Branes and Anomalies

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M theory involves two BPS, extended, dual branes M2, M5

We don't really have a useful formulation of M theory or know its fundamental symmetries and principles

Anomalies provide a robust way to probe some of the structure - they can be computed in the UV or IR and have a topological nature.

* Q_5^3 d.o.f. on Q_5 M5's (w/ Freed, Moore + Minasian)

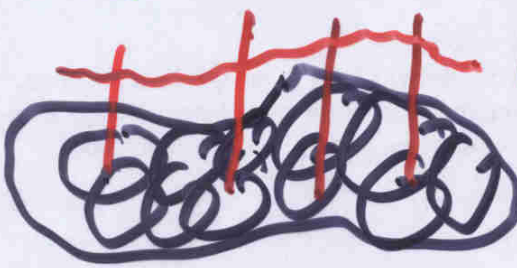
* Info on d.o.f. on self-dual string bndy of Q_2 M2's ending on Q_5 M5s (in progress w/ D. Berman)

1. Anomalies and Inflow

Chiral fermions coupled to a gauge field A:

$$e^{iS_{eff}(A)} = \mathrm{Pf}\,\not{D}_A \sim \sqrt{\mathrm{Det}\,\not{D}_A}$$

For each A, a phase. Is it well defined?



$\mathrm{Pf}\,\not{D}_A$ ~ section of a line bundle

Vanishing anomaly ⇒ trivialize bundle ⇒ no non-trivial topology in 2 param. families of A's

⇒ Gauge anomaly in 4k dim related to chiral anomaly in 4k+2 dim.

A physical model:

Non-chiral fermions coupled to a gauge field and an axion string in 3+1 dimensions.

$$\mathcal{L} = \bar{\Psi}\, i \not{D}_A \Psi - \bar{\Psi}(\Phi_1 + i\gamma_5 \Phi_2)\Psi + \mathcal{L}_\Phi + \mathcal{L}_A$$

Axion string: $\Phi = f(\rho)\, e^{i\theta}$

$f(\infty) = v$

$$v\bar{\Psi} e^{i\theta\gamma_5}\Psi \quad \Longrightarrow \quad \frac{1}{8\pi^2}\int \theta \,\mathrm{Tr}\, F\wedge F$$

$\partial_\mu j^\mu_5 \sim \mathrm{Tr}\, F\wedge F$

The topology of the string leads to an anomaly as follows.

Thin string: $\int_{S_1} d\theta = 2\pi = \int_{D_2} d^2\theta$

If $H_1 = d\theta \qquad H_3 = {}^*H_1$

$$dH_1 = d^*H_3 = 2\pi\,\delta_2(W_2 \hookrightarrow M_4) = 2\pi\,\delta(x)\delta(y)\,dx\wedge dy$$

String w.s.

Now let $\omega_4 = \mathrm{Tr}\, F\wedge F$

$$\text{descent}\begin{cases} \omega_4 = d\omega_3^{(0)} \\ \delta_\Lambda \omega_3^{(0)} = d\omega_2^{(1)} \end{cases}$$

Then

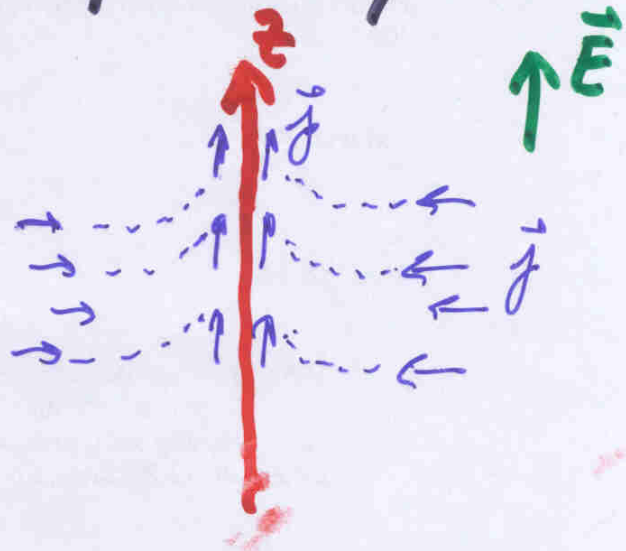
$$\delta_\Lambda \int_{M_4} \theta\,\omega_4 = \delta_\Lambda \int_{M_4} H_1 \wedge \omega_3^{(0)} = \int_{M_4} H_1 \wedge d\omega_2^{(1)}$$

$$= \int_{M_4} \delta_2(W_2 \hookrightarrow M_4)\wedge \omega_2^{(1)} = \int_{W_2} \omega_2^{(1)}$$

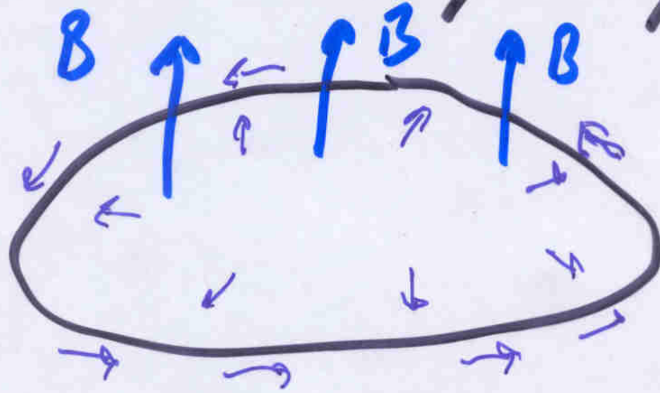
$\Rightarrow$ An anomaly on the string derived by descent from chiral anomaly in $D=4$.

