PENTAQUARKS:

CHIRAL SOLITON MODELS CONFRONT EXPERIMENTS

Igor Klebanov.

The possibility of "exotic" pentaquark baryons $qqqq\bar{q}$ was raised in the earliest days of the quark model.

Recently, several experiments reported evidence for a $S=1$, $B=1$ particle Θ^+ with mass 1530-1540 MeV. Its minimal quark content is $\bar{s}udud$

The first paper is by Nakano et al (Spring-8 in Osaka) which used photoproduction off ^{12}C nuclei.

$$\gamma n \to K^- \Theta^+ \to K^- K^+ n$$

Observe a narrow peak in the inv. mass of K^+n; width < 20 MeV.

Photoproduction of Θ^+ found at several other facilities (CLAS, SAPHIR, ...)

Also, DIANA at ITEP observed

$$K^+ Xe \to \Theta^+ + X \to p K^0 + X$$

but old studies of $K^+ d$ scattering did not reveal a Θ^+.

Also, recent searches at HERA-B and BES do not show any peak in $p K_S$ or $n K^+$ distribution.

What about the theory?

The rigid rotator quantization of Skyrmions predicts an exotic $\overline{10}$ multiplet to which Θ^+ belongs.

In 1997 Diakonov, Petrov and Polyakov used a similar model to predict a narrow $\Theta^+(1530)$.

If, however, the Skyrme model is treated in a consistent semi-classical regime (large N_c), then it is impossible to find a simple relation between properties of exotic and non-exotic baryons.

The existence or non-existence of exotics is a subtle dynamical question. For standard Skyrme model parameters, the Θ^+ does not exist. This was known in the 80's, and recently confirmed in Itzhaki, IK, Rastelli, Ouyang, hep-ph/0309305 solitons (such as gauge theory monopoles), which in the weak coupling limit become heavy, rigid, almost classical objects. In fact, the idea that baryons could be solitons in mesonic theories was developed by Skyrme long before quarks and QCD.

The Skyrme Model

Pion interactions for energies up to ~ 600 MeV are well-described by the non-linear chiral lagrangian:

$$\mathcal{L} = \frac{f_\pi^2}{16} \, tr\left(\partial_\mu U \partial_\mu U^+\right) + \frac{1}{32e^2} \, tr\left[\partial_\mu U U^+, \partial_\nu U U^+\right]^2,$$

where $U(\vec{x}, t) = \exp\left(\frac{2i \vec{\tau} \cdot \vec{\pi}}{f_\pi}\right) \subset SU(2)$.

Transformation properties of U can be understood through $U^\beta_\alpha \sim \langle \bar{q}^\beta_{R\,i} \, q_{L\alpha}^{\;i} \rangle$, where $q_L = \frac{1+\gamma_5}{2} q$, $q_R = \frac{1-\gamma_5}{2} q$.

Under $SU(2)_L$, $q_{L\alpha} \to A_\alpha{}^\beta q_{L\beta}$;

Under $SU(2)_R$, $q_{R\alpha} \to B_\alpha{}^\beta q_{R\beta}$.

$\Rightarrow U \to A U B^{-1}$. The vacuum value $U=1$ breaks $SU(2)_L \times SU(2)_R$ down to the diagonal $(A=B)$ $SU(2)_V$ subgroup, the isospin.

If we expand U in $\vec{\pi}$ fluctuations as $U \approx 1 + \frac{2i\vec{\tau}\cdot\vec{\pi}}{f_\pi} + \dots$, and assign $f_\pi \sim \sqrt{N}$, $1/e \sim \sqrt{N}$, then the behaviour of pion amplitudes is consistent with the large-N rule $g_{2+n} \sim N^{-\frac{n}{2}}$. For example, the leading term in

$$\frac{1}{32e^2} tr[\partial_\mu U U^+, \partial_\nu U U^+]^2 \sim \frac{1}{e^2 f_\pi^4}(\partial_\mu \vec{\pi} \times \partial_\nu \vec{\pi})^2,$$

