WRAPPED BRANES AND NON-PERTURBATIVE GAUGE THEORY

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OUTLINE

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- BRANE CONSTRUCTION ON $\mathcal{N}=1$ THEORIES
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- THE SUPERPOTENTIAL
- REMARKS

* hep-th/0403035
w/ S. de Haro

INTRODUCTION

Q: HOW MANY PEOPLE HAVE A CONSTRUCTION, A PROJECTION AND A DUALITY NAMED AFTER THEM?

INTERSECTING AND/OR WRAPPED BRANES PROVIDE A POWERFULL AND QUITE GENERIC CONSTRUCTION OF GAUGE THEORIES

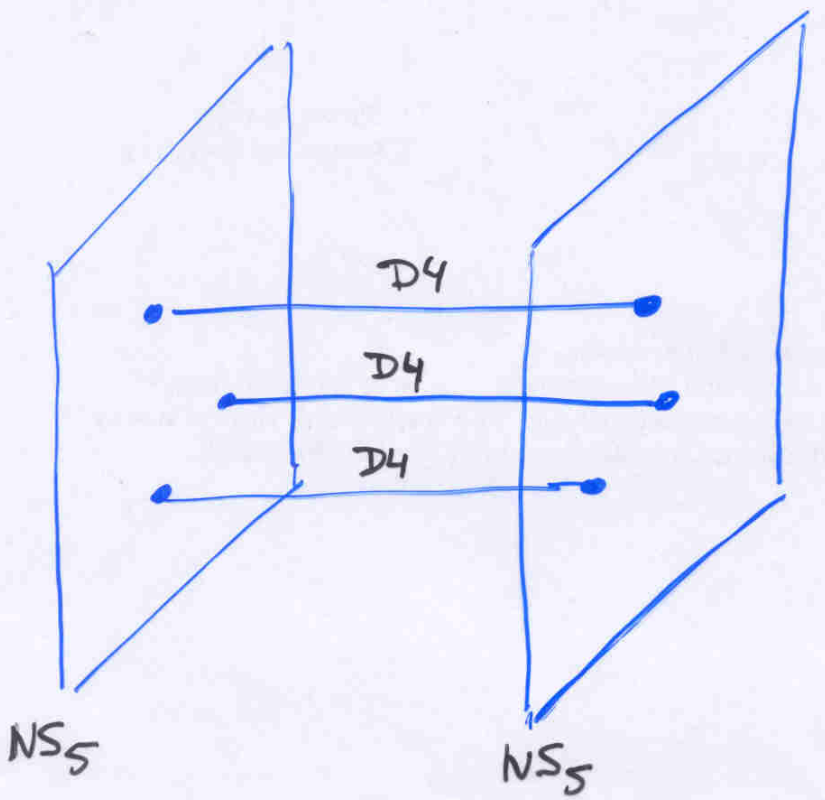
→ HOW TO UNDERSTAND THE NON-PERTURBATIVE DYNAMICS IN TERMS OF $S_i = \mathrm{Tr}_i(\lambda^\alpha \lambda_\alpha)$ OF $\mathcal{N}=1$ THEORIES IN THIS CONTEXT? COMPUTE $W[S_i]$?

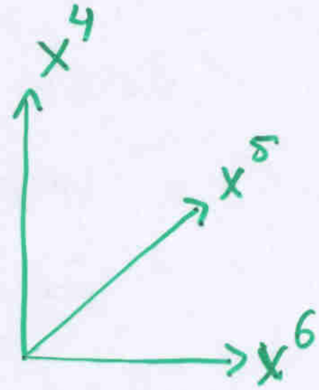
– ALSO MOTIVATED BY TRYING TO UNDERSTAND THE RELATION BETWEEN S_i AND INTEGRABLE SYSTEMS, AND BY STUDYING THE EMBEDDING OF TOPOLOGICAL STRING THEORY IN ORDINARY STRING THEORY

