Minimal String Theory

Nathan Seiberg

Strings, Gauge Fields and Duality
A conference to mark the retirement of
Professor David Olive
University of Wales, Swansea
March, 2004

David Olive is one of the founding fathers of modern theoretical physics.

His pioneering work in field theory and string theory has blazed the trail for many developments. It is both deep and visionary.

He introduced some of the crucial ingredients of string theory, including supersymmetry and duality, and his work on solitons, two dimensional CFT and integrable systems is an essential part of modern string theory.

We have gathered here to mark David's retirement.

This retirement is only from administrative duties. Now he is going to have more time for research, and will undoubtedly produce more spectacular results, which will set the direction of our field for many years to come.

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Klebanov, Maldacena and N. S., hep-th/0309168

N.S. and Shih, hep-th/0312170

Motivation

- Simple (minimal) and tractable string theory

- Explore D-branes, nonperturbative phenomena

- Other formulations of the theory – matrix models, holography

Descriptions

- Worldsheet: Liouville ϕ + minimal CFT (2d gravity)

- Spacetime: Dynamics in one (Euclidean) dimension ϕ

- Matrix model: Eigenvalues λ

Nonlocal relation between ϕ and λ.

Here: A Riemann surface emerges from the worldsheet description. It leads to a “derivation” of the matrix model and sheds new light on the nonlocal relation between ϕ and λ.

Roughly, ϕ and λ are conjugate (T-dual) (Moore and N.S.)

Outline

- Review of minimal CFT
- Review of Liouville theory
- Minimal string theory
- Review of D-branes in Liouville theory

- D-branes in minimal string theory

- Geometric interpretation

- Matrix Model

- Conclusions

Review of minimal CFT

Labelled by $p < q$ relatively prime

$$c = 1 - \frac{6(p-q)^2}{p\,q}$$

Finite set of Virasoro representations

$$\Delta(\mathcal{O}_{r,s}) = \frac{(rq - sp)^2 - (p-q)^2}{4p\,q}$$

$$1 \le r < p \ , \quad 1 \le s < q \ , \quad sp < rq$$

Review of Liouville theory

Worldsheet Lagrangian

$$(\partial\phi)^2 + \mu\, e^{2\,b\,\phi}$$

Will set in the second term (cosmological constant) $\mu = 1$

Central charge

$$c = 1 + 6Q^2$$
$$Q = b + \frac{1}{b}$$

Virasoro primaries

$$\Delta(e^{2\alpha\phi}) = -\left(\frac{Q}{2} - \alpha\right)^2 + \frac{Q^2}{4}$$

Degenerate representations labelled by integer $r, s \geq 1$

$$2\alpha_{r,s} = \frac{1}{b}(1 - r) + b(1 - s)$$

have special fusion rules and allow to solve the theory (Dorn, Otto, Zamolodchikov, Zamolodchikov, Teschner)

Minimal String Theory

Combine the minimal CFT (“matter”) with Liouville and ghosts.

Total $c = 26$ sets $b^2 = \frac{p}{q}$

Simplest operators in the BRST cohomology are “tachyons”

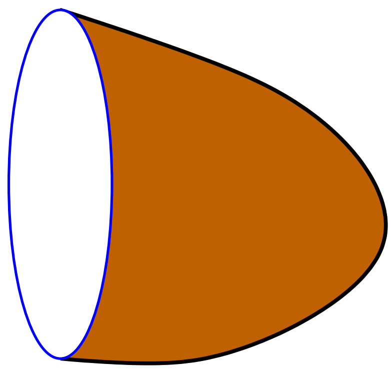
$$\mathcal{T}_{r,s} = c\,\bar{c}\,\mathcal{O}_{r,s} e^{2\,\beta_{r,s}\,\phi}$$

$$2\beta_{r,s} = \frac{p + q - (rq - sp)}{\sqrt{p\,q}}$$

$$1 \leq r < p\ , \qquad 1 \leq s < q\ , \qquad rq > sp$$

Review of Branes in Liouville

FZZT branes (Fateev, Zamolodchikov and Zamolodchikov, Teschner) – macroscopic loops in the worldsheet



