

One of David Olive's many profound contributions was the formulation of the $\mathcal{N}=4$ SYM. This maximally supersymmetric 4-d gauge theory has many miraculous properties.

It is superconformal for any value of the complex coupling

$$\frac{4\pi}{g_{\text{YM}}^2} + i \frac{\theta}{2\pi}$$

and exhibits exact Montonen-Olive duality.

Nevertheless, the theory has many non-trivial observables, such as the anomalous dimensions of gauge invariant operators, Wilson loops, entropy and viscosity

at finite temperature, etc.

String theory (and theorists) have a special bond to the $N=4$ SYM which dates back to its discovery.

Gliozzi, Scherk and Olive wrote in their classic paper "Supersymmetry, Supergravity and the Dual Spinor Model":

We show that in the limit where both the slope and the size of the 6-dimensional compact go to zero, the dual model of open strings turns into a supersymmetric $U(1)$ theory (1 spin 1, 3 scalars, 3 pseudo-scalars, 4 Majorana spin $\frac{1}{2}$) with $SU(4)$ symmetry.

$N=4$ supersymmetric YM theory with gauge group $U(N)$ is realized on N parallel D3-branes of the type IIB string theory, which



create the metric

$$ds^2 = h^{-\frac{1}{2}} dx_{11}^2 + h^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2)$$

$$h(r) = 1 + \frac{L^4}{r^4}$$

This has led to formulation of the $AdS_5 \times S^5$ dual of the $N=4$ SYM.

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Another recent realization of the $N=4$ SYM is in terms of topological open strings on twistor space.

Witten