A: GKO, GSO, MO : PROBABLY JUST ONE

BRANE CONSTRUCTION OF N=2 THEORIES

Witten





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x^0, x^1, x^2, x^3

THE D4-BRANES SUPPORT A PURE $N=2$ $U(N)$ GAUGE THEORY, $N = \#$ D4 BRANES

POINT ON COULOMB BRANCH $\longleftrightarrow$

EIGENVALUES OF Φ : ADJOINT SCALAR SUPERFIELD $\longleftrightarrow$

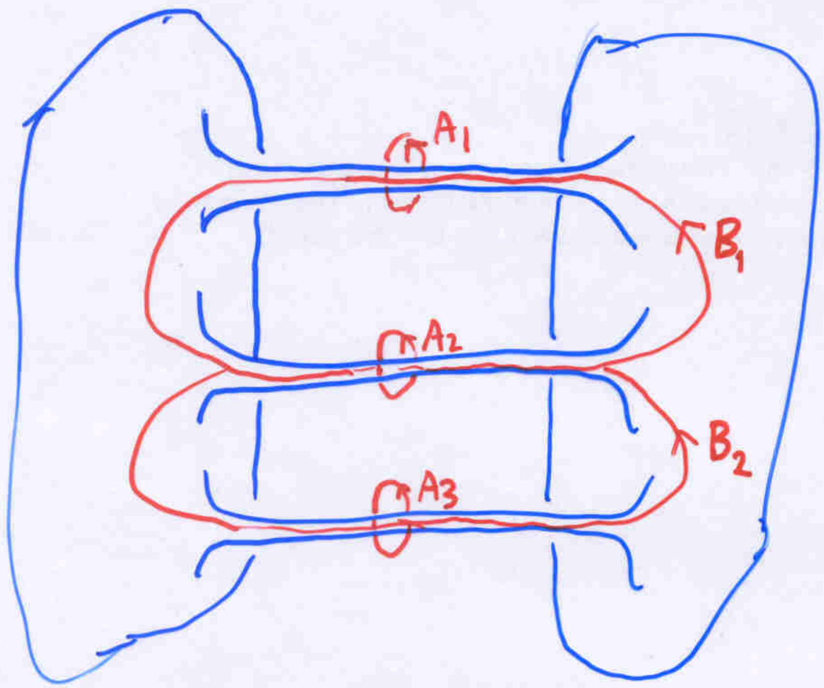
LOCATIONS OF D4-BRANES IN x^4, x^5

LIFT TO M-THEORY:

EXTRA CIRCLE APPEARS, RADIUS R

D4 BRANE $\Longleftrightarrow$ M5 BRANE WRAPPED ON S^1

NS5 BRANE $\longleftrightarrow$ M5 BRANE NOT WRAPPED ON S^1



SINGLE M5-BRANE WRAPPED ON

RIEMANN SURFACE Σ

$$v = x^4 + ix^5$$

$$t = \exp\left(-(x^6 + ix^{10})/R\right)$$

$$\Sigma: \quad t^2 - 2P_N(v)t + \Lambda_{N=2}^{2N} = 0 \equiv \text{Seiberg-Witten curve}$$

$$P_N(v) = \det(v - \Phi)$$

THE (2,0) THEORY ON THE M5 BRANE REDUCES TO THE LOW-ENERGY EFFECTIVE ACTION OF QUANTUM N=2 SYM

"MQCD"

