David has found many fundamental results;

vertices with loops in the bosonic string, fermion scattering in string theory, N = 4 Yang-Mills theory, formulation of the superstring, Montonen - Olive duality, role of lattices and algebras in string theory, GKO construction ....

- understand the arguments you use completely
- tackle original problems and keep going until they are solved.
- trust in the elegance of the mathematics
- utilize deep mathematics to solve important physical problems.

Symmetries of M^E Theories

- show that $D = 11$ supergravity is a non-linear realization.

- Explain the appearance of E_{11}

- IIA and IIB also have a conjectured underlying E_{11}, implying a correspondence between the non-linear realizations.

- Consider non-linear realizations of

$\mathcal{G}^{+++}$; $A^{+++}_{D-3} \longleftrightarrow$ gravity

$D^{+++}_{D-2} \longleftrightarrow$ closed bosonic string

$\vdots$

- How does space-time enter

- Relation between E_{11} and half-B.P.S solutions.

Very Extended Algebras.

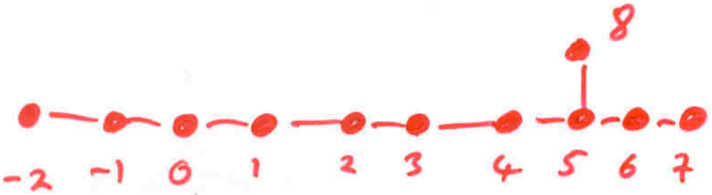
Gaberdiel, Olive, West.
hep-th/0205068

Consider a finite dimensional semi-simple Lie algebra g with simple roots α_a, $a = 1, \dots, r$ spanning a lattice Λ_g

extension	rank	root added	Lattice	det
affine, g^+	$r+1$	$\alpha_0 = k - \theta$	$\Lambda_g \oplus \Pi^{(1,1)}$. $x \cdot k = 0$	0
over extended, g^{++}	$r+2$	$\alpha_{-1} = -(k+\bar{k})$	$\Lambda_g \oplus \Pi^{(1,1)}$	$-\det A_g$
very extended, g^{+++}	$r+3$	$\alpha_{-2} = k - (\ell + \bar{\ell})$	$\Lambda_g \oplus \Pi^{(1,1)} \oplus \Pi^{(1,1)}$ $(\ell - \bar{\ell}) \cdot x = 0$	$-2 \det A_g$

$$k^2 = \bar{k}^2 = 0 \;,\; k \cdot \bar{k} = 1 \;,\; \ell^2 = \bar{\ell}^2 = 0 \;,\; \ell \cdot \bar{\ell} = 1$$

Example



$e_{11} = e_8^{+++}$

if $n = 24$, k_{27}. $= d_{24}^{+++}$

Given any finite dimensional semi-simple Lie algebra $\mathcal{G}$ we can construct its very extension $\mathcal{G}^{+++}$ and then the corresponding non-linear realization with respect to the appropriate sub-algebra

This is a theory involving gravity plus other massless fields (possibly dilatons, forms) and an infinite number of other fields

Kleinschmidt, E. Schnakenburg, West hep-th/0307024

Are these consistent theories of gravity?

Gravity in D dimensions

Lambert - West
hep-th/0107209.

Conjectured that gravity has a Kac-Moody symmetry

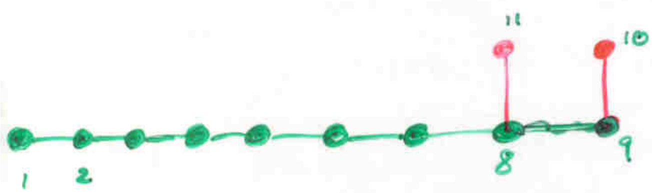
consistent with E_{11}

II A and II B Supergravity

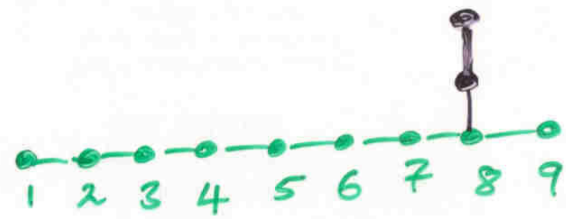
Schnakenburg West

II B can be described as a non-linear realization and the conjectured Kac-Moody algebra is E_{11}. Similarly for II A.