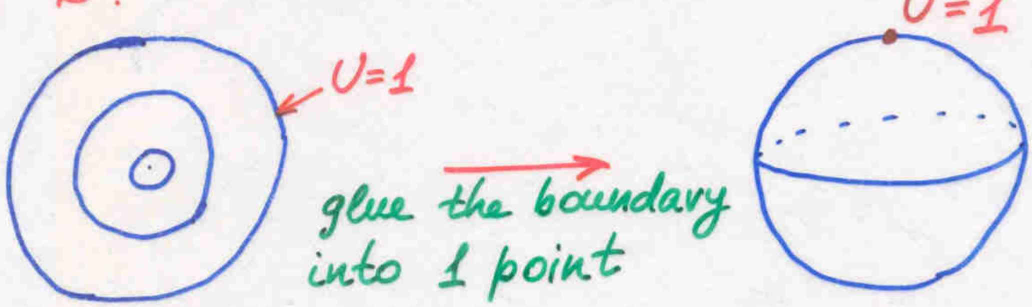
consistent with $g_4 \sim \frac{1}{N}$.

Skyrme showed that, in addition to small fluctuations about $U=1$, the chiral lagrangian has topologically stable solutions with

U taking values all over $SU(2)$.

Parametrize $SU(2)$ as $U = u_0 + i\,\vec{u}\cdot\vec{\tau}$;
$UU^\dagger = 1 \Rightarrow u_0^2 + \vec{u}^2 = 1 \Rightarrow SU(2)$ group manifold has topology of a 3-sphere S^3.

Consider static configurations where, as $r\to\infty$, $U\to 1$ sufficiently rapidly. This identifies all points with $r\to\infty$ into one point which is mapped into $U=1$. Topology R^3 is replaced by S^3.



We have reduced the problem to maps from S^3(space) to $S^3(SU(2))$, which fall into homotopy classes $\pi_3(SU(2))$ characterized by an integer (the winding number).

$$B = \frac{1}{24\pi^2}\int d^3x\,\epsilon^{ijk}\,\mathrm{tr}\left(\partial_i U U^\dagger \partial_j U U^\dagger \partial_k U U^\dagger\right)$$

Skyrme proposed that the winding number be identified with the baryon number.

It is invariant under any smooth deformations of $U(\vec{x}) \Rightarrow$ conserved in any physical process. Define baryon number current

$$B^{\mu} = \frac{1}{24\pi^2} \epsilon^{\mu\alpha\beta\gamma} \, tr \left(\partial_\alpha U U^+ \partial_\beta U U^+ \partial_\gamma U U^+ \right).$$

Check that $\partial_\mu B^\mu = 0$ identically, without any use of eqs. of motion $\Rightarrow B = const.$

The simplest non-trivial configuration is a one-to-one map with $B=1$. To construct a solution, Skyrme proposed the hedgehog ansatz

$$U_0(\vec{x}) = exp\left(i F(r) \vec{\tau} \cdot \hat{r} \right),$$

with $F(0) = \pi$ and $F(r) \to 0$ as $r \to \infty$.

It's a one-to-one map: $U_0(0) = -1$, $U_0(\infty) = 1$.

The formula for the winding number yields

$$B = -\frac{2}{\pi} \int_0^\infty \sin^2 F \cdot F' dr = 1.$$

For any $F(r)$, U_0 has a symmetry under combined spatial and isospin rotations:

$$H = M + \frac{1}{8\Omega}\sum_{i=0}^{3} -\frac{\partial^2}{\partial a_i^2} = M - \frac{1}{8\Omega}\nabla^2,$$

where ∇^2 is the laplacian on the 3-sphere $\Sigma a_i^2 = 1$. Classically, the spin and isospin are

$$J_k = \Omega \, tr(\tau_k A^{-1}\dot{A})$$

$$I_k = \Omega \, tr(\tau_k A \frac{d}{dt} A^{-1}).$$

They satisfy $\vec{J}^2 = \vec{I}^2$. Upon quantization,

$$J_k = \frac{i}{2}\left(a_k \frac{\partial}{\partial a_0} - a_0 \frac{\partial}{\partial a_k} - \epsilon_{klm} a_l \frac{\partial}{\partial a_m}\right)$$

$$I_k = \frac{i}{2}\left(a_0 \frac{\partial}{\partial a_k} - a_k \frac{\partial}{\partial a_0} - \epsilon_{klm} a_l \frac{\partial}{\partial a_m}\right)$$

You can check the operator identity

$$\vec{J}^2 = \vec{I}^2 = -\frac{1}{4}\nabla^2 \Rightarrow H = M + \frac{1}{2\Omega}\vec{J}^2;$$

All wave functions satisfy $I = J$ rule.

