DAVID OLIVE Symposium

(Swansea, March 24 2004)

Circa 1967:

I 'High Energy' Physics

- There are only three forces worth considering.
- GRAVITY is (a) Irrelevant (too weak)
  (b) Too difficult (quantum?)
- COSMOLOGY is branch of Astrology.

II General Relativity

- Only Einstein's equations are worth considering (maybe with Maxwell's)
- Quantum mechanics is embarrassing

THINGS WERE EVEN WORSE IN SOME PLACES!
(DAMTP, Berkeley, ...)

- Quantum Field Theory had failed (QED successes were "accidental").
- Only the Strong Force can be studied since there are no massless hadrons!
  (Success of QED are "mysterious")

The S-MATRIX ruled and QFT was banished

(although ϕ^3 field theory was useful as a tool)

The Bible: 'The Analytic S Matrix'

by Geoffrey Chew

The New Testament: 'The Analytic S-Matrix'

by R.J. Eden, P.V. Lanshoff, D.I. OLIVE, J.C. Polkinghorne

$\boxed{\text{Chapter 4}}$ is surely by David and is

remarkable (the others are worth reading - once).

Unitarity + analyticity

BUT !

Born out of despair, the S-Matrix approach led directly to VENEZIANO's MODEL and STRING THEORY

DAVID made some of the most important contributions in the earliest days of string theory

e.g. Fermion vertex; loop calculations; very early insights into connections with Yang-Mills theory and Gravity; SUSY in string theory

These and his many subsequent achievements are characterised by powerful mathematics applied to remarkable insight with great elegance.

THANK YOU, DAVID

" One of the most remarkable discoveries in elementary particle physics is the existence of the complex plane." !

Opening line of Eden, Landshoff, Olive + Polkinghorne

IIB SUSY AND LOW ENERGY ACTION

- MONTONEN - OLIVE duality and type IIB superstring

  $\frac{1}{\alpha'}$, R^4 effective interaction; AdS/CFT; instantons

- Flux contributions to R^4 interaction and parallel D3-branes (with C. Stahn)

- Plane-wave D-instantons and $\mathcal{N}=4$ SUSY Yang-Mills instantons in BMN limit. (with S. Kovacs and A. Sinha)

Montonen-Olive Duality

Generalised electromagnetic duality

- acts on electric, magnetic fluxes in

$\mathcal{N}=4$ Super Yang-Mills

Coupling $\tau \equiv \tau_1 + i\tau_2 = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}$

$SL(2,\mathbb{Z})$: $\tau \to \frac{a\tau+b}{c\tau+d}$ $(ad-bc=1)$

(i) Higgs $\phi=0$ Superconformal point

(ii) $\phi\neq 0$ Spont. symm. breaking

Massive Dyonic states (p,q) $SL(2,\mathbb{Z})$ doublet

electric magnetic charges

Embed in IIB superstring

$\mathcal{N}=4$ Super Yang-Mills $\Rightarrow$ D3-brane

Dirac-Born-Infeld effective action

AGSEN44/SB

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Further additional papers herewith for the meeting of SENATE to be held on 10 March 2004.

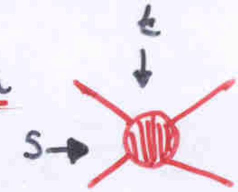
9 March 2004

Steven Boarder
Committee Secretary

AGENDUM

2 UWS Strategic Direction proposals
.1 Report from Senior Management Team, (paper L8032 attached);
.4 Faculty discussions and motions – further report clarifying the views of the Faculty of Business, Economics and Law (paper L8036 attached).

$SL(2,\mathbb{Z})$ and IIB effective action



Tree-level e.g. 4 graviton scattering

(Virasoro 1968)

$$A_4 = e^{-2\phi}\, K \;\frac{1}{stu}\, \exp \sum_{n=1}^{\infty} \frac{\zeta(2n+1)}{n+\frac{1}{2}}\, \alpha'^{\,2n+1}\left(s^{2n+1}+t^{2n+1}+u^{2n+1}\right)$$

dilaton

Linearised R^4 $(R \sim k_\mu k_\nu \zeta_{\mu\nu})$

$s+t+u=0$

$$= e^{-2\phi} K \left(\frac{1}{stu} + \alpha'^3 \zeta(3)\left(s^3 \cancel{+t^3+u^3}\right) + \ldots\right)$$

