Strings as Consistent Quantum Theories:

The search for Dual Loops

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Olivefest 2004

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The Early Days

1968 Gabriele Veneziano

1969 Koba - Nielsen
N-point amplitude

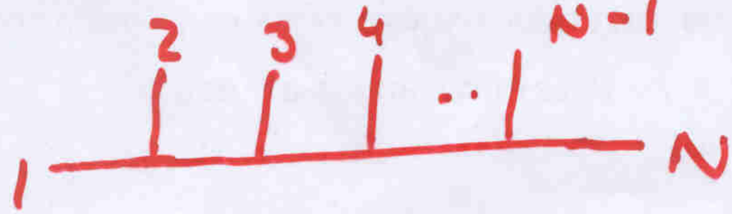
Fubini, Veneziano, Nambu

factorization, Vertex op.

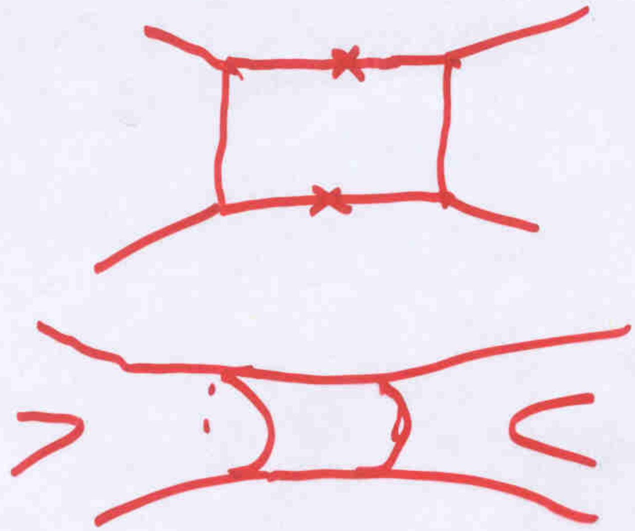
1971 Ramond, Neveu - Schwarz,
Thorn Superstring

1972 Brower, Goddard - Thorn
No ghost theorem

Status: We knew N-point tree diagrams.



1971 Lovelace checked



Ugly divergencies

David told him:

Go to $d = 26$. Take away 2 partition functions in the measure

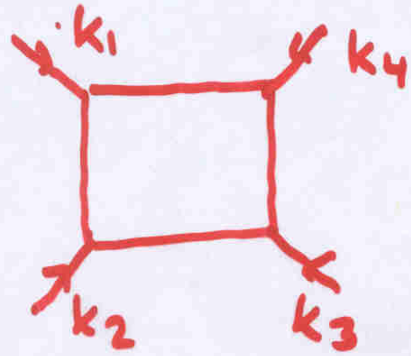
Note No action, No string picture, just tree diagrams

However, 't Hooft had made Yang-Mills theory popular

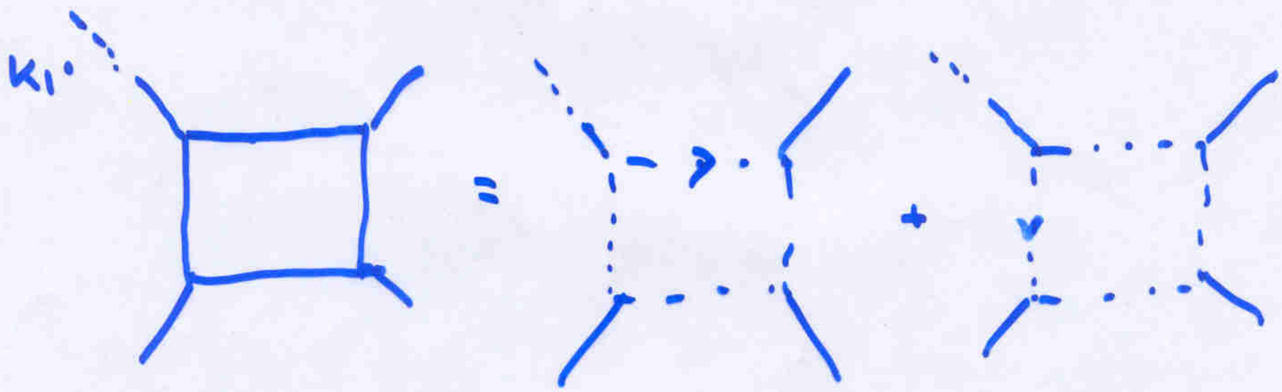
Feynman - Faddeev - Popov ghosts understood.