Both have A_9 as their gravity sector. In E_{11} there are two such sub-algebras.



II A



II B.

Note They have an A_9 in common.

Schnakenburg West
hep-th/0107181.

The Closed Bosonic String

The effective action can be constructed as a non-linear realisation for the algebra with generators

$K^a{}_b$, R $R_{a_1 \ldots a_{24}}$; $R_{a_1 a_2}$, $R_{a_1 \ldots a_{22}}$

$h_a{}^b$; ϕ $\phi_{a_1 \ldots a_{24}}$; $B_{a_1 a_2}$ $B_{a_1 \ldots a_{22}}$

gravity the spin 0 the 2 form

The corresponding Kac-Moody algebra has rank 27 with Dynkin diagram

Predicts symmetry of reduced theory on an n torus to be.

$$\frac{O(n,n)}{O(n)\otimes O(n)} \quad \text{for } n = 3, \ldots, 22$$

$$\frac{O(24,24)}{O(24)\times O(24)} \quad \text{for } n = 23$$

Non-linear Realizations

A non-linear realization of a group G with respect to a subgroup H is invariant under.

$$g \rightarrow g_0 g h$$

where $g_0 \in G$ is a rigid transformation and $h \in H$ is a local transformation.

The Cartan forms transform as

$$\nu = g^{-1} dg \longrightarrow h^{-1} \nu h. + h^{-1} dh$$

If the generators of H are H_I and T_a are the remaining generators of G we may choose

$$g = \exp(\phi^a T_a.)$$

- There is a 1-1 relation between generators outside H and fields of the theory.

- The dynamics is essentially fixed if H is large enough.

Gravity as a Non-linear Realization

Borisov Ogievetsky

Consider $IGL(D)$ with subgroup the Lorentz group $SO(1, D-1)$.

generator	P_a ,	$K^a{}_b$
field	x^a ,	$h_a{}^b$
	(space-time)	(graviton)

The group element is

$$g = e^{x^a P_a} \, e^{h_a{}^b(x) \, K^a{}_b}$$

We take the simultaneous non-linear realization with the conformal group $SO(2, D)$ or equivalently a non-linear realization of the two groups

Leads uniquely to Einstein's theory.

$D = 11$ Supergravity as a Non-linear Realization

hep-th/0005270.

field $\quad h_a{}^b, \quad A_{a_1 \cdot a_3}, \quad A_{a_1 \cdots a_6}$

generator $\quad K^a{}_b, \quad R^{a_1 \cdot a_3}, \quad R^{a_1 \cdots a_6}$

— The $K^a{}_b$ generate $SL(11)$ or A_{10}

— The $R^{a_1 \cdot a_3}$, $R^{a_1 \cdots a_6}$ are tensors under $SL(11)$ ie

$$[K^a{}_b, R^{c_1 c_2 c_3}] = \delta_b^{c_1} R^{a c_2 c_3} + \cdots$$

— The generators $K^a{}_b$, $R^{a_1 \cdot a_3}$, $R^{a_1 \cdots a_6}$ belong to a specific algebra, $\hat{G}_{11}$

i.e $\quad [R^{c_1 \cdot c_3}, R^{c_4 \cdot c_6}] = 2 R^{c_1 \cdots c_6}$

The non-linear realization is constructed from

$$g = e^{x^a P_a} \, e^{h_a{}^b(x) K^a{}_b} \, e^{A^{c_1 \cdot c_3} R_{c_1 \cdot c_3} + A^{c_1 \cdot c_6} R_{c_1 \cdot c_6}}$$

Carrying out the simultaneous realization with the conformal group $SO(2,11)$ we find the PRECISE field equations of $D=11$ supergravity.