- Einstein - Hilbert terms (tree-level supergravity)

$$\frac{1}{(\alpha')^4}\int d^{10}x \sqrt{g}\, e^{-2\phi} R$$

$SL(2,\mathbb{Z})$ invariant

- $O((\alpha')^3)$ terms

$$\frac{\zeta(3)}{\alpha'}\int d^{10}x \sqrt{g}\, e^{-2\phi} C^4$$

NOT $SL(2,\mathbb{Z})$ invariant

A specific contraction of four WEYL curvatures

$$\tau \equiv C^{(0)} + i e^{-\phi}$$

absent in Einstein frame

SL(2,Z) COMPLETION

$$\frac{1}{\alpha'}\int d^{10}x\,\sqrt{g}\; e^{-\phi/2}\; f^{(0,0)}(\tau,\bar\tau)\; C^4$$

with
$$f^{(0,0)} = \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m+n\tau|^3}$$

modular weight $(0,0)$ - scalar

c.f. $$f^{(w,\bar w)} \xrightarrow[SL(2,\mathbb{Z})]{} (c\tau+d)^{w}(c\bar\tau+d)^{\bar w}\, f^{(w,\bar w)}$$

Tree 1-loop

$$e^{-\phi/2} f^{(0,0)} \simeq 2\zeta(3)\, e^{-2\phi} + \frac{\pi^2}{3}$$
$$+\, e^{-\phi/2}\left(\sum_{k>0} \mu(k)\, e^{2\pi i k\tau}\left(1+O(\tau_2^{-1})\right) + c.c.\right)$$

Series of D-instantons charge k

- Many terms $O(1/\alpha')$ related by SUSY

e.g. $$\left(\partial G\,\partial G\,\partial G^*\partial G^* + \lambda\lambda\,\lambda^*\lambda^*\right) f^{(0,0)}$$

$H_{RR}+iH_{NSNS}$ $\lambda_1+i\lambda_2$ dilatini

$$+\; G^8 f^{(4,-4)} + \lambda^{16} f^{(12,-12)} + \lambda^8 C^2 f^{(6,-6)} + \ldots$$

Relative coeffs? COMPLETE $O(1/\alpha')$ action?

Match with Yang-Mills via AdS/CFT

$$e^{-\phi} \equiv g_s = g^2/4\pi$$

$$\alpha'/L^2 = (g^2 N)^{1/2}$$

e.g. $AdS_5 \times S^5$

$$ds^2 = \frac{L^2}{\rho^2}(dx^2 + dy^2)$$

x^μ $(\mu = 0,1,2,3)$ $\quad$ y^i $(i=1,\dots,6)$

$\rho = |y|$

$$\frac{1}{\alpha'} e^{-\phi/2} L^2 \longrightarrow N^{1/2}$$

SU(N) Yang-Mills

$\Rightarrow$ Instanton measure:

$$N^{1/2}\, \mu(K)\, \frac{d^4 x_0\, d\rho_0}{\rho_0^5}\, d^5\hat{\Omega}\, d^8\eta^A_\alpha\, d^8\bar{\xi}^A_{\dot\alpha}$$

'exact' moduli $\qquad$ $AdS_5 \times S^5 \times$ Poincaré SUSY's $\times$ Conformal SUSY's

Verified by explicit evaluation of ADHM measure in large-N limit (saddle pt.)

(Dorey, Hollowood, Khoze, Mattis + Vandoren) **

- Integrate out (quasi) moduli:

$$\underbrace{w_{\dot\alpha u},\ \bar{w}^{\dot\alpha u}}_{\text{bosons}},\ \underbrace{\nu^A_u,\ \bar{\nu}^{A u}}_{\text{fermions}} \qquad (K=1)$$

$u = 1,\dots,N$ $\qquad$ $A = 1,2,3,4$ SU(4)

Hypermultiplets

(Orientation of SU(2) in SU(N) moduli)

MBG + C Stahn hep-th/0308061

Generalise to background with flux F_5

e.g. Parallel D3-branes $SU(N) \to S(U(M_1)\times\ldots\times U(M_\ell))$

(not conformal) $\sum_{r=1}^{\ell} M_r = N \qquad M_r \gg 1$

$$ds^2 = H^{-\frac{1}{2}} dx^2 + H^{\frac{1}{2}} dy^2 \qquad H = \sum_{r=1}^{\ell} \frac{M_\ell}{|y-y_r|^4} + 1$$

$$F_5 = (1 + *)\, dC^{(4)}$$

$$\hookrightarrow C^{(4)} = H^{-1}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

C^4 interaction with five-form flux

$M = 0, 1, \ldots, 9$

$$\int d^{10}x \sqrt{g} \int d^{16}\theta \left[\theta \Gamma^{MNP}\theta\, \theta\Gamma^{QRS}\theta\, \mathcal{R}_{MNPQRS}\right]^4 f^{(0,0)}(\tau, \bar\tau)$$

$$\mathcal{R}_{MNPQRS} = \frac{1}{8} g_{PS}\, C_{MNQR} + \frac{i}{48} D_M F_{NPQRS} + F_5^2 \text{ terms} \quad (+\, G_3 G_3^*)$$

Motivated by nonlinear SUSY (but NOT proved!)