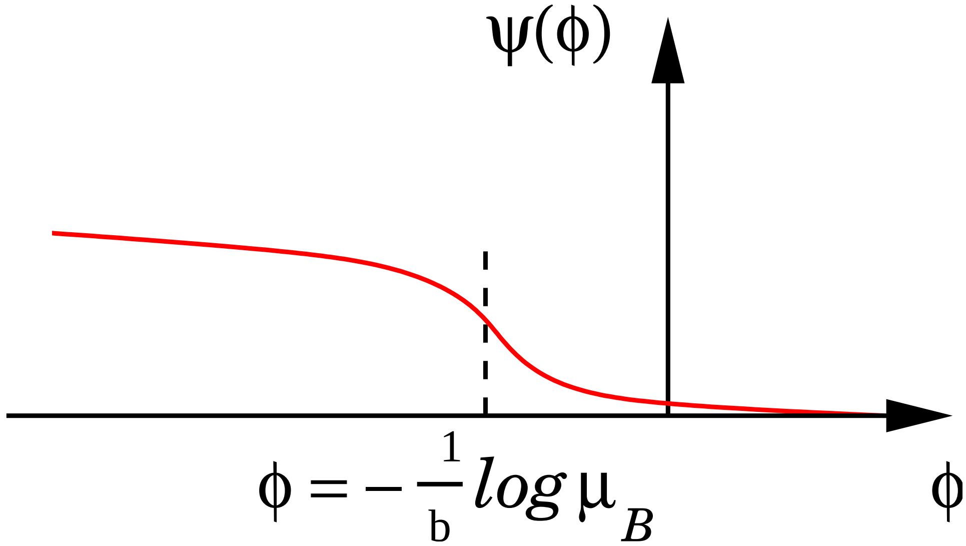
Labelled by the “boundary cosmological constant”

$$\delta S = \mu_B \oint e^{b\phi}$$

Minisuperspace wavefunction

$$\Psi(\phi) = \langle \phi | \mu_B \rangle = e^{-\mu_B e^{b\phi}}$$

The brane comes from infinity and dissolves at $\phi \approx -\frac{1}{b} \log \mu_B$.



In Cardy's formalism a brane is labelled by a representation in the open string channel

$$\mu_B = \cosh \pi\, b\, \sigma \qquad \longleftrightarrow \qquad \Delta = \frac{1}{4}\sigma^2 + \frac{Q^2}{4}$$

The boundary state is

$$|\sigma\rangle = \int_0^\infty dP \, \cos(2\pi P \sigma) \Psi(P) |P\rangle\rangle$$

$|P\rangle\rangle$ is a closed string (Ishibashi) state.

For the degenerate representations

$$\sigma = i\left(\frac{m}{b} + nb\right)$$

Subtracting the null vectors in the representation leads to the ZZ (Zamolodchikov and Zamolodchikov) branes

$$|m,n\rangle = |\sigma(m,n)\rangle - |\sigma(m,-n)\rangle$$

Same

$$\mu_B = (-1)^m \cos \pi\, n\, b^2$$

at $\sigma(m, \pm n)$ (Martinec).

These branes are localized in the strong coupling region $\phi \to +\infty$.