Our first attempt

Consider a naive loop in Y-M.

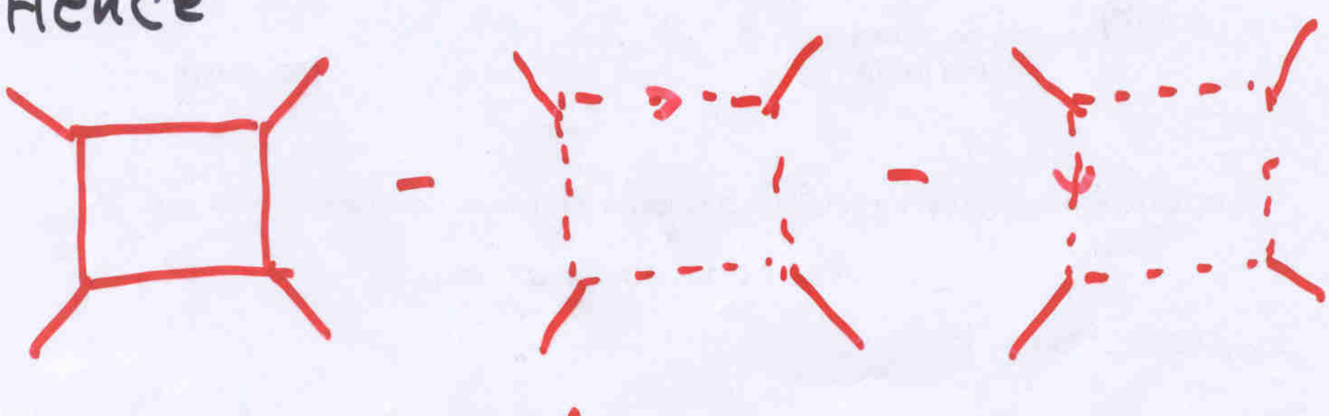


$$k_1 \cdot A_4 = 0$$



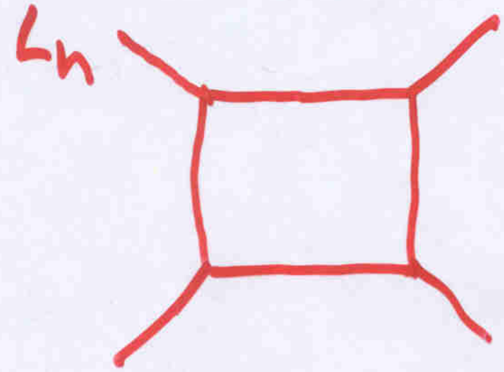
interpret as ghost loops

Hence



unitary

Try the same technique for a dual loop.



$$(L_n - L_0 + 1) z^{L_0 - 1} V = z^{L_0 + n - 1} V (L_n - L_0 + 1)$$
$$- z \frac{d}{dz} \left((1 - z^n) z^{(L_0 - 1)} V \right)$$

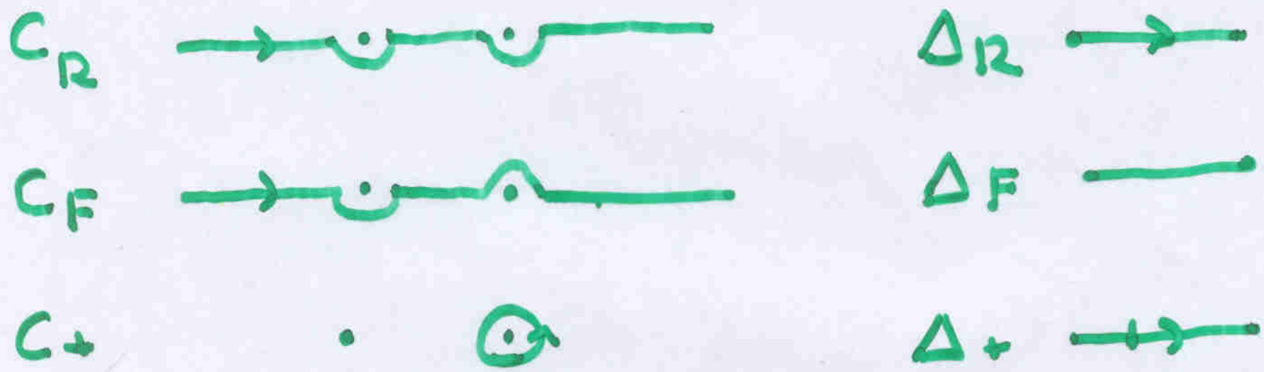
However, the procedure did not work. What was the problem?