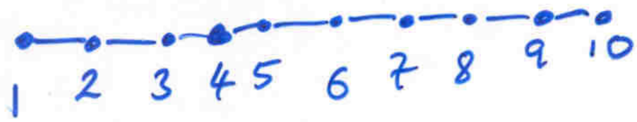
$D = 11$ supergravity is the non-linear realization of $G_{11} = \{ P_a, \hat{G}_{11} \}$
where $\hat{G}_{11} = \{ K^a{}_b, R^{a_1 a_2 a_3}, R^{a_1 \dots a_6} \}$

together with the conformal group $SO(2,11)$.

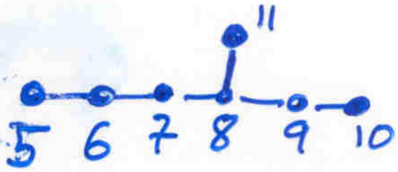
- What is the significance of $\hat{G}_{11}$

The algebra $\hat{G}_{11}$ contains. hep-th/0104081

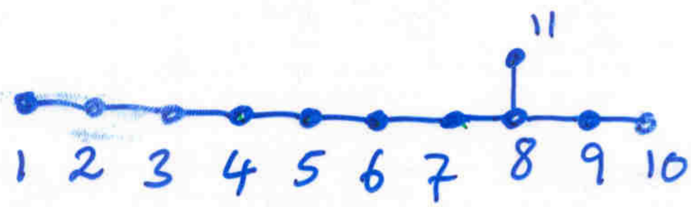
— A_{10}



and the Borel subgroup of E_7.



If the theory is invariant under a Kac-Moody algebra it must contain E_{11}



- The D = 11 theory contains the symmetries found in the dimensional reductions

- Weyl group of E_{11} is the U-duality group

Englert, Houart, Taormina, West.
hep-th/0304206.

Generators in E_{11} with A_{10} gravity line

level	p_i	β	
0	[1, 0, 0, 0, 0, 0, 0, 0, 0, 1]	(0,0,0,0,0,0,0,0,0,1,0)	$K^a{}_b$
1	[0, 0, 0,0,0,0,0,1,0,0]	(0,0,0,0,0,0,0,0,0,0,1)	$R^{c_1c_2c_3}$
2	[0, , 0,0,0,1,0,0,0,0,0]	(0,0,0,0,0,0,1 2 3 2 1 2)	$R^{c_1\ldots c_6}$
3	[0, 1,0,0,0,0,0,0,0,1]	(0,0,0,1,2,3,4,5,3,1,3)	$R^{c_1\ldots c_8,b}$
		(0,1,2,3,4,5,6,7,8,5,2,4)	R^a
4	[0, 0,0,0,0,0,0,0,0,1]		
4	[0,1, 0,0,0,0,1,0,0,]	(0,0,0,1 2 3 4 5,6,4,2,4)	$R^{c_1c_2c_3}{}_{b_1b_2}$
4	[1,0, 0,0,0,0,0,0,0,2]	(0,0,1 2 3 4 5 6 7 4,1,4)	$R^{(ab)}{}_c$

⋮

The A_{10} weight is $\Lambda = \sum_j p_j \lambda_j$

The E_{11} root is $\beta = \sum_a n_a \alpha_a = n_{11}\alpha_{11} + \sum_{i=1}^{10} \alpha_i n_i$

The level is n_{11}

Relations between D = 11, IIA and IIB.

All have E_{11} as a symmetry algebra and so there is a 1-1 correspondence between the fields in the three non-linear realizations.

For example. given a field in IIB $\longleftrightarrow$ a IIB generator in E_{11} $\longleftrightarrow$ root in E_{11} $\longleftrightarrow$ a D = 11 generator in E_{11} $\longleftrightarrow$ field in D = 11.

i.e.

$\hat{h}_9{}^{10}$ in IIB $\leftrightarrow$ $\hat{K}^9{}_{10}$ $\leftrightarrow$ $(0,0,\ldots,1)$ of E_{11}

$\longleftrightarrow$ $R^{9\,10\,11}$ $\longleftrightarrow$ $A_{9\,10\,11}$ in D = 11.

or

$\hat{A}_{2\ldots 10}$ in IIA $\leftrightarrow$ $\hat{R}^{2\ldots 10}$ $\leftrightarrow$ $(0,1,2,3,4,5,6,7,4,14)$

of E_{11} $\longleftrightarrow$ $R^{11\,11}{}_{1}$ $\longleftrightarrow$ $A_{11\,11}{}^{1}$ of D = 11

suggests massive IIA is contained in the D = 11 theory at higher levels.

or

$A_{1\ldots 10}$ in IIB $\longleftrightarrow$ . .