Branes in Minimal String Theory

FZZT branes: Tensor a Liouville brane labelled by σ and a matter brane

ZZ branes: Tensor a Liouville brane labelled by (m, n) and a matter brane

Simplification: the independent ZZ branes are

$$1 \leq m < p \ , \quad 1 \leq n < q \ , \quad np < mq$$

Geometric Interpretation

The disk amplitude $Z(\mu_B)$ is not a single valued function of

$$x \equiv \mu_B = \cosh \pi b \, \sigma \ , \qquad b^2 = \frac{p}{q}$$

Instead, x and

$$y \equiv \partial_{\mu_B} Z(\mu_B) = \cosh \frac{\pi \sigma}{b}$$

satisfy

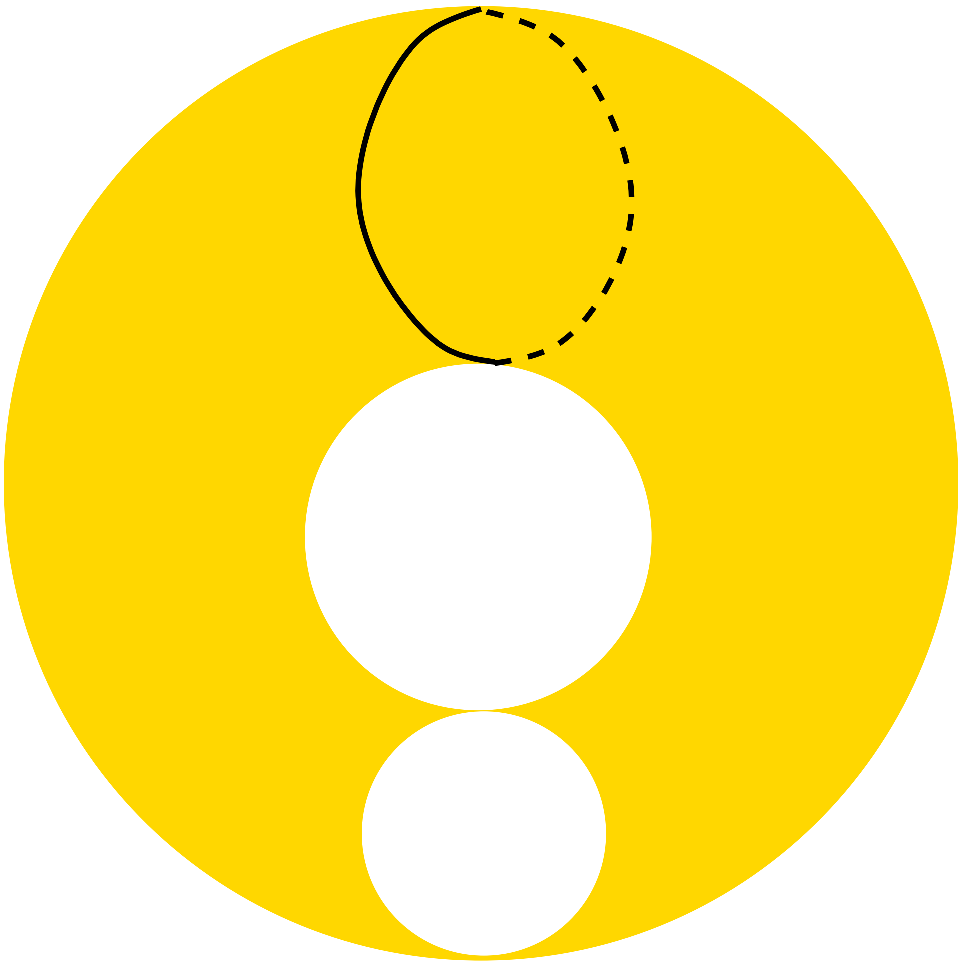
$$T_p(y) = T_q(x)$$

where

$$T_p(y = \cos \theta) = \cos p \, \theta$$

are Chebyshev polynomials

This is a genus $\frac{(p-1)(q-1)}{2}$ Riemann surface with $\frac{(p-1)(q-1)}{2}$ pinched A-cycles



Line integrals of $y\,dx$ lead to branes:

