Lattice QCD calculations for high-precision tests of the Standard Model of Particle Physics

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The Standard Model of Particle Physics

- Theory to describe the (known) elementary particles and their interactions
- quarks: \( u, c, t \) (up, charm, top) and \( d, s, b \) (down, strange, bottom)
  - gauge bosons: \( \gamma \) (photon), \( g \) (gluon), \( W^\pm, Z \) bosons
- leptons: \( e, \mu, \tau \) (electron, muon, tau) and \( \nu_e, \nu_\mu, \nu_\tau \) (electron neutrino, muon neutrino, tau neutrino)
- fundamental interactions
  - electro-magnetism, mediated by photons
  - weak interaction, mediated by \( W^\pm, Z \) bosons, e.g. \( \beta \)-decays
  - strong interaction, mediated by gluons, e.g. binds Nuclei together
- Higgs boson, discovered 2012 at LHC, predicted over 50 years ago (Noble Prize 2013 for Higgs, Englert)
Physics Beyond the Standard Model

- very successful theory, but:
- open questions
  - What is dark matter? Or dark energy?

- Why is there more matter than antimatter in the Universe?
- Why are there three generations of fermions?
- Why is there such a hierarchy of masses?
- ...

- search for physics beyond the Standard Model
  - high-energy searches in colliders, e.g. LHC
  - new physics enters in low energy as small corrections due to quantum loops
    → high precision tests to find deviations from SM predictions
QCD and confinement

- Quantum Chromo Dynamics (QCD) theory of the strong interaction
- strong coupling $\alpha_s$

\begin{align*}
\alpha_s(M_Z) &= 0.1181 \pm 0.0011 \\
\text{pp} \rightarrow \text{jets} &\text{ e.w. precision fits (N}^3\text{LO)} \\
0.1 &\quad 0.2 &\quad 0.3 \\
\alpha_s(Q^2) &\quad 1 &\quad 10 &\quad 100 \\
Q \text{ [GeV]} &\quad 1000 \\
\text{DIS jets (NLO)} &\quad \tau \text{ decays (N}^3\text{LO)} \\
\text{Heavy Quarkonia (NLO)} &\quad e^+e^- \text{ jets & shapes (res. NNLO)} \\
\text{e.w. precision fits (N}^3\text{LO)} &\quad p\bar{p} \rightarrow \text{jets (NLO)} \\
\text{pp} \rightarrow \text{tt (NNLO)}
\end{align*}

- quarks and gluons confined to bound states (hadrons)

[Particle Data Group (PDG), Phys. Rev. D 98, 030001 (2018)]
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\[ \text{e}^+\text{e}^- \text{ precision fits (N}^3\text{LO)} \]

\[ \tau \text{ decays (N}^3\text{LO)} \]

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![plot of $\alpha_s(Q^2)$ vs Q [GeV]]

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QCD and confinement

- Quantum Chromo Dynamics (QCD) theory of the strong interaction

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\[
\alpha_s(M_Z) = 0.1181 \pm 0.0011
\]

- quarks and gluons confined to bound states (hadrons)

- each additional gluon line or quark-antiquark pair comes with $\alpha_s$ ($\alpha_s \sim O(1)$ at small energies)

$\rightarrow$ Monte Carlo sampling

[Particle Data Group (PDG), Phys. Rev. D 98, 030001 (2018)]
QCD on the lattice

- Wick rotation \( (t \rightarrow -ix_0) \) to Euclidean space-time
- Discretize space-time by a hypercubic lattice \( \Lambda \)
- Quantize QCD using Euclidean path integrals

\[
\langle A \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}] \mathcal{D}[U] e^{-S_E[\psi, \bar{\psi}, U]} A(U, \psi, \bar{\psi})
\]

- can be split into fermionic and gluonic part

- Calculate gluonic expectation values using Monte Carlo techniques:

\[
\langle \langle A \rangle_F \rangle_G = \int \mathcal{D}[U] \langle A \rangle_F P(U) \approx \frac{1}{N_{\text{cfg}}} \sum_{n=1}^{N_{\text{cfg}}} \langle A \rangle_F
\]

average over gluonic gauge configurations \( U \) distributed according to

\[
P(U) = \frac{1}{Z} (\det D)^{N_f} e^{-S_G[U]}
\]

- extrapolate to the continuum \( (a \rightarrow 0) \) and infinite volume \( (V \rightarrow \infty) \)
Computational Challenges

- two energy scales in the problem, box size $L$, lattice spacing $a$
  \[ \mathcal{O}(1/L) \ll E \ll \mathcal{O}(1/a) \]
- typical size of a lattice
  \[ N = L^3 \times T = 64^3 \times 128 \sim \mathcal{O}(10^7 - 10^8) \]
- Dirac-operator $D$: matrix of size $N \times N$
- calculate quark propagators $\rightarrow$ need the inverse $D^{-1}$
  $\rightarrow$ solve the Dirac equation using appropriate sources $\eta$
  \[ D \phi = \eta \]
- solve numerically using Conjugate Gradient
Overview