Need $\delta^{(3)}\psi \sim (\alpha')^3\, C^3 D\epsilon, \ \ldots$ (M.B.G + S.Sethi)

where $\delta\Phi = \delta^{(0)}\Phi + (\alpha')^3 \delta^{(3)}\Phi + \ldots$

Action $S = S^{(0)} + (\alpha')^3 S^{(3)} + \ldots$

$\delta S = 0 \qquad \delta^{(0)} S^{(3)} + \delta^{(3)} S^{(0)} = 0$ (A)

$[\delta, \delta]\Phi = $ eqs. of motion $\Phi \qquad \Rightarrow [\delta^{(0)}, \delta^{(3)}]\Phi = \frac{\delta S^{(3)}}{\delta \Phi}$ (B)

BUT

$$\det \mathring{e}\,({}^{0}R)^2 = H^{-1}\,\partial_y^2\,\partial_y^2\,H \qquad \nabla^2 H = 0$$

Let $R = {}^{0}R + \hat{R}$ ← fluctuation

⇒ Induced interactions:

e.g. $$\int d^{10}x\ \underbrace{H^{-1}\,\partial_y^2\,\partial_y^2\,H}_{{}^{0}R^2}\ \Lambda^8\ f^{(4,-4)}_{(\tau,\bar{\tau})}$$

Contains D-instantons

How does this measure factor arise in the dual Yang-Mills theory on $|y^i| \to \infty$ boundary?

Non-renormalisation of D3-brane (at $O(1/\alpha')$)

Background geometry:

$$^{\circ}R_{\mu\nu i\rho\sigma j} = \frac{1}{144}\left(2\, g_{\mu[\rho}\, g_{\sigma]\nu} - i\, \epsilon_{\mu\nu\rho\sigma}\right) B_{ij}$$

$$^{\circ}R_{ijklmn} = -\frac{1}{48}\left(6 B_{il}\, g_{jm}\, g_{kn} - i\, B_i^{\,p}\, \epsilon_{jklmnp}\right)$$

$$B_{ij} = 2 D_i D_j A \quad (\text{where } H = \exp 2A)$$

Self-dual:

Let $\quad R_{abcd} \equiv \left(\Gamma^{MNP}\right)_{ab} \left(\Gamma^{QRS}\right)_{cd} R_{MNPQRS}$

$a = 1, \ldots 16 \quad SO(9,1)$ spinor

Decompose: $\quad \underset{16}{a} = \underset{8}{(A, \dot\alpha)} + \underset{8}{(\bar A, \alpha)}$

$SO(9,1) \to SO(3,1) \times SU(4)$

8 components

$$^{\circ}R \equiv {}^{\circ}R_{(A,\dot\alpha);(B,\dot\beta);(C,\dot\gamma);(D,\dot\delta)}$$

i.e. $\left({}^{\circ}R\right)^4 \equiv {}^{\circ}R_{[a_1 b_1 c_1 d_1} \cdots\cdots {}^{\circ}R_{a_4 b_4 c_4 d_4]} = 0$

$\equiv \int d^{16}\theta\, \left[{}^{\circ}R\, \theta^4\right]^4$

and $\left({}^{\circ}R\right)^3 = 0$

It follows that R^4 does not alter the classical equations for the dilaton, metric or F_5

(Constrained) Instanton Measure

$$\phi^i_{uv} = y^i_u \, \delta_{uv} \qquad SU(N) \to S(U(M_1) \times \ldots U(M_\ell))$$

$$Z = g^4 \int \underbrace{d^8\eta^A_\alpha \, d^4 x_0^\mu}_{\text{SUSY Poincaré}} \hat{Z}$$

$$\hat{Z} = -\frac{1}{16\pi^2} \int d^6\chi \; \partial^2_\chi \partial^2_\chi I_N(\chi)$$

χ^i vector modulus (couples to fermion bilinear)

where

$$I_N(\chi) = \sum_{u=1,\ldots,N} \frac{1}{x_u} \prod_{v \neq u} \frac{x_u^2}{x_v^2 - x_u^2}$$

with $x_u = (\chi - y_u)^2$ (Dorey, Hollowood, Khoze)

Consider $N \to \infty$ with ℓ groups of degenerate eigenvalues with M_r in each group $r = 1, \ldots, \ell$

Use saddle pt. to give:

$$I_{\{M_r\}}(x_1, \ldots, x_\ell) \simeq -\frac{1}{8\pi^2\sqrt{\pi}} \left(\sum_r \frac{M_r}{|\chi - y_r|^4} \right)^{1/2}$$

$$\simeq H^{1/2}$$

in agreement with $O(\frac{1}{\alpha'})$ SUGRA calculation.

(correlation fns ?)