This is cancelled because there are chiral fermion zero modes on the string coupled to A w/ precisely the opposite anomaly.

Picture is



This is realized physically in QHE



change $\vec{B} \Rightarrow \vec{E} \Rightarrow \vec{j}_{Hall}$

z.m. on 'string' ~ edge states in QHE

2. M5-brane anomaly story

Analogous to axion string

- zero modes are tensor mult. of (2,0) susy in $D=6$ w/ chiral fermions, asd tensor

- Fivebrane is magnetic source for field strength $G_4 = dC_3$

$$dG_4 = \delta_5 (W_6 \hookrightarrow M_{11})$$

8-form in curvature

- Analog of $\int \Theta\, \omega_4$ is $\int C_3 \wedge I_8(R)$

- Analog of gauge symmetries are diffeomorphisms mapping $W_6 \to W_6$

M5 breaks $SO(10,1) \to SO(5,1) \times SO(5)$

$$TM_{11}\Big|_{W_6} = TW_6 \oplus N$$

normal bundle

- diffeos of W_6
- diffeos acting as $SO(5)$ gauge transf. on $SO(5)$ conn. on N

(think of K.K. reduction on transverse S^4)

Witten: $$\int_{W_6} I_6^{(1)\,z.m.} + I_6^{(1)\,\text{inflow}} = \int_{W_6} \left[\frac{p_2(N)}{24}\right]^{(1)} \neq 0$$

Is M-theory inconsistent?

Anomaly cancellation (FHMM)

In D=11 SUGRA there is a C-S like term

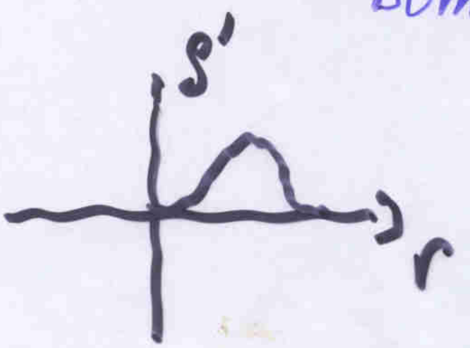
$$\frac{-1}{6(2\pi)^3}\int_{M_{11}} C_3\wedge dC_3\wedge dC_3 \qquad G_4 = dC_3$$

It is ind. of metric, but rather singular in the presence of a 5-brane w/ δ-func. source.

There is a well-defined math formalism for smoothing out the 5-brane (Bott + Tu).

$$dG_4 = d\rho\wedge e_4/2$$

bump form   global angular form



$$\int d\rho = 1$$

Now $dG_4 \neq 0$ so

$$G_4 = dC_3 - dg \wedge e_3^{(0)}/2$$

fluctuations

where
$$\begin{cases} e_4 = d e_3^{(0)} \\ \delta e_3^{(0)} = d e_2^{(1)} \end{cases}$$

(like $H_3 = dB_2 - \omega_3^{(0)}$ in Green-Schwarz)

$$\Rightarrow \quad \boxed{\delta C_3 = - dg \wedge e_2^{(1)}/2}$$

$\Rightarrow$ SO(5) variation of $\int C_3 \wedge G_4 \wedge G_4$

A careful computation shows the variation is

$$-\int_{W_6} \frac{[p_2(N)]^{(1)}}{24}$$

canceling the anomaly!

For a "flat" 5-brane w/ no SO(5) gauge field

$$\frac{e_4}{2} \propto \epsilon_{a_1 \ldots a_5} d\hat{y}^{a_1} \ldots d\hat{y}^{a_4} \hat{y}^{a_5} \quad \text{w/ } \hat{y}^a = y^a/r$$

$\hat{y}^a$ - coor. normal to W_6

When the SO(5) gauge field $\Theta_\mu^{ab} \neq 0$ this must be made covariant

$$d\hat{y}^a \to (d\hat{y}^a - \Theta^{ab} \hat{y}^b)$$

and e_4 has terms involving

$$F^{ab} = d\Theta^{ab} - \Theta^{ac} \wedge \Theta^{cb}$$

to ensure $de_4 = 0$

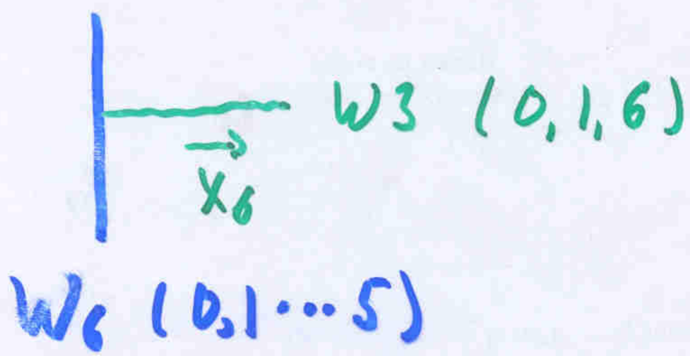
So what?