$\longleftrightarrow$ $A^{11\,11}{}_{10}$ in D = 11.

vels of the two decompositions into A_9 subalgebras discussed in the

is the data obtained for the IIA case. All the listed levels are

A_9 weight	E_8^{+++} element α	α^2	$ht(\alpha)$	μ	
[0,0,0,0,0,0,0,1,0]	(0,0,0,0,0,0,0,0,0,0,1)	2	1	1	$R^{c_1c_2}$
[0,0,0,0,0,0,0,0,1]	(0,0,0,0,0,0,0,0,0,1,0)	2	1	1	R^{c_1}
[0,0,0,0,0,0,1,0,0]	(0,0,0,0,0,0,0,1,1,1,1)	2	4	1	$R^{c_1c_2c_3}$
[0,0,0,0,1,0,0,0,0]	(0,0,0,0,0,1,2,3,2,1,2)	2	11	1	$R^{c_1\cdots c_5}$
[0,0,1,0,0,0,0,0,0]	(0,0,0,1,2,3,4,5,3,1,3)	2	22	1	$R^{c_1\cdots c_7}$
[1,0,0,0,0,0,0,0,0]	(0,1,2,3,4,5,6,7,4,1,4)	2	37	1	$R^{c_1\cdots c_9}$
[0,0,0,1,0,0,0,0,0]	(0,0,0,0,1,2,3,4,3,2,2)	2	17	1	$R^{c_1\cdots c_6}$
[0,0,1,0,0,0,0,0,1]	(0,0,0,1,2,3,4,5,3,2,3)	2	23	1	$R^{c_1\cdots c_7,b}$
[0,1,0,0,0,0,0,0,0]	(0,0,1,2,3,4,5,6,4,2,3)	0	30	1	$R^{c_1\cdots c_8}$
[0,1,0,0,0,0,0,1,0]	(0,0,1,2,3,4,5,6,4,2,4)	2	31	1	$R^{c_1\cdots c_8,b_1b_2}$ new fields
[1,0,0,0,0,0,0,0,1]	(0,1,2,3,4,5,6,7,4,2,4)	0	38	1	
[0,0,0,0,0,0,0,0,0]	(1,2,3,4,5,6,7,8,5,2,4)	-2	47	2	
[1,0,0,0,0,0,1,0,0]	(0,1,2,3,4,5,6,8,5,2,5)	2	41	1	
[0,0,0,0,0,0,0,1,0]	(1,2,3,4,5,6,7,8,5,2,5)	0	48	1	
[0,0,0,0,0,1,0,0,0]	(1,2,3,4,5,6,8,10,6,2,6)	2	53	1	
[0,1,0,0,0,0,0,0,1]	(0,0,1,2,3,4,5,6,4,3,3)	2	31	1	
[1,0,0,0,0,0,0,0,0]	(0,1,2,3,4,5,6,7,5,3,3)	0	39	0	
[0,1,0,0,0,0,1,0,0]	(0,0,1,2,3,4,5,7,5,3,4)	2	34	1	
[1,0,0,0,0,0,0,0,2]	(0,1,2,3,4,5,6,7,4,3,4)	2	39	1	
[1,0,0,0,0,0,0,1,0]	(0,1,2,3,4,5,6,7,5,3,4)	0	40	1	
[0,0,0,0,0,0,0,0,1]	(1,2,3,4,5,6,7,8,5,3,4)	-2	48	2	
[0,1,0,0,1,0,0,0,0]	(0,0,1,2,3,5,7,9,6,3,5)	2	41	1	
[1,0,0,0,0,0,1,0,1]	(0,1,2,3,4,5,6,8,5,3,5)	2	42	1	
[1,0,0,0,0,1,0,0,0]	(0,1,2,3,4,5,7,9,6,3,5)	0	45	1	
[0,0,0,0,0,0,0,1,1]	(1,2,3,4,5,6,7,8,5,3,5)	0	49	1	
[0,0,0,0,0,0,1,0,0]	(1,2,3,4,5,6,7,9,6,3,5)	-2	51	3	
[1,0,0,0,0,0,1,0,0]	(0,1,2,3,4,5,6,8,6,4,4)	2	43	1	
[0,0,0,0,0,0,0,0,2]	(1,2,3,4,5,6,7,8,5,4,4)	2	49	1	
[0,0,0,0,0,0,0,1,0]	(1,2,3,4,5,6,7,8,6,4,4)	0	50	0	