There are 2 consistent ways to quantize a soliton; since A and $-A$ give the same U

1). $\Psi(A) = \Psi(-A)$; i.e. $\Psi(A) = (a_0 + i a_1)^l$, where l is even. Then $I = J = \frac{l}{2} \Rightarrow$ soliton is a boson.

2). $\Psi(A) = -\Psi(-A)$; i.e. $\Psi = (a_0 + i a_1)^{\ell}$, ℓ-odd $\Rightarrow$ the soliton is a fermion. In the 3-flavor Skyrm model, the soliton is required to be a fermion for N-odd; to be a boson - for N-even.

In the 2-flavor case we simply choose 2).

The allowed quantum numbers are $I = J = \frac{1}{2}, \frac{3}{2}, \ldots$

A few representative wave functions are

$$|p\uparrow\rangle = \frac{1}{\pi}(a_1 + i a_2); \quad |p\downarrow\rangle = -\frac{i}{\pi}(a_0 - i a_3)$$

$$|\Delta^{++}, S_z = \tfrac{3}{2}\rangle = \frac{\sqrt{2}}{\pi}(a_1 + i a_2)^3$$

$$|\Delta^{+}, S_z = \tfrac{1}{2}\rangle = -\frac{\sqrt{2}}{\pi}(a_1 + i a_2)\left[1 - 3(a_0^2 + a_3^2)\right].$$

The quantum numbers of nucleons and deltas have been successfully reproduced. The states with $I = J = \frac{5}{2}, \ldots$ are presumably artifacts of the rigid rotator approximation, since for them the rotational energy $\sim$ static soliton energy.

$$M_N = M + \frac{1}{2\Omega} \times \frac{3}{4}; \quad M_\Delta = M + \frac{1}{2\Omega} \times \frac{15}{4}$$

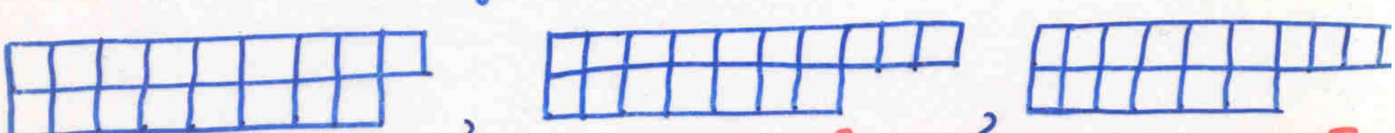
$$M = 36.5\,\frac{f_\pi}{e}; \quad \Omega = \frac{2\pi}{3e^3 f_\pi} \times 50.9.$$

Fitting f_π and e to the 2 masses, we find $f_\pi = 129$ MeV ; $e = 5.45$, about 30% off the observed values.

Within this fit, $M(5/2) = M + \frac{1}{2\Omega}\frac{35}{4} \approx 1730$ M

For this state, $E_{rot} > M \Rightarrow$ the semiclassical approximation is invalid. Since $M \sim O(N)$ and $E_{rot} \sim O(1/N)$, the larger is N, the more states are subject to semiclassical treatment. we expect $I = J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots O(N)$.

Now consider the large-N quark model. Low-lying baryons have all quarks in the same spatial state. The total symmetry of the spin-flavor state requires the identical spin and flavor Young tableaux



$I = J = \frac{1}{2}$, $I = J = \frac{3}{2}$, $I = J = \frac{5}{2}$

etc., the total number of boxes $= N$ (in this case $N = 1$

It is not hard to guess a correction to the e.o.m., which breaks P_0 and $(-1)^{N_B}$ separately:

$$\partial_\mu\left(\frac{1}{8}f_\pi^2 U^+\partial_\mu U\right)+\ldots+\lambda\epsilon^{\mu\nu\alpha\beta}U^+\partial_\mu U\ldots U^+\partial_\beta U=0.$$

What is the action from which this follows?
It is subtle and cannot be written as a 4-dimensional integral:

$$S_{WZ}=-\frac{in}{240\pi^2}\int d^5x\,\epsilon^{\mu\nu\alpha\beta\gamma}\mathrm{tr}(U^+\partial_\mu U\ldots U^+\partial_\gamma U)$$

The integral is over a 5-dimensional disc, whose boundary is space-time. The crucial fact is that the coefficient n must be an integer. This is because the choice of domain is not unique.