AGREEMENT ONLY FOR BPS QUANTITIES

$$B_{\mu i} = \sum_{K} w_i^{(k)} A_\mu^{(k)}$$

$B_{\mu i}$: self-dual 6d two form

$w_i^{(k)}$: holomorphic one-forms on Σ

$A_\mu^{(k)}$: 4d gauge fields

PERIOD MATRIX

$$\tau_{Kl} = \int_{B_K} w^{(l)}$$

YIELD LOW-ENERGY GAUGE COUPLINGS

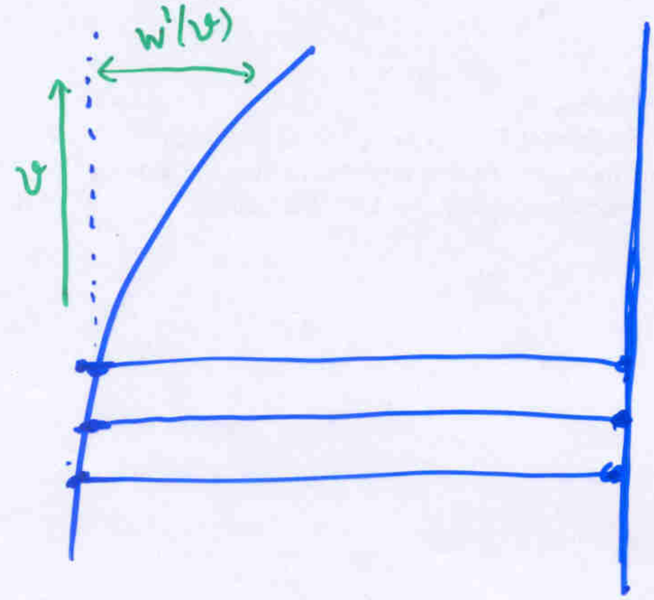
BREAK TO $\mathcal{N}=1$

$$\Delta \mathcal{L} = \mathrm{Tr}\left(\int d^4x\, d^2\theta\, W[\Phi]\right)$$

$W[\Phi]$: NON-TRIVIAL SUPERPOTENTIAL

Dijkgraaf-Vafa

BEND BRANES IN $w = x^8 + i x^9$ DIRECTION



JdB, Oz

D4-BRANES DO NOT FIT ; OPEN STRING MASS (NS_5-D4) $\sim W'(v)$

D4-BRANES ONLY FIT WHEN

$$\langle \Phi \rangle = \begin{pmatrix} p_1 & & & & & & \\ & \ddots & & & & & \\ & & p_1 & & & & \\ & & & p_2 & & & \\ & & & & \ddots & & \\ & & & & & p_2 & \\ & & & & & & \ddots \\ & & & & & & & p_n \\ & & & & & & & & \ddots \\ & & & & & & & & & p_n \end{pmatrix} \begin{matrix} \}N_1 \\ \}N_2 \\ \\ \}N_n \end{matrix}$$