More later.

Feynman's Tree Theorem

Consider three different kinds of Green functions, Δ_R, Δ_F, Δ_+

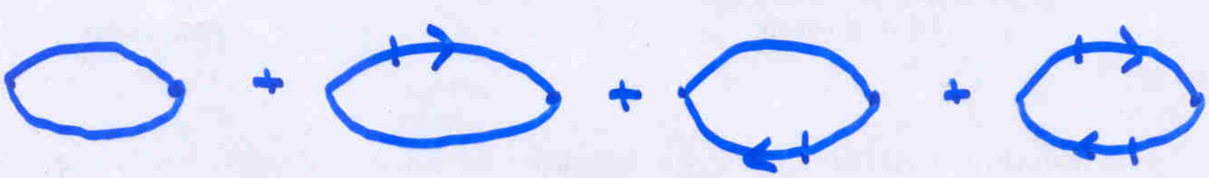
$$\Delta_i = -\frac{1}{(2\pi)^4}\int_{C_i} d^4k \, \frac{e^{ik\cdot x}}{k^2-m^2}$$

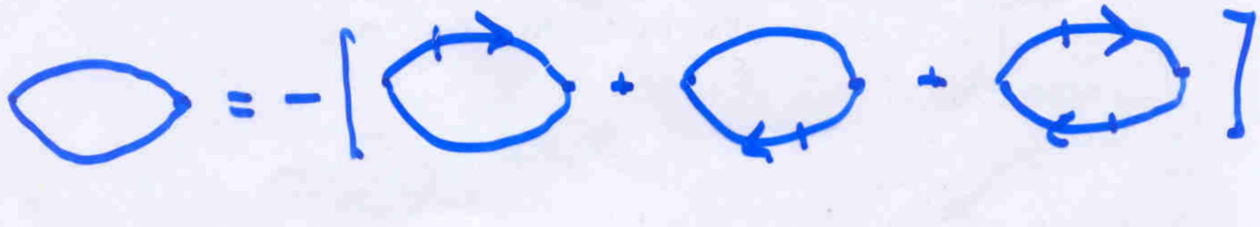


$$\Delta_R(x) = \Delta_F(x) + \Delta_+(x)$$

Consider now the loop

$\int d^4x \, \Delta_R(x)\Delta_R(-x) =$  $= 0$

$=$ 

$\Rightarrow$ 

More general

$$\int \frac{i\,d^4k}{(2\pi)^4} \prod_i \frac{g_i}{k_i^2 - m_i^2 - i\epsilon} = -\sum_{\substack{\text{ways of} \\ \text{cutting}}} \int \frac{i\,d^4k}{(2\pi)^4} \prod_i \Delta_i(k_i)\, g_i$$

Has all the correct singularities

Feynman's proposal for Yang-Mills Theory

Use the r.h.s and introduce a projection operator when a propagator is cut.

— . . . —

Take over this scheme to dual models

We need a projection operator onto the physical states

P_N

Pole when $P_N^2 = 2(N-1)$

Projection to full Fock space

$$\mathcal{M}_N = \oint \frac{dz}{2\pi i}\, z^{L_0(P_N)-2}$$

Projection onto physical states

$$\mathcal{J}(k) = \oint \frac{dy}{2\pi i}\, y^{\mathcal{L}_0 - H - 1}$$

$$\mathcal{L}_0 = \sum_{n=1}^{\infty} \sum_{i=1}^{d-2} A_n^{i+}(k)\, A_n^{i}(k)$$