Q  $\vec{x},t$

domain of integration

 Q'

another choice of domain

If, instead of Q, we choose Q', then

$$\Delta S_{WZ}=-\frac{in}{240\pi^2}\int_{Q+Q'} d^5x\,\epsilon^{\mu\nu\alpha\beta\gamma}\mathrm{tr}(U^+\partial_\mu U\ldots U^+\partial_\gamma U)$$

Since the path integral only involves $e^{iS_{WZ}}$, the non-singlevaluedness is harmless if $\Delta S=2\pi\times$intege[r]
This is why the coefficient of S_{WZ} must be quantized!

In addition, the W-Z term imposes powerful constraints on the $SU(3)_f \times SU(2)_{spin}$ quantum numbers of baryons. In the collective coordinate quantization, $U(\vec{x},t) = A(t)\, U_0(\vec{x}) A^{-1}(t)$, the effective lagrangian for $A(t)$ is

$$L = -\frac{\Omega}{2}\sum_{j=1}^{3}(tr\,\lambda_j A^{-1}\dot{A})^2 - \frac{\Phi}{2}\sum_{a=4}^{7}(tr\,\lambda_a A^{-1}\dot{A})^2 -$$

$$-i\frac{N}{2\sqrt{3}}\,tr(\lambda_8 A^{-1}\dot{A}),$$ where λ are Gell-Mann matrices, Ω and Φ are functionals of $F(r)$. Numerically,

$$\Omega = \frac{106}{f_\pi e^3}, \quad \Phi = \frac{39}{f_\pi e^3}.$$

Convenient coordinates are given by $A^{-1}\dot{A} = \frac{i}{2}\sum_{i=1}^{8}\lambda_i \dot{a}_i$

$$L = \frac{\Omega}{2}\sum_{j=1}^{3}(\dot{a}_j)^2 + \frac{\Phi}{2}\sum_{a=4}^{7}(\dot{a}_a)^2 + \frac{N}{2\sqrt{3}}\,\dot{a}_8$$

Let π_i be the momenta conjugate to a_i. Then

$$H = \frac{1}{2\Omega}\sum_{j=1}^{3}\pi_j^2 + \frac{1}{2\Phi}\sum_{a=4}^{7}\pi_a^2,$$

and there is a constraint originating from the W-Z term: $\pi_8 \Psi(A) = \frac{N}{2\sqrt{3}}\Psi(A)$

Since Π_i are generators of the <u>right</u> rotations on A, the constraint implies

$$\Psi(Ae^{i\alpha Y}) = e^{i\alpha\frac{N}{3}}\Psi(A),$$ where

$Y = \frac{\lambda_8}{\sqrt{3}}$ is the conventionally normalized hypercharge. The wave functions can be written as

$$\Psi(A) = \langle I, I_3, Y | D^{(p,q)}(A) | I', I_3', Y' \rangle,$$

which are components of the group element A in the (p,q) representation of $SU(3)$. Due to the group property $D^{(p,q)}(CD) = D^{(p,q)}(C)\, D^{(p,q)}(D)$ and the constraint, the <u>right</u> hypercharge index $Y' = \frac{N}{3}$. $\Rightarrow$ (p,q) must contain a state with hypercharge $\frac{N}{3}$. The allowed wave functions are

$$\langle I, I_3, Y | D^{(p,q)}(A) | I', I_3', \tfrac{N}{3} \rangle$$

What are their quantum numbers under $SU(3)_f \times SU(2)_{spin}$?

1) Under $SU(3)_f$, $U \to FUF^{-1} \Rightarrow A \to FA \Rightarrow$ the wave functions transform as

$$D^{(p,q)}_{(I,I_3,Y),(I',I_3',\frac{N}{3})}(A) \to D^{(p,q)}_{(I,I_3,Y),(I'',I_3'',Y'')}(F) \times$$

$$D^{(p,q)}_{(I'',I_3'',Y''),(I',I_3',\frac{N}{3})}(A)\,;$$ $\Rightarrow$ the left set of indeces (I, I_3, Y) are the flavor indeces which transform in the (p,q) representation of $SU(3)$

2) Under $SU(2)_{spin}$, $U_0 \to R^{-1} U_0 R \Rightarrow A \to AR^{-1} \Rightarrow$

$\Rightarrow$ by a similar argument, the right indeces (I', I_3') label the angular momentum representation: physical $J = I'$; physical $J_3 = -I_3'$.