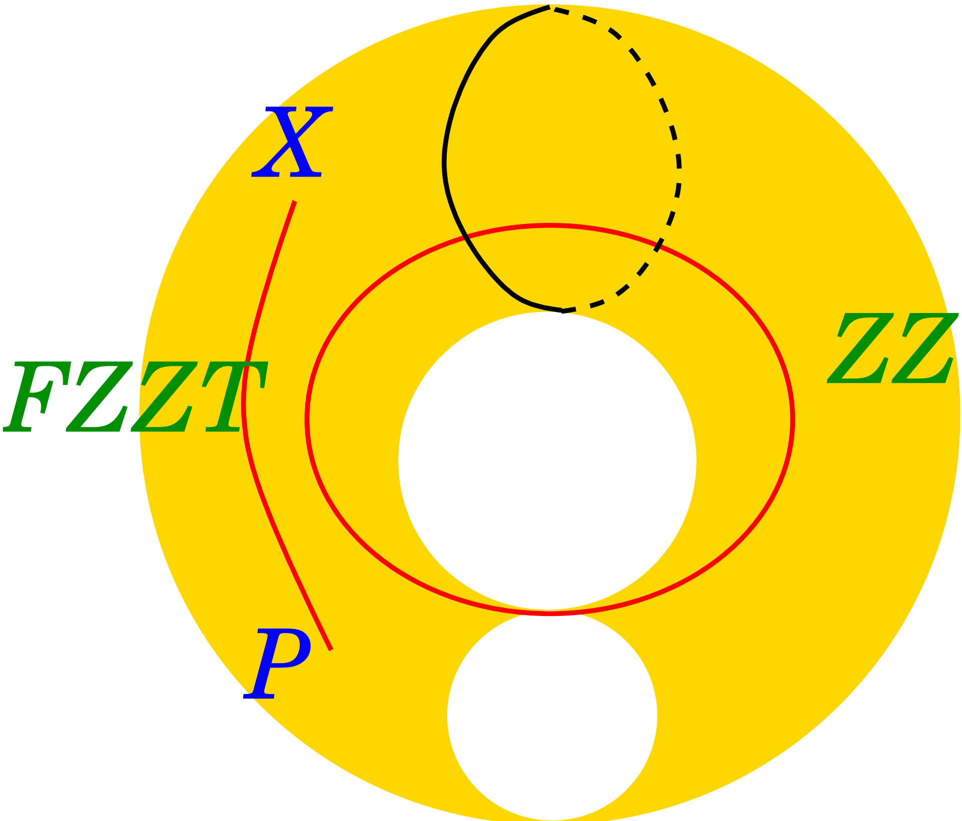
An FZZT brane is an open line integral

$$Z(x) = \int_P^x y\,dx'$$

A ZZ brane is a difference between two FZZT branes, and hence it is an integral along a closed contour. It passes through a singularity (will show); it is an integral along a B-cycle

$$Z(m,n) = \oint_{B_{m,n}} y\,dx$$

FZZT and ZZ branes on the Riemann surface:



$x_{m,n}$ at the singularities are the values of μ_B of the ZZ branes.

Matrix Model

Consider $(p = 2, q = 2l + 1)$, which corresponds to the one matrix model

Our surface is

$$2y^2 - 1 = T_q(x)$$

It has two copies of the complex x plane which are connected along a cut $(-\infty, -1)$ and l singularities (pinched cycles)

$$\left(x_n = \cos \frac{2\pi n}{q} \ , \ y_n = 0 \right) , \quad n = 1, ..., l$$

Interpretation:

Discontinuity along the cut

$$\rho(x) = \mathrm{Im}\sqrt{2 + 2\,T_q(x)}$$

is the eigenvalue density.

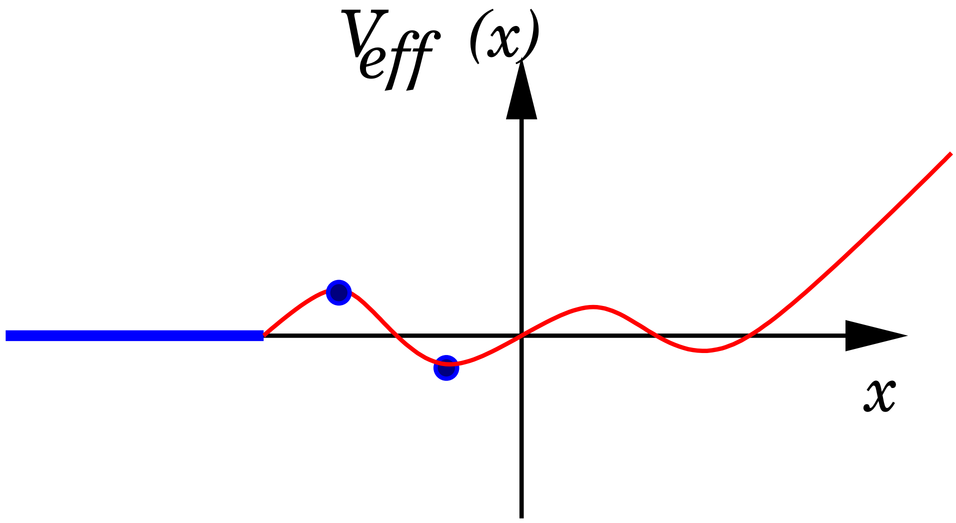
y is the force on an eigenvalue. $y = 0$ at the singularities.

The disk amplitude of FZZT brane

$$Z(x) = \int^x y\,dx' = V_{eff}(x)$$

is the effective potential of a probe eigenvalue.

ZZ brane: Eigenvalue at a stationary point of $V_{eff}(x)$ (where $y = 0$).



Nonperturbative instability

Conclusions

- We translated the features of the minimal string theory to simple properties of an underlying Riemann surface.

- D-branes are contour integrals of a certain one form:

  – FZZT branes are open contours

  – ZZ branes are associated with the B-cycles.

This gives a worldsheet "derivation" of the matrix model, and adds a new perspective to the understanding that

the eigenvalues are associated with D-branes (Polchinski, McGreevy, Verlinde, Klebanov, Maldacena, N.S., Martinec...)