all extra fields and their duals except $R^{c_1\cdots c_9}$ which is needed for massive IIA.

on E_8^{+++} with such a field where one does not need to include

s the data obtained for the IIB case.

A_9 weight	E_8^{+++} element α	α^2	$ht(\alpha)$	μ	
[0,0,0,0,0,0,0,0,0]	(0,0,0,0,0,0,0,0,0,1,0)	2	1	1	R
[0,0,0,0,0,0,0,1,0]	(0,0,0,0,0,0,0,0,1,0,0)	2	1	1	$R^{a_1a_2}$
[0,0,0,0,0,0,0,1,0]	(0,0,0,0,0,0,0,0,1,1,0)	2	2	1	$R^{a_1a_2}$
[0,0,0,0,0,1,0,0,0]	(0,0,0,0,0,0,1,2,2,1,1)	2	7	1	$R^{a_1\cdots a_4}$
[0,0,0,1,0,0,0,0,0]	(0,0,0,0,1,2,3,4,3,1,2)	2	16	1	$R^{a_1\cdots a_6}$
[0,0,0,1,0,0,0,0,0]	(0,0,0,0,1,2,3,4,3,2,2)	2	17	1	$R^{a_1\ldots a_6}$
[0,1,0,0,0,0,0,0,0]	(0,0,1,2,3,4,5,6,4,1,3)	2	29	1	$R^{a_1\cdots a_8}$
[0,0,1,0,0,0,0,0,1]	(0,0,0,1,2,3,4,5,4,2,2)	2	23	1	$R^{a_1\ldots a_7, b}$
[0,1,0,0,0,0,0,0,0]	(0,0,1,2,3,4,5,6,4,2,3)	0	30	1	$R^{a_1\cdots a_8}$
[0,1,0,0,0,0,0,0,0]	(0,0,1,2,3,4,5,6,4,3,3)	2	31	1	$R^{a_1\cdots a_8}$ new
[0,0,0,0,0,0,0,0,0]	(1,2,3,4,5,6,7,8,5,1,4)	2	46	1	$R^{a_1\cdots a_{10}}$
[0,1,0,0,0,0,0,1,0]	(0,0,1,2,3,4,5,6,5,2,3)	2	31	1	$\vdots$
[1,0,0,0,0,0,0,0,1]	(0,1,2,3,4,5,6,7,5,2,3)	0	38	1	
[0,0,0,0,0,0,0,0,0]	(1,2,3,4,5,6,7,8,5,2,4)	-2	47	2	
[0,1,0,0,0,0,0,1,0]	(0,0,1,2,3,4,5,6,5,3,3)	2	32	1	
[1,0,0,0,0,0,0,0,1]	(0,1,2,3,4,5,6,7,5,3,3)	0	39	1	
[0,0,0,0,0,0,0,0,0]	(1,2,3,4,5,6,7,8,5,3,4)	-2	48	2	
[0,0,0,0,0,0,0,0,0]	(1,2,3,4,5,6,7,8,5,4,4)	2	49	1	
[1,0,0,0,0,0,1,0,0]	(0,1,2,3,4,5,6,8,6,2,4)	2	41	1	
[0,0,0,0,0,0,0,1,0]	(1,2,3,4,5,6,7,8,6,2,4)	0	48	1	
[0,1,0,0,0,1,0,0,0]	(0,0,1,2,3,4,6,8,6,3,4)	2	37	1	
[1,0,0,0,0,0,0,1,1]	(0,1,2,3,4,5,6,7,6,3,3)	2	40	1	
[1,0,0,0,0,0,1,0,0]	(0,1,2,3,4,5,6,8,6,3,4)	0	42	1	
[0,0,0,0,0,0,0,0,2]	(1,2,3,4,5,6,7,8,6,3,3)	0	48	0	
[0,0,0,0,0,0,0,1,0]	(1,2,3,4,5,6,7,8,6,3,4)	-2	49	3	
[1,0,0,0,0,0,1,0,0]	(0,1,2,3,4,5,6,8,6,4,4)	2	43	1	
[0,0,0,0,0,0,0,1,0]	(1,2,3,4,5,6,7,8,6,4,4)	0	50	1	