The constraint may be rephrased: an allowed $SU(3)_f \times SU(2)_{spin}$ rep. must contain a state with $Y = \frac{N}{3}$ and $I = J$. The smallest reps are the octet: $[(1,1);\ J=\frac{1}{2}] \Rightarrow$ nucleon; the decuplet: $[(3,0);\ J=\frac{3}{2}] \Rightarrow$ delta.

What are the low-lying states in the large-N QCD? The rule is identical to the 2-flavor case.

The spin and flavor parts must have identical Young tableaux (with N boxes each):

, etc.

$\Uparrow$ The large-N analogue of octet

$\Uparrow$ The large-N analogue of decuplet

Check that each of these reps satisfies the Skyrme model constraint. Once again, the low-lying states in the Skyrme model agree with the quark model. In addition, there are higher Skyrmion reps which cannot be constructed out of quarks alone. They are exotic. For $N=3$, in addition to $(\overset{(1,1)}{8}, \frac{1}{2})$ and $(\overset{(3,0)}{10}, \frac{3}{2})$, there are $(\overset{(0,3)}{\overline{10}}, \frac{1}{2})$, $(\overset{(2,2)}{27}, \frac{1}{2})$ and $(\overset{(2,2)}{27}, \frac{3}{2})$

The exotic multiplets have higher masses. We will see that rigid rotator methods are not trustworthy in describing the exotic baryons.

Let us express the rotational corrections to the masses in a simpler form.

For a general N_c, the octet $(1,1)$ generalizes to $(1, \frac{N_c-1}{2})$; the decuplet $(3,0)$ to $(3, \frac{N_c-3}{2})$.

The splittings between the non-exotic multiplets are $\mathcal{O}(1/N_c)$.

The first exotic multiplet $\overline{10}$ becomes $(0, \frac{N_c+3}{2})$.

Its energy is $\mathcal{O}(1) = \frac{N_c}{4\Phi}$ higher than of non-exotics. Kaplan, IK; Cohen

Same order as for meson fluctuations around soliton!

States of strangeness $\mathcal{O}(1)$ are small $\mathcal{O}(1/\sqrt{N_c})$ fluctuations.

The small fluctuation ("bound state") approach is valid for any m_K, as long as N_c is large!

Using $U = \sqrt{U_\pi}\, U_K \sqrt{U_\pi}$ in the $SU(3)\times SU(3)$ chiral lagrangian ($U_K = \exp[2i\lambda_a K^a/f_\pi]$), one finds the kaon-skyrmion interaction lagrangian (Callan, IK)

$$\mathcal{L} = (D_\mu K)^\dagger D^\mu K - m_K^2 K^\dagger K + \dots$$

$$+ i \frac{N_c}{f_\pi^2} B^\mu \left(K^\dagger D_\mu K - (D_\mu K)^\dagger K\right)$$

↑ from the WZW term; it attracts antikaons, but repels kaons.