$k \cdot P_N = 0$

DDF operators
Not covariant

A direct computation gives

$$E = \mathcal{L}_0 - H = (D_0 - 1)(L_0 - 1) + \sum_{n=1}^{\infty} (D_{-n} L_n + L_{-n} D_n)$$

$$D_n = \left\langle \frac{1}{k \cdot P(z)} \right\rangle_n$$

$$\mathcal{J}^2 = \mathcal{J}$$

Consider

$$\langle \psi | \, m_N (\mathcal{J} - 1) \, | \phi \rangle$$

$$L_n | \phi \rangle = (L_0 - 1) | \phi \rangle = 0$$

Start with

$$\langle \psi | m_N (y^E - 1) | \phi \rangle$$

$$y^E - 1 = E \int_1^y dz \, z^{E-1}$$

Let $\overleftarrow{L_{-n}}$ $\overrightarrow{L_n}$

$$\Rightarrow \langle \psi | m_N (y^E - 1) | \phi \rangle = 0$$

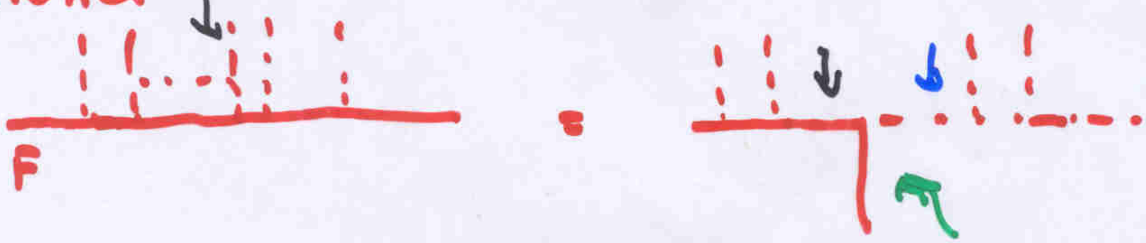
$$\Rightarrow \langle \psi | m_N (I - 1) | \phi \rangle = 0$$

NO GHOSTS.

Interlude

The projection operators for R-N-S models could be constructed in the same way. Many results

Ramond



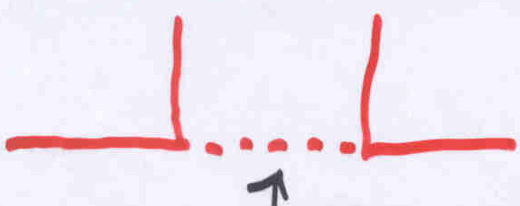
Introduce projection operator

Fermion emission vertex

Move it through vertex

⇒ fermion mass = 0

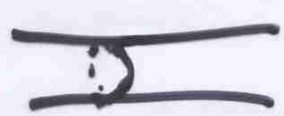
⇒ bosons satisfy N-S model!



projection operator

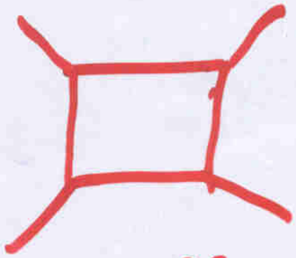
⇒ 4 fermion amplitude

Also



open-closed string vertex

Back to tree theorem

 =

$$- \sum_{\substack{\text{ways of}\\ \text{cutting}}} \int \frac{i\, d^{26}k}{(2\pi)^{26}} \mathrm{Tr}\left(\Delta_1 V_1 \ \ldots \ \Delta_4 V_4 \right)$$

$$\Delta_i(k) = \begin{cases} -\frac{1}{2} \int_0^1 \frac{dz_i}{z_i}\, z_i^{L_0 - 1} & \text{uncut} \\ \sum_{n_i=0}^{\infty} \Big[2\pi i\, \theta(k_i^0)\, \delta\left(k_i^2 - 2(N_i - 1)\right) \times \oint \frac{dz_i}{2\pi i z_i}\, z_i^{H - N_i} \Big] & \text{cut} \end{cases}$$