It is natural to consider the non-linear realization of $E_{11} \otimes_s l_1$ i.e.

$$g = \exp(x^a P_a + z_{a_1 a_2} Z^{a_1 a_2} + \dots)$$

$$\exp(h_a{}^b K^a{}_b) \exp(A_{c_1 c_2 c_3} R^{c_1 c_2 c_3}) \dots$$

- fields depend on $x_a, z^{a_1 a_2}, \dots$

- strings and branes occur on an equal footing

- can find action of Weyl group of E_{11} on even E_{11} on solutions.

Kleinschmidt West
hep-th/0312247.

- In general G^{+++} contains no generator which can be identified with P_a. However E_{11} contains one at level $7+11s$, $s \in \mathbb{Z}_+$.

- Given any representation of A_{10} in E_{11} with Dynkin indices p_j $\exists$ a representation with Dynkin indices p_j $j \geq 2$, $p_1 + m$. but also one with Dynkin indices p_j, $j \neq 9$, $p_j + m$. ie central charge coordinate derivatives

- In fact the A representations in l_1 generally occur in the adjoint representation of G^{+++}. They also have the correct A representation to act as brane charges.

For E_{11}

we can add the l_1 representation. The root string is

root l_1, $l_1 - \alpha_1$, ..., $l_1 - \alpha_1 \cdots - \alpha_8 - \alpha_{11}$, ...

generator P_1, P_2 $\qquad Z^{10\,11}$

The l_1 representation contains

$$P_a, \; Z^{a_1 a_2}, \; Z^{a_1 \dots a_5}, \; Z^{a_1 \dots a_7, b}, \dots\dots$$

In fact $Z^{a_1 a_2}$, $Z^{a_1 \dots a_5}$ are the central changes in the $D = 11$ supersymmetry algebra.

We know (hep-th/991226 Barwald West) and (hep-th/0005270) that M-theory should possess an $SL(32)$ symmetry that rotates the central changes.

The Cartan involution invariant subalgebra of E_{11} contains $SL(32)$ which can be identified with the above $SL(32)$, but

$$[S^{a_1 a_2 a_3}, P_b] = \delta_b^{[a_1} Z^{a_2 a_3]}, \dots$$

Where is Space-time

Our derivation of E_{11} deliberately excluded considerations about P_a. We can either

- add generator $P_a, \ldots$ in addition to those of E_{11}
- hope that space-time is somehow included in E_{11}.

For gravity as a non-linear realization we added the translations P_a. They belong to the vector or l_1 representation of A_{D-1}

$$(l_a, \alpha_b) = \delta_{ab}.$$

There are two approaches to the way space-time enters

(A) We put in space-time by hand

ie $g = g(x^\mu)$, $h = h(x^\mu)$, $\phi_a = \phi_a(x^\mu)$.

ie $\dfrac{SU(2) \times SU(2)}{SU(2)}$. the π's or scalars in $D=4$, $N=8$ supergravity with $\dfrac{E_7}{SU(8)}$.

Cremmer-Julia

(B) The generators of space-time $P_\mu, \ldots$ are part of G

ie $T_a = \{ P_\mu, \ldots, \hat{T}_a \}$

then

$$g = e^{x^\mu P_\mu} \ldots e^{\phi^a(x) \hat{T}_a} \ldots$$

Volkov, ...

ie superspace, conformal symmetry

- non-linear realization were extended to include differential forms

Cremmer, Julia, Lu, Pope.