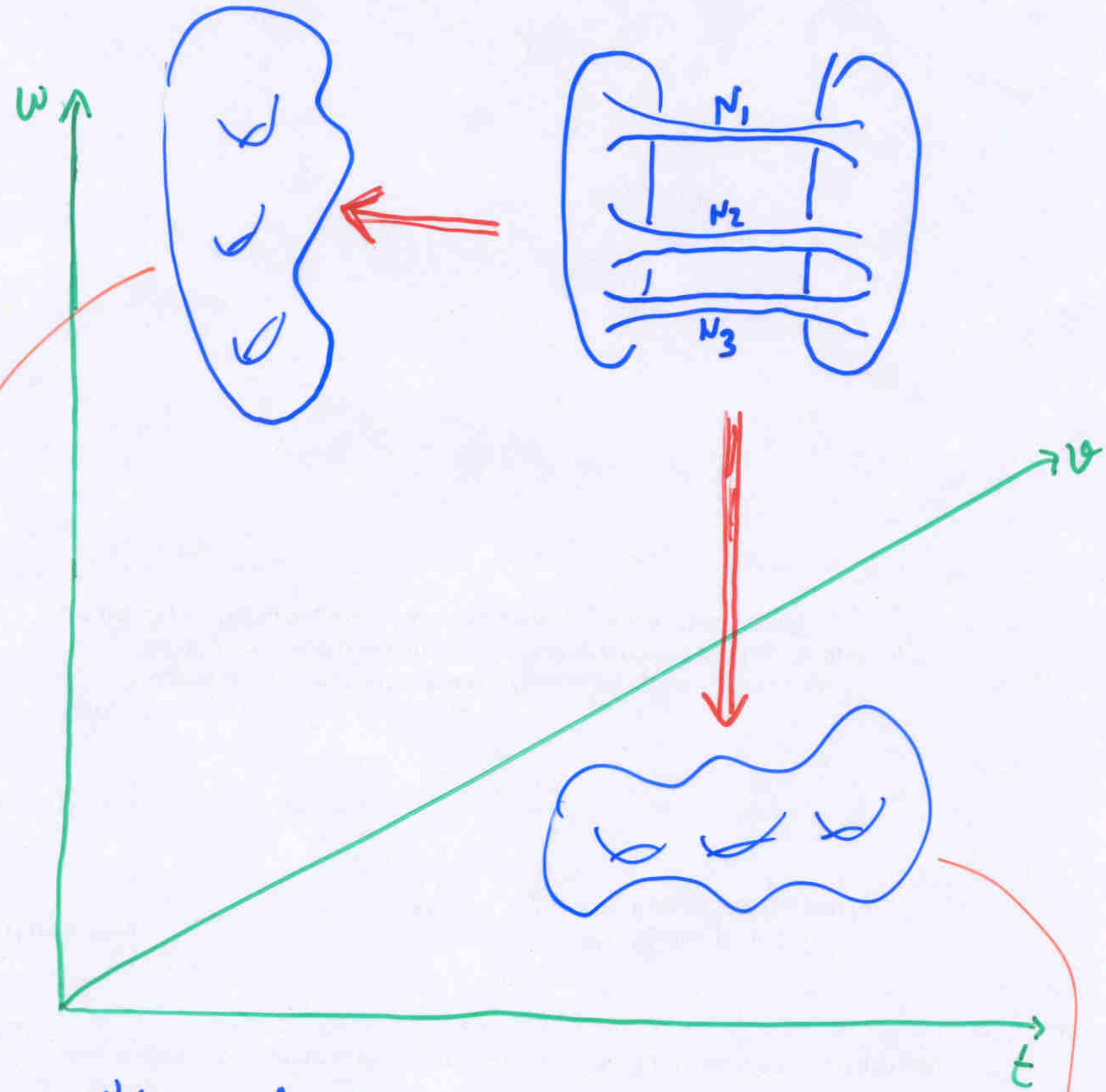
WITH p_i THE ROOTS OF $W'(v) = 0$

Field theory & branes agree

THIS BREAKS THE GAUGE GROUP TO

$U(N_1) \times \ldots \times U(N_n)$

LIFT TO M-THEORY : $\Sigma \subset \mathbb{R}^5 \times S^1$



$$w^2 - 2W_m'(v)w - f_{m-1}(v) = 0$$

$$t^2 - 2P_N(v)t + \Lambda_{\mathcal{N}=2}^{2N}$$

COMPATIBILITY OF

$$\begin{cases} w^2 - 2W_m'(v)w - f_{m-1}(v) = 0 \\ t^2 - 2P_N(v)t + \Lambda_{N=2}^{2N} = 0 \end{cases}$$

IMPLIES

$$P_N^2(v) - \Lambda_{N=2}^{2N} = S_{N-n}^2(v)\left[G_n^2(v) + f_{n-1}(v)\right]$$

$$W_m'(v)^2 + f_{m-1}(v) = H_{m-n}^2(v)\left[G_n^2(v) + f_{n-1}(v)\right]$$

THESE ALGEBRAIC EQUATIONS DETERMINE COMPLETELY THE LOCATION OF THE ISOLATED QUANTUM VACUA OF THE THEORY

* TWO NATURAL DIFFERENTIALS

Dijkgraaf-Vafa
Cachazo Douglas Seiberg Witten
Cachazo Seiberg Witten

$$\frac{dt}{t} = \mathrm{Tr}_{\text{gauge theory}}\left(\frac{dv}{v-\Phi}\right)$$

$$w\,dv = 2\,\mathrm{Tr}_{\text{matrix theory}}\left(\frac{dv}{v-M}\right) = \frac{-1}{16\pi^2}\mathrm{Tr}_{\text{gauge theory}}\left(\frac{W_\alpha W^\alpha\, dv}{v-\Phi}\right)$$

$$\int dM\, \mathrm{Tr}\left(\frac{1}{v-M}\right) e^{\frac{-1}{g_s}\mathrm{Tr}(W(M))}$$