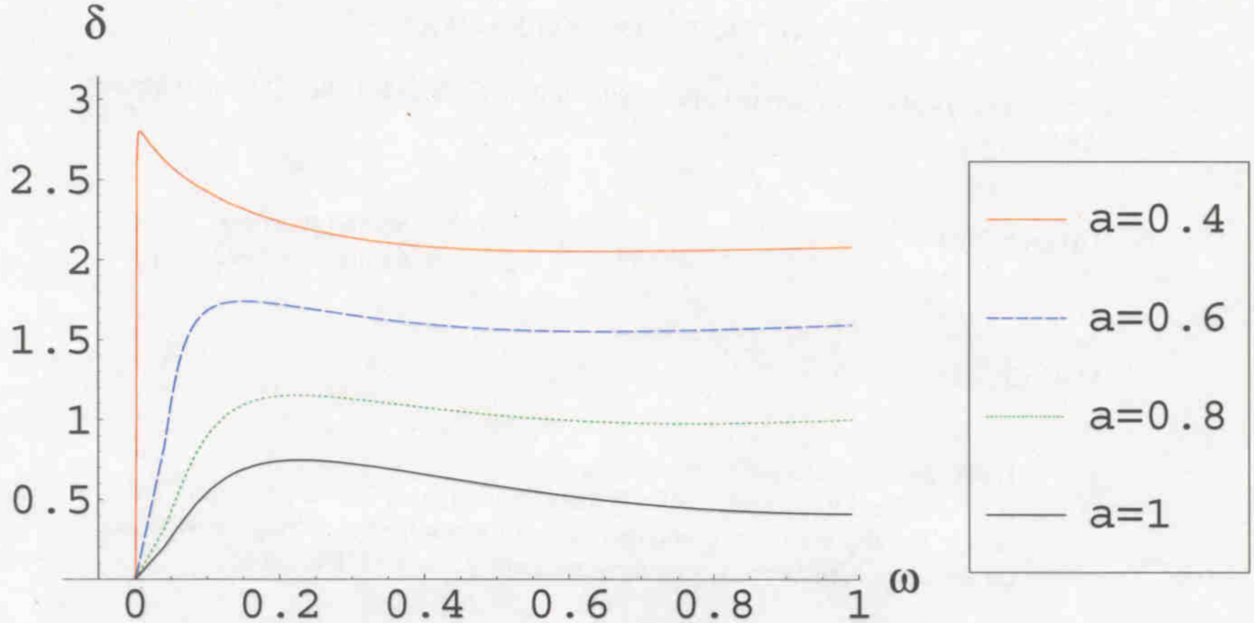
For $S=-1$, energies are found from

$$[f(r)\,\omega^2 + 2\lambda(r)\,\omega + \mathcal{O}]\,k = 0 .$$

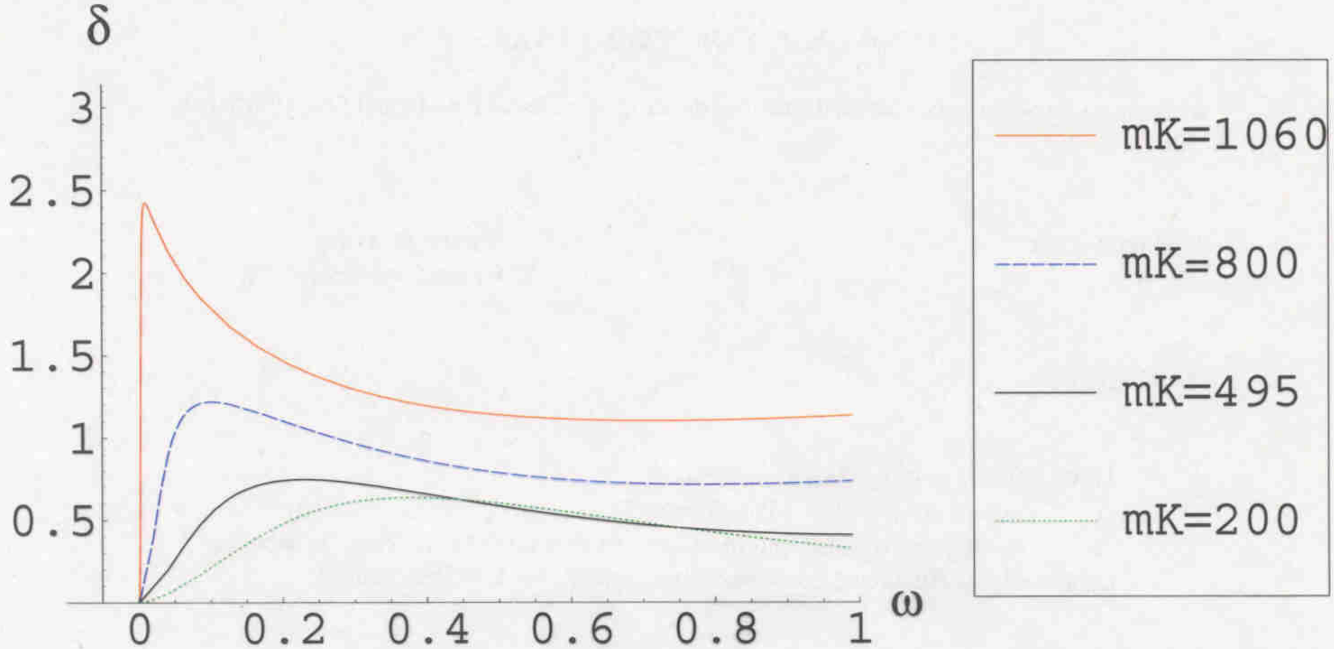
For $m_K = 0$, find a zero mode which becomes a bound state for $m_K > 0$.