Perhaps Space-time is in E_{11}

Damour, Henneaux and Nicolai considered a non-linear realization of E_{10} with fields that depend only on time t i.e.

$$g = e^{h^a{}_b(t) K_a{}^b} e^{R^{c_1c_2c_3} A_{c_1c_2c_3}(t)} \cdots$$

It is hoped that the dynamics encodes the spatial dependence.

In fact E_{10} has generators

$R_b{}^{a_1a_2a_3}, \ldots$ and so fields $A^{a_1a_2a_3}{}_b, \ldots$

which may be the spatial derivatives.

Houart and Englert took a non-linear realization of E_{11}, but the fields depend on an auxiliary parameter τ. and thought that E_{11} and indeed g^{+++} would contain P_a.

Assuming space-time enters through the l_1 representation we can also make a 1-1 correspondence between the generalized coordinates

in IIA

the IIA $\tilde{X}^\mu = X^\mu$ $\mu \neq D = 11$, $\mu = 0, \ldots$ etc.

$\hat{Z} = X^{11}$

"

and IIB

the IIB $\tilde{X}^\mu = X^\mu$ $\mu \neq 10, 11$ $= 0, \ldots, 9$

$\tilde{Z}^{10} = X^{10}$

It is likely that if one makes all the non-linear realizations these changes are completely equivalent.

Programme

last updated 20/03/04

	Wednesday 24th March	Thursday 25th March	Friday 26th March	Saturday 27th March
09.00		**N. Nekrasov** *Spacetime foam and strings*	**B. Julia** *Non-abelian p-forms, algebraic surfaces and M-theory symmetries*	**[illegible]** *The importance of hermiticity and unitarity in integrable models*
09.45		**J. Harvey** *Branes and anomalies*	**C. Montonen** *Unitarity and analyticity when Lorentz invariance is broken*	**E. Corrigan** *Some aspects of affine Toda theory*
10.30		Coffee	Coffee	Coffee
11.00		**A. Tseytlin** *Aspects of AdS/CFT duality*	**D. Fairlie** *What did David do before the string model?*	**W. Nahm** *Monopoles and Olive-Montonen duality*
11.45		**J. de Boer** *Wrapped branes and non-perturbative gauge theories*	**L. Brink** *Strings as consistent quantum theories: the search for dual loops*	**P. West** *Symmetries of M theories*
12.30	Lunch	Lunch	Lunch	Lunch
14.00	**I. Klebanov** *Pentaquarks: chiral soliton models confront experiments*	**N. Seiberg** *Minimal string theory*	**D. Tong** *Instantons, vortices, monopoles and kinks*	
14.45	**C. Hull** *Duality and geometry*	**H. Ooguri** *Large N duality*	**B. Acharya** *Observations on the space of 4d String and M theory compactifications*	
15.30	Coffee	Coffee	Coffee	
16.00	**M. Green** *Supersymmetry and the low energy effective action*	**O. Aharony** *The Hagedorn/deconfinement phase transition in weakly coupled large N gauge theories*	**E. Rabinovici**	
16.45				

E_{11} and Solutions

hep-th/0402140

Given any solution of $D=11$ supergravity

i.e. $e_\mu{}^a = (e^h)_\mu{}^a$, $A^T_{a_1a_2a_3}$, . . .

we can write down the corresponding

E_{11} group element

$$g = e^{\sum_a h_a{}^a K^a{}_a} \, e^{\sum_{a<b} h_a{}^b K^a{}_b} \, e^{A^T_{a_1a_2a_3} R^{a_1a_2a_3}} \; \ldots\ldots$$

All the usual half B.P.S solutions

of $D=11$, IIA and IIB supergravity

are given by

$$g = e^{-\frac{1}{2}\ln N \beta\cdot H} \, e^{(1-N) E_\beta}$$

where β is a root of E_{11} and $\beta\cdot H = \beta^i H_i$

Example. Consider the M2 brane solution of D=11 supergravity

$$ds^2 = N_2^{-2/3}(-dx_1^2 + dx_2^2 + dx_3^2) + N_2^{1/3}(dx_4^2 + \dots + dx_{11}^2)$$

Duff Stelle.