BY COMPARING WE OBSERVE THAT IN THE MINIMA

(ALSO NATURAL BY STUDYING DIMENSIONS IN TWISTED (2,0) THEORY)

$$N_i = \frac{1}{2\pi i} \oint_{A_i} \frac{dt}{t} \qquad \tau = \oint_{B_i} \frac{dt}{t}$$

$$S_i = \frac{1}{2\pi i} \oint_{A_i} w\, dv \qquad \frac{\partial \mathcal{F}}{\partial S_i} = \oint_{B_i} w\, dv \qquad (*)$$

HERE, $S_i \sim \mathrm{Tr}_{U(N_i)} (W^\alpha W_\alpha)$ IS THE GAUGINO CONDENSATE SUPERFIELD IN THE (CLASSICALLY) UNBROKEN $U(N_i)$

THERE IS A QUANTUM LOW-ENERGY EFFECTIVE SUPERPOTENTIAL $W[S_i]$, COMPUTED E.G. USING A MATRIX MODEL. MINIMIZING $\left(\frac{\partial W[S_i]}{\partial S_i} = 0 \right)$ PUTS S_i EQUAL TO THE VALUES GIVEN IN $(*)$

<u>QUESTION:</u> WHAT IS THE M5-BRANE CONFIGURATION DESCRIBING S_i AWAY FROM SUSY MINIMA, AND HOW DO WE COMPUTE $W[S_i]$ FROM IT ?

NAIVE GUESS:

S_i ARE HOLOMORPHIC (CHIRAL) SUPERFIELDS $\Rightarrow$ LOOK FOR HOLOMORPHIC DEFORMATIONS.

THERE ARE NONE!

CLUE: LOOK AT
$$\begin{cases} N_i = \dfrac{1}{2\pi i} \oint_{A_i} \dfrac{dt}{t} & (1) \\ S_i = \dfrac{1}{2\pi i} \oint_{A_i} w\, dv & (2) \end{cases}$$

IGNORING t, THERE ARE HOLOMORPHIC DEFORMATIONS OF THE CURVE IN (w, v) SPACE AND ENOUGH TO PARAMETRIZE ALL S_i

THERE IS THEN A UNIQUE NONHOLOMORPHIC t SUCH THAT (1) IS STILL SATISFIED

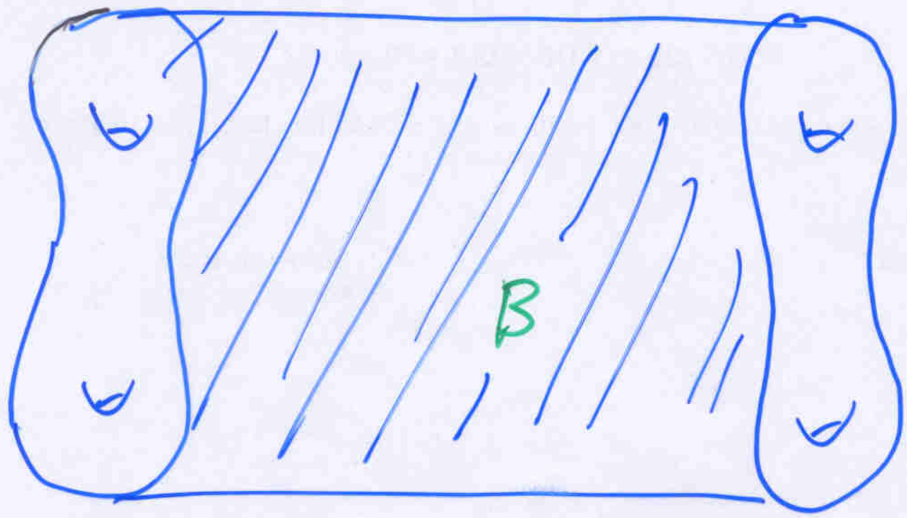
BECAUSE $t = \exp\left[-\dfrac{x^6 + i x^{10}}{R}\right]$ THE FORM $\dfrac{dt}{t}$ NEEDS TO HAVE INTEGER PERIODS, WHETHER t IS HOLOMORPHIC OR NOT.