Reproduce $\Lambda, \Sigma, \Sigma^*$.

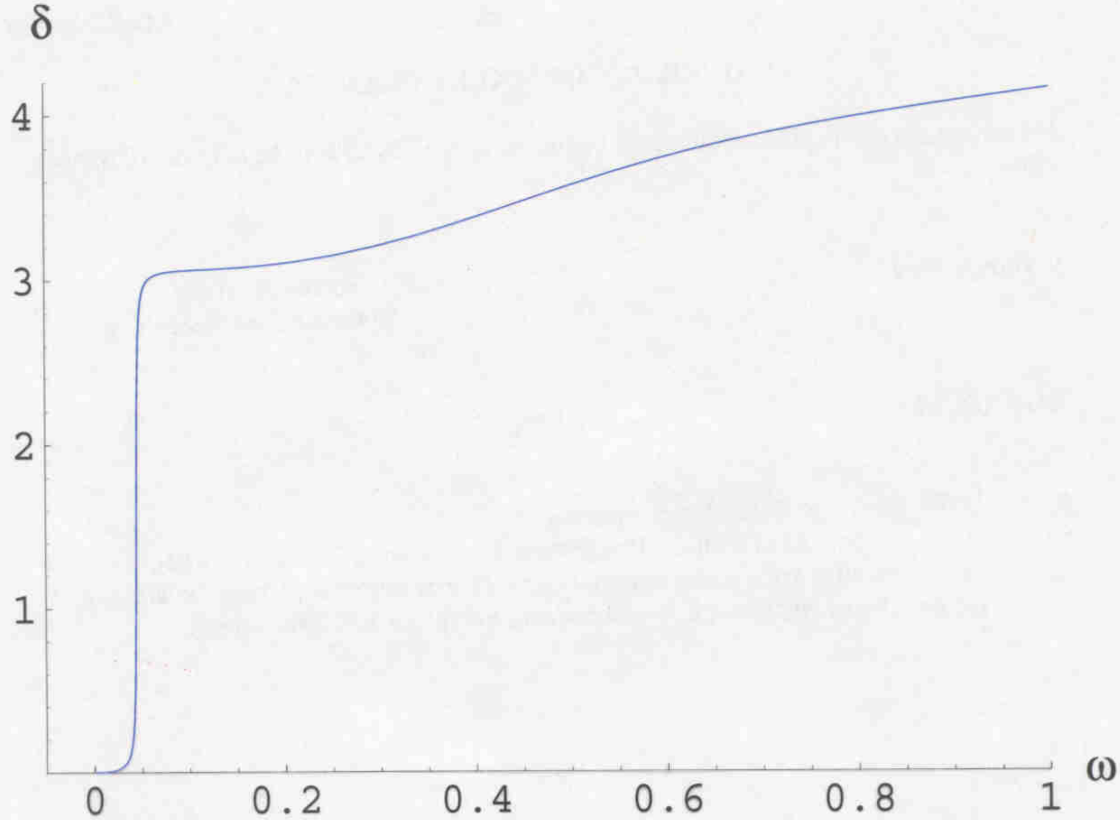
For $m_K = 495$ MeV, also have a



Phase shifts as a function of energy in the L=1, T=1/2, S=+1 channel, for various choices of the parameter *a* (strength of the WZ term.) The energy ω is measured in units of ef_π (e=5.45, f_π=129 MeV) and the phase shift δ is measured in radians. ω=0 corresponds to the K-N threshold.



Phase shifts as a function of energy in the L=1, T=1/2, S=+1 channel, for various values of the kaon mass mK. The energy ω is measured in units of ef_π (e=5.45, f_π=129 MeV) and the phase shift δ is measured in radians. ω=0 corresponds to the K-N threshold.



Phase shift as a function of energy in the L=2, T=3/2, S=-1 channel. The energy ω is measured in units of ef_π (with the kaon mass subtracted, so that ω=0 at threshold) and the phase shift δ is measured in radians. Here e=5.45 and f_π= 129 MeV.

This is the D-wave resonance that describes $\Lambda(1520)$.