and $$A^{W}_{123} = N_2^{-1} - 1$$

The vierbein takes the values

$$e_1^{\ 1} = (e^h)_1^{\ 1} = N_2^{-\frac{1}{3}} \quad \dots \quad , (e^h)_4^{\ 4} = N_2^{1/6}, \dots, \quad A^T_{123} = 1 - N_2$$

The corresponding E_{11} group element is

$$g = \exp\left(-\tfrac{1}{2}\ln N_2 \left(\tfrac{2}{3}(K^1_{\ 1} + K^2_{\ 2} + K^3_{\ 3}) - \tfrac{1}{3}(K^4_{\ 4} + \dots + K^{11}_{\ 11})\right)\right)$$
$$\cdot \exp\ (1 - N_2)\, R^{123}.$$

Now

$$R^{91011} = E_{11} \quad \text{and} \quad \alpha_{11} \cdot H = H_{11} = -\tfrac{1}{3}(K^1_{\ 1} + \dots + K^8_{\ 8}) + \tfrac{2}{3}(K^9_{\ 9} + \dots + K^{11}_{\ 11})$$

acting with $K^9_{\ 6}$ we find

$$R^{123} = E_\beta \quad \text{with} \quad \beta = \alpha_{11} + \alpha_1 + 2\alpha_2 + \dots + \alpha_{10}$$

and

$$g = \exp(-\tfrac{1}{2}\ln N\ \beta \cdot H)\ \exp\ (1 - N)\, E_\beta$$

Higher Order Branes

Given any E_{11} generator we can construct a "half-BPS brane".

In $D=11$ at level 4 we have the generator R^a whose solution is

$$ds^2 = N^{-2/3}(-dx_1^2) + N^{\frac{1}{3}}(dx_2^2 + \quad + dx_{11}^2)$$

and the generator $R^{(ab)}_{c}$ whose solution is

$$ds^2 = N^{-5/3}(-dx_1^2) + N^{1/3}(dx_2^2 + \cdot + dx_{10}^2) + N^{4/3}\, dx_{11}^2$$

While in IIB we have the generator $R^{a_1 \dots a_{10}}$ which leads to the space-filling brane predicted from world-sheet considerations.

The formula gives all the IIA and IIB branes in both the NS-NS and R-R sector.

In IIA we also find the generator $R^{a_1 \dots a_9}$ whose corresponding solution is

$$ds^2 = N^{1/8}\left(-(dx_1)^2 + dx_2^2 + \dots + dx_9^2\right) + N^{9/8}(dx_{10})^2 .$$

and $e^A = N^{5/4}$, $A_{1\dots 9} = N^{-1} - 1$.

which is the eight brane of Witten Polchinski.

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[illegible]				

Conclusion

- There exists growing evidence for an E_{11} symmetry underlying an extension of $D=11$, IIA and IIB supergravity

- are the higher order fields dynamical and do they ensure the consistency of the theory

- need generalized coordinates in addition to those of space-time. ie $x^\mu, z_{ab}, z_{a_1\ldots a_5}, \ldots$ in $D=11$.

- higher one to one correspondence between $D=11$, IIA and IIB

  $A_{a_1\ldots a_9}$ of IIA $\longleftrightarrow$ $A^{(ab)}_{c}$ of $D=11$.

- Given any finite dimension semi-simple Lie algebra g we can construct its very extension g^{+++} and its corresponding non-linear realization

  A^{+++}_{D-3} is gravity + higher modes.

  suggests all theories are different limits of a single theory

- what is the underlying theory.

Open questions

- what theory is to possess E_{11} or what role do the higher order fields play

- How does space-time enter? P_a was excluded in E_{11} considerations

- How does the conformal group fit with E_{11}

- How do we incoorperate fermions.

Mark how the lark and linnet sing;
with rival notes
They strain their warbling throats
To welcome in the spring.
But in the close of night,
When Philomel begins her heav'nly lay,
They cease their mutual spite,
Drink in her music with delight,
And list'ning and silent obey.

So ceas'd the rival crew when David came,
They sung no more, or only sung his fame.
Struck dumb, they all admir'd the matchless man;
Alas, too soon retir'd the matchless man.

after John Dryden