Since the CS term is cubic in C_3, this implies that the $SO(5)$ anomaly of Q_5 M5's scales as Q_5^3 - an interesting challenge for a microscopic description.

Are there other aspects of M5 dynamics that can be used to study this issue?

M2 - M5 anomalies

It is well known that an M2 can end on an M5:



This configuration preserves

$$\underbrace{\overset{(0,1)}{SO(1,1)} \times \overset{(2,3,4,5)}{SO(4)_T}}_{SO(5,1)} \times \underbrace{\overset{(7,8,9,10)}{SO(4)_N}}_{SO(5)_N}$$

For $Q_5 = 1$ this is described in the M5 w-volume theory as a self-dual soliton string (Howe, Lambert + West)

The fermion zero modes can be found from broken susy's and transform as

$$(2,1,1,2)^{1/2} \oplus (1,2,2,1)^{-1/2}$$

$SO(4)_T \times SO(4)_N$

and have an anomaly which comes from descent of

$$\chi(A_T) - \chi(A_N)$$

(Brax + Mourad 9707246)

Euler class

How is this cancelled?

$SO(4)_T$

When a M2 ends on an M5 so its w-volume W_3 has a bndy ∂W_3 there is a coupling

$$\underbrace{\int_{W_3} C_3}_{\text{Spacetime 3-form}} - \int_{\partial W_3} \underbrace{B_2}_{\text{5-brane w-vol 2-form}}$$

w/ gauge invariance

$$C_3 \to C_3 + d\Lambda_2$$

$$B_2\big|_{\partial W_3} \to B_2\big|_{\partial W_3} - \Lambda_2\big|_{\partial W_3}$$

Now $$\int_{\partial W_3} B_2 = \int_{W_6} B_2 \wedge \underbrace{\eta_{t4}(\partial W_3 \hookrightarrow W_6)}_{\text{Poincare Dual}}$$

Now IBP and use $\eta_4 \sim \chi(A_T)$ to get a coupling

$$\int_{W_6} H_3 \wedge \chi^{(0)}(A_T)$$

which has a variation cancelling the anomaly.

$SO(4)_N$

This involves a coupling studied by Ganor + Motl and by Intriligator

$$\int_{W_6} H_3 \wedge \Omega_3(\hat{\phi}, A_N)$$

$d\Omega_3$ is e_4 of earlier, but w/ $\hat{y}^i \to \hat{\phi}^i$ = normalized M5 w-volume scalars

One can show in the SU string bkgnd that

$$\Omega_3(\hat{\Phi}, A_N) \sim \chi^{(0)}(A_N)$$

and the coupling

$$\alpha_e \int_{W_6} H_3 \wedge \chi^{(0)}(A_N)$$

cancels the anomaly in the usual way.

For general Q_2, Q_5 there is a coefficient $\alpha_e(Q_5)$ in front of this term and the anomaly of the z.m. must scale as

$$\alpha_e(Q_5)\, Q_2$$

↑

from $dH_3 \sim Q_2 \delta^4$

Intriligator:

Coulomb branch of (2,0) theory $\quad G \to H \times U(1)$

$$d_e = \frac{1}{4}\left(|G| - |H| - 1\right)$$

$G = SU(N)$, $H = SU(N-1)$ $\quad d_e \sim N$

$G = SU(N)$, $H = U(1)^{N-1}$ $\quad d_e \sim N^2$

So the anomaly scales as $Q_5 Q_2$ or $Q_5^2 Q_2$.

The 1st agrees w/ SUGRA calc. of Berman, the 2nd is generic. This needs further study.

There are also implications for other systems:

Reduce M to IIA in 2 ways

- D2 ending on NS5 has $SO(4)_T$ anomaly
- D2 ending on D4 has $SO(4)_N$ anomaly

$\therefore$ Need D4 coupling

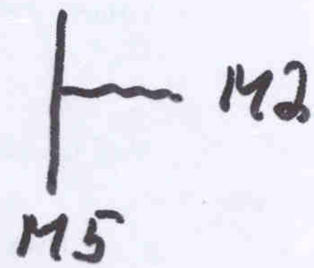
$$\sim \int_{\Sigma_5} F_2 \wedge \chi^{(0)}(F_N)$$

distinct from usual anomalous couplings

$$\int C \wedge \mathrm{Tr}\, e^F \sqrt{\hat{A}(T)/\hat{A}(N)}$$

Conclusions

* Q_5^3 d.o.f. on Q_5 M5's

* Analysis of d.o.f. on 

* New (1-loop) anomalous D-brane couplings