C_2 \text{ PHASE} \end{cases}$$

IN THE LIMIT $\hat{t}_1, \hat{t}_2 \to 0$,

BOTH C_1 AND C_2 PHASES COEXIST

WITH THE GEOMETRIC PHASE



EACH OF THESE C PHASES IS PARAMETRIZED AS

C_1 PHASE :

$$\sigma_1 = \sigma \, , \quad \sigma_2 = -\sigma$$

$$x_0 + t_1/2 = -\log \sigma$$

$$x_1 = x_2 = x_3 = x_4 = -\log \sigma$$

C_2 PHASE :

$$\sigma_1 = \sigma \, , \quad \sigma_2 = -\sigma$$

$$x_0 + t_1/2 = -\log \sigma + \pi i$$

$$x_1 = x_2 = x_3 = x_4 = -\log \sigma$$

THE TWO PHASES ARE MAPPED INTO EACH OTHER

BY $x_0 \to x_0 + \pi i$.

THE TWO C-PHASES ON CLOSED STRING WORLDSHEET ARE EXCHANGED BY πi SHIFT OF x_0.

SINCE THE MIRROR SYMMETRY IS

$$x_0 \to x_0 + 2\pi i \underset{\text{T-DUAL}}{\longleftrightarrow} \phi_0 \to e^{2\pi i} \phi_0 ,$$

IN THE ϕ_i VARIABLES, THE TWO PHASES ARE DISTINGUISHED BY HOLONOMIES OF GAUGE FIELD FOR THE CYCLE $\phi_0 \to e^{2\pi i} \phi_0$

$$\exp\left(\oint A \right) = \pm 1 .$$

IN THE IR LIMIT, THIS CYCLE FLOWS TO THE GENERATOR OF $\pi_1 (S^3/Z_2)$.

THIS IS WHAT WE WANTED TO SHOW.