Sketch of the evalution 1 cut

$$I = \oint \frac{dy}{2\pi i y} \oint \frac{dz_1}{2\pi i z_1} \prod_{i=2}^{4} \int_0^1 \frac{dz_i}{z_i}$$

$$\mathrm{Tr}\left(y^E z_1^{L_0 - 1} V_1 z_2^{L_0 - 1} \ldots \right)$$

Consider $f(y) = \mathrm{Tr}\left(y^{a^\dagger a} \right)$

$$\frac{1}{f} \frac{\partial f}{\partial y} = \frac{1}{1-y} \quad \Rightarrow \quad f(y) = \frac{\text{const}}{1-y}$$

$$f(y) = \mathrm{Tr}\left(y^E z_1^{L_0 - 1} V_1 \ldots\right)$$

$$\frac{\partial f}{\partial y} = \mathrm{Tr}\left(E\, y^{E-1} z_1^{L_0 - 1} V_1 \ldots\right)$$

Move $L_n \rightarrow$

$\leftarrow L_{-n}$

Long calculation

$$\Rightarrow I = \oint \frac{dz_1}{2\pi i z_1} \prod_{i=2}^{N} \int_0^1 \frac{dz_i}{z_i} \prod_{n=1}^{\infty} (1 - z_{1n})^2$$

$$\mathrm{Tr}\left(z_1^{L_0 - 1} V_1 \ldots V_4\right)$$

new term

We have to do this for every term in the sum

Expand every expression in terms of all particle propagators

Final expression

$$-\sum_{\substack{\text{ways of} \\ \text{cutting}}} \int \frac{i\, d^{26}k}{(2\pi)^{26}} \sum_{\text{particle}} T_{n_1 n_2 n_3 n_4}$$

IF we can interchange the two sums THEN we can use Feynman's prescription to get a Feynman loop and then sum up again over all particles

$$A_N = \int \frac{i\,d^{26}k}{(2\pi)^{26}} \prod_{i=1}^{4} \left[-\frac{1}{2} \int_0^1 \frac{dz_i}{z_i} \right]$$

$$\prod_{n=1}^{\infty} (1 - z_{1N})^2$$

$$\mathrm{Tr}\, \left(z_1^{L_0 - 1} V_1 \ldots z_N^{L_0 - 1} V_N \right)$$

Correct result

What about closed strings? Infinite over counting. Miss modular invariance!

Dual Models (String Theory) could have a perturbation series.

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June 5, 1973

Dr. L. Brink
Dr. D. Olive
CERN
Theory Division
1211 Geneva 23
Switzerland

Gentlemen:

I was pleased to see my tree theorem being of use. (Ref. TH.1620-CERN). At last (after ten years) I have written a more detailed account of it in the book "Magic Without Magic - John Archibald Wheeler," edited by Kaluder, published by Freeman, 1972. Maybe you would be interested, but your article already contains almost everything I know, and it is all explained with clarity and simplicity. Thank you.

Yours sincerely,

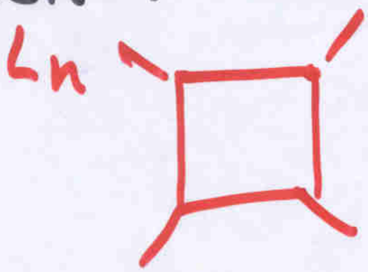
R. P. Feynman

Richard P. Feynman

RPF;ht

Encl.

What was wrong in diagrammatic approach ?



Pre BRST !

Correct $L_n = L_n^{(\alpha)} + L_n^{(ghosts)}$

We lacked the <u>two</u>-dimensional ghost fields

1986 David with Freeman use BRST

$$Q = \sum_{-\infty}^{\infty} [L_{-m}^{(\alpha)} + \tfrac{1}{2} L_{-m}^{(c)} - \delta_{n,0}] c_m$$

In propagators use full L_0

$$L_0 |N,P\rangle = (N - 1 - \tfrac{P^2}{2}) |N,P\rangle$$

$$\hat{E} = N_{tr}(k) - N$$

$$\hat{E} = E - \sum_{n=1}^{\infty} n \left(c_{-n} \bar{c}_n + \bar{c}_{-n} c_n \right)$$

$$\hat{E} = \{ Q, S \}$$

$$\hat{E} \, | N, p \rangle = Q S \, | N, p \rangle$$

vanishes on non-trivial cohomology classes of Q.

No ghost theorem trivial

For the loop, need to evaluate

$$f(y) = \mathrm{Tr} \left(y^{QS + SQ} \ldots\ldots \right)$$

Take a derivative. Move Q through.

$$y \frac{\partial f}{\partial y} = 0 \implies \mathcal{I} = 1$$

in loop.

Partition functions from ghosts