PROPOSAL:

DEFORM Σ HOLOMORPHICALLY IN THE w, v DIRECTION AND THEN DEFINE A (NON-)HOLOMORPHIC t BY THE EQUATIONS $\frac{1}{2\pi i}\oint_{A_i} \frac{dt}{t} = N_i$. (& SIMILAR FOR B_i)

THIS YIELDS A NON-SUPERSYMMETRIC M5-BRANE THAT EXPLICITLY BREAKS SUSY. TO RESTORE SUPERSYMMETRY, WE NEED TO MAKE IT HOLOMORPHIC AND WE RECOVER THE PREVIOUS VACUA

THE SUPERPOTENTIAL



Σ' $\qquad$ $\partial B = \Sigma' \# (-\Sigma_0)$ $\qquad$ Σ_0 : REFERENCE SURFACE

WITTEN:

$$W(\Sigma') - W(\Sigma_0) = \frac{1}{2\pi i} \int_B \Omega^{(3,0)}$$

$$\Omega^{(3,0)} = R\, dv \wedge dw \wedge \frac{dt}{t}$$

PARTIAL INTEGRATION $\qquad$ SUBTLE

$$W = \frac{R}{2\pi i} \int_{\Sigma} \frac{dt}{t} \wedge w\, dv$$

MIRROR TO $\int H \wedge \Omega$

$$= R \sum_{i=1}^{n} N_i \frac{\partial F_0}{\partial S_i} - RNS \log \frac{\Lambda_{N=2}}{\Lambda_0}$$

bilinear identities

cf: Dijkgraaf-Vafa

AGREES !

EXAMPLE (MASS DEFORMED $U(N)$ THEORY)

$$W = \frac{1}{2} m \Phi^2$$

$$(W_1')^2 + f_0 \equiv m^2 v^2 + 2mS$$

THIS GUARANTEES $\frac{1}{2\pi i}\oint_A w\,dv = S$

$$t(v,\bar{v}) = \left[v + \sqrt{v^2 + \frac{2S}{m}}\right]^a \left[\bar{v} + \sqrt{\bar{v}^2 + \frac{2\bar{S}}{m}}\right]^b$$

$$\begin{cases} a - b = N \\ N \log\left(\dfrac{-2\Lambda_0^2 m}{\Lambda_{N=1}^3}\right) = \log\left(\dfrac{-2m\Lambda_0^2}{S}\right)^a \cancel{\log}\left(\dfrac{-2m\bar{\Lambda}_0^2}{\bar{S}}\right)^b \end{cases}$$

$\Rightarrow$ PRECISELY WHEN $S = \Lambda_{N=1}^3$ WE GET $b = 0$.