- use Monte Carlo methods to calculate QCD observables at low energies on a space-time lattice
- compare results from calculations (i.e. Standard Model predictions) with experimental results
  → low-energy tests of the Standard Model
  → high precision to find hints for (small) deviations from Standard Model
- code development and optimisation
  ▶ GRID https://github.com/paboyle/Grid
  ▶ Hadrons (Grid-powered Workflow Management System) [P. Boyle et al]
  ▶ [A. Portelli et al]

Outline

- The anomalous magnetic moment of the muon
- Flavour physics
  ▶ CKM matrix and leptonic Meson decays
  ▶ rare Kaon decays
- Conclusions
Outline

Standard Model of Particle Physics

The anomalous magnetic moment of the muon

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  CKM matrix and leptonic Meson decays
  rare Kaon decays

Conclusions
Magnetic moment of leptons (e, μ, τ)

- magnetic moment $\vec{\mu}$ of the lepton $\ell$ due to its spin $\vec{s}$ and electric charge $e$

$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$

- torque $\vec{\tau} = \vec{\mu} \times \vec{B}$

- gyromagnetic factor (g-factor)

- without quantum fluctuations for a lepton one finds $g = 2$

- deviation from the value “2” due to quantum loops
  $\rightarrow$ anomalous magnetic moment of lepton $\ell$

$$a_\ell = \frac{g_\ell - 2}{2}$$
The anomalous magnetic moment of the muon

\( a_\mu : \) Experiment vs. Theory

- \( a_\mu = (g_\mu - 2)/2 \)
- Measured and calculated very precisely \( \rightarrow \) test of the Standard Model
- Experiment: polarized muons in a magnetic field \[ \text{[Bennet et al., Phys.Rev. D73, 072003 (2006)]} \]
  \[ a_\mu = 11659208.9(5.4)(3.3) \times 10^{-10} \]

- New experiments at Fermilab and JPARC \( \rightarrow \) reduce error by \( \approx 4 \)
  \( \rightarrow \) first result from Fermilab expected 2019

\[ \omega_a = a_\mu \frac{eB}{m_\mu} \]
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[http://muon-g-2.fnal.gov/bigmove/gallery.shtml]

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[Credit: Brookhaven National Laboratory]  
[Credit: Fermilab]
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  $\text{em} \quad (11658471.895 \pm 0.008) \times 10^{-10} \quad [\text{Kinoshita et al., Phys.Rev.Lett. 109, 111808 (2012)}]$
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\[ \nu_\mu \]

\[ \mu \]

\[ \mu \]

\[ Z \]

\[ H \]
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  - HVP \( (692.3 \pm 4.2 \pm 0.3) \times 10^{-10} \) [Davier et al., Eur.Phys.J. C71, 1515 (2011)]
  - HVP(\( \alpha^3 \)) \( (-9.84 \pm 0.06) \times 10^{-10} \) [Hagiwara et al., J.Phys. G38, 085003 (2011)]
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- Standard Model
  - $em$ (11658471.895 ± 0.008) x $10^{-10}$ [Kinoshita et al., Phys.Rev.Lett. 109, 111808 (2012)]
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  - LbL \[ (10.5 \pm 2.6) \times 10^{-10} \] \[ [\text{Prades et al., Adv.Ser.Direct.High Energy Phys. 20, 303 (2009)}] \]

- Comparison of theory and experiment: \( 3.6\sigma \) deviation

\[ \Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 28.8(6.3)^{\text{Exp}}(4.9)^{\text{SM}} \times 10^{-10} \]

- new physics?
Hadronic Vacuum Polarisation (HVP) from the R-ratio

- current best theoretical estimate uses experimental data
- optical theorem

\[ R(s) = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons}, s)}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-, s)} \]

- R-ratio

- first principles calculation of HVP \( \rightarrow \) lattice QCD
The anomalous magnetic moment of the muon

Hadronic Vacuum Polarisation (HVP) from the lattice

- calculate hadronic part on the lattice

\[ C_{\mu \nu}(t) = \sum_{\vec{x}} \langle J_{\mu}(t, \vec{x}) J_{\nu}(0) \rangle \]

- vector two-point function

- electromagnetic current

\[ J_{\mu} = \frac{2}{3} u\gamma_{\mu} u - \frac{1}{3} d\gamma_{\mu} d - \frac{1}{3} s\gamma_{\mu} s + \ldots \]


\[ a_{\mu} = \sum_{t} w_{t} C_{ii}(t) \quad \text{for } i = 0, 1, 2 \]
Hadronic Vacuum Polarisation (HVP) from the lattice

- calculate hadronic part on the lattice

- vector two-point function

\[ C_{\mu\nu}(t) = \sum_{\vec{x}} \langle J_{\mu}(t, \vec{x}) J_{\nu}(0) \rangle \]

- electromagnetic current

\[ J_{\mu} = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s + \ldots \]


\[ a_\mu = \sum_t w_t C_{ii}(t) \quad i = 0, 1, 2 \]


- for the first time included electromagnetic corrections in the calculation

\[ \rightarrow \text{quarks have electric charge} \]

\[ \rightarrow O(\alpha) \text{ with } \alpha \approx 1/137 \text{ fine structure constant} \]
Comparison of results

\[ e^+ e^- \rightarrow \text{hadrons} \]

\[ R\text{-ratio/lattice combined} \]

\[ \text{lattice} \]

\[ a_\mu^{\text{HVP}} \cdot 10^{10} \]

Jegerlehner 2017
Davier \textit{et al} 2017
Teubner \textit{et al} 2017
RBC/UKQCD 2018
BMW 2017
CLS Mainz 2017
HPQCD 2016
ETMC 2018

Most precise determination to date!
Comparison of results

\[ e^+ e^- \rightarrow \text{hadrons} \]

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Vera G"ulpers (University of Southampton)
Outline

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**Flavour physics**
- CKM matrix and leptonic Meson decays
- rare Kaon decays

Conclusions
**Introduction**

- charged weak interaction ($W^\pm$): changes “up”-type quarks into a “down”-type quarks
- mixes different generation of quarks
- quark-mixing Cabibbo–Kobayashi–Maskawa (CKM) matrix

\[
V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]
Unitarity of the CKM matrix

- within the Standard Model CKM matrix is unitary $V_{CKM} V_{CKM}^\dagger = 1$
- example

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

\[ \begin{vmatrix}
V_{ud} & V_{ub}^* \\
V_{cd} & V_{cb}^*
\end{vmatrix} = 1 \]

\[ \begin{vmatrix}
V_{td} & V_{tb}^*
\end{vmatrix} \]

[PDG]
\( V_{us} \) from leptonic Kaon decays

- leptonic Kaon decay \( K^+ \rightarrow \ell^+ \nu_\ell \)

\[
\begin{align*}
\text{K}^+ & \quad \quad \text{u} \quad \quad \rightarrow \quad \text{W}^+ \\
\text{W}^+ & \quad \quad \text{u} \quad \quad \rightarrow \quad \ell^+ \\
\text{u} & \quad \quad \text{f} \quad \quad \rightarrow \quad \nu_\ell
\end{align*}
\]

- effective weak Hamiltonian

\[
\begin{align*}
\overline{\text{u}} & \quad \quad \text{W}^+ \\
\text{W}^+ & \quad \quad \text{f} \quad \quad \rightarrow \quad \ell^+ \\
\text{f} & \quad \quad \text{G}_F \quad \quad \rightarrow \quad \nu_\ell
\end{align*}
\]

- decay rate (can be measured experimentally)

\[
\Gamma(K^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2 |V_{us}|^2 f_K^2}{8\pi} m_K m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2
\]

- known factors (Fermi constant \( G_F \), masses \( m \))
- kaon decay constant \( f_K \), can be calculated on the lattice
- CKM matrix element \( V_{us} \)

\( \rightarrow \) need both - experiment to determine \( \Gamma \) and lattice to determine \( f_K \)
\( f_K / f_{\pi} \) from the lattice

\[ f_{K^\pm} / f_{\pi^\pm} \]

- **1% precision → electro-magnetic corrections become important**


- **RBC/UKQCD work in progress, see poster by J. Richings**
RBC/UKQCD Flavour Physics Program

- leptonic and semi-leptonic Kaon decays (in progress: include electromagnetic corrections) → determine $V_{us}$

- charm physics
  - $D$ meson decays constants [J. T. Tsang et al., JHEP 1712 (2017) 008]

  \[
  \begin{aligned}
  &D/D_s \\
  \rightarrow & \text{determine } V_{cd}, V_{cs}
  \end{aligned}
  \]

  - semi-leptonic $D$ decays, determination of charm quark mass

- bottom physics, leptonic and semi-leptonic decays

- $K \rightarrow \pi\pi$, e.g.
  → Ken Wilson Lattice award 2012

- $K^0 - \bar{K}^0$ mixing (including beyond the Standard Model, poster by J.Kettle)
  → ...

- rare Kaon decays
rare Kaon decays

- rare Kaon decay $K \rightarrow \pi e^+ e^-$
- Flavor changing neutral current
  $\rightarrow$ forbidden in Standard Model at “tree-level”, i.e.
rare Kaon decays

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- Flavor changing neutral current
  $\rightarrow$ forbidden in Standard Model at “tree-level”, i.e.

$$\begin{align*}
K^+ &\rightarrow \pi^+ e^+ e^- \\
\downarrow &
\end{align*}$$

$\Rightarrow$ no Standard Model process
rare Kaon decays

- rare Kaon decay $K \rightarrow \pi e^+e^-$
- Flavor changing neutral current
  - forbidden in Standard Model at “tree-level”, i.e.
    - $\rightarrow$ no Standard Model process
  - $s$ to $d$ transition in Standard Model only via loop-processes, e.g.
    - extremely rare in the Standard Model $\rightarrow$ sensitive to new physics
rare Kaon decays

- rare Kaon decay $K \rightarrow \pi e^+ e^-$
- Flavor changing neutral current
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$\Rightarrow$ no Standard Model process
rare Kaon decays

- rare