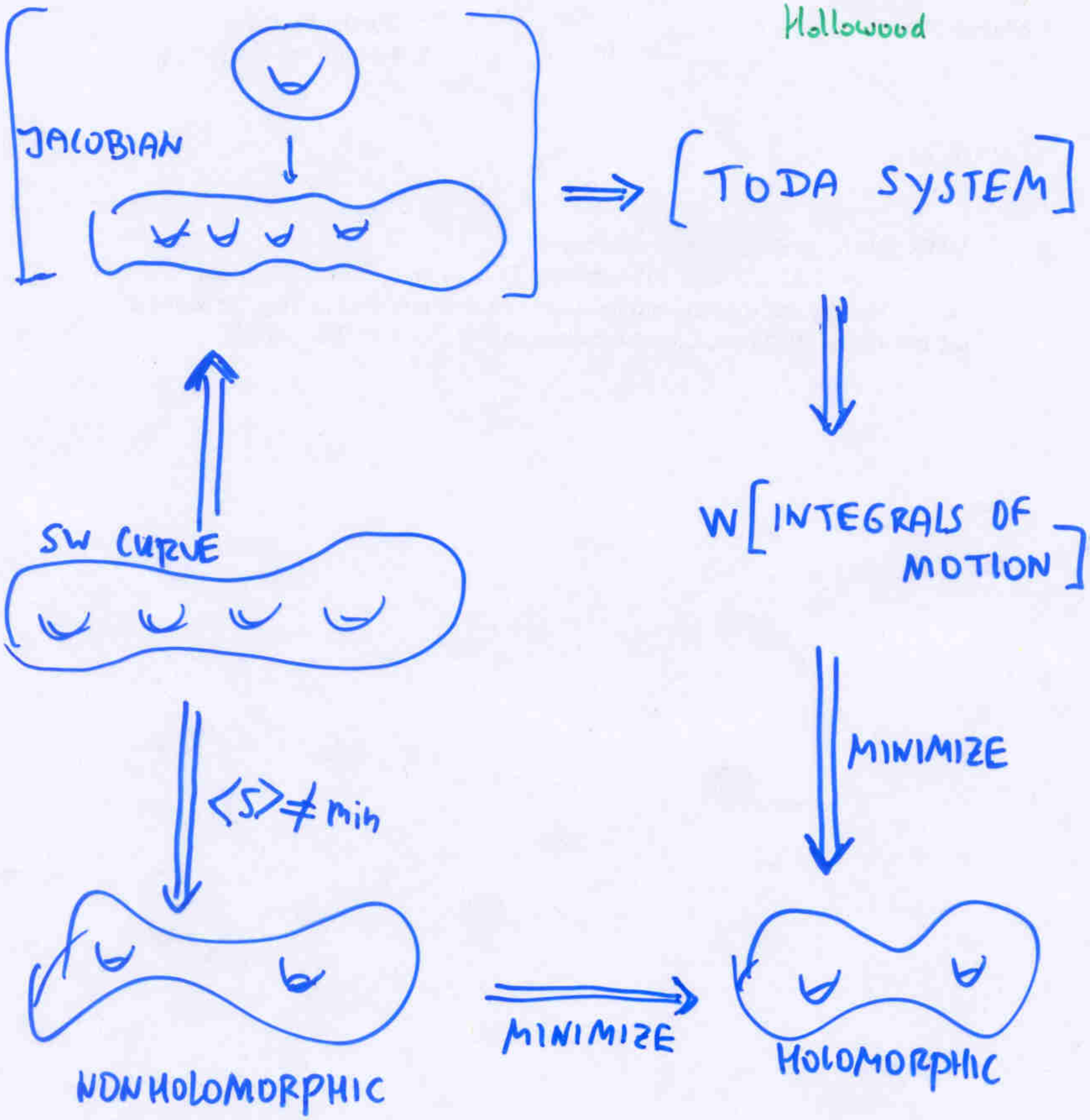
- CAN NOW EASILY GENERALIZE TO MANY OTHER GAUGE THEORIES FOR WHICH THERE IS A BRANE CONSTRUCTION

- CAN ALSO UNDERSTAND SUPERPOTENTIAL FROM THE DECONSTRUCTION OF THE (2,0) THEORY

REMARKS

RELATION TO THEORY ON $\mathbb{R}^3 \times S^1$ AND INTEGRABLE SYSTEMS ?

Dorey
JdB, Boels, Duivenvoorden, Wijnhout
Hollowood



$\Rightarrow$ STUDY TODA SYSTEM FOR NONHOLOMORPHIC SURFACES & GAUGE THEORIES W/MATTER.

TOPOLOGICAL STRINGS

B-MODEL ON $ux + H(v,w)=0$ $\qquad H = w^2 + W'(v)^2$

NONTOP IIB	TOP B-MODEL
$ux + H(v,w)=0$ + D5-branes on $\mathbb{P}^1$ $\downarrow$ T-DUALITY	$ux + H(v,w)=0$ + D1-branes on $\mathbb{P}^1$
IIA WITH D4 & NS_5-BRANES $\downarrow$ LIFT	$\downarrow$ BACKREACT
M-THEORY ON Σ: $\tilde{H}(v,w)=0$ VARIABLE t APPEARS $\oint_{A_i} \frac{dt}{t} = N_i$ $\oint_{A_i} w\,dv = S_i$	$ux + \tilde{\tilde{H}}(v,w)=0$ NO VARIABLE t $\oint_{A_i} w\,dv = g_s N_i$ cf matrix model

TOPOLOGICAL TWISTING EXCHANGES

$$g_s \frac{dt}{t} \Longleftrightarrow w\,dv$$

$$g_s H \Longleftrightarrow \Omega$$

CRAZY NEW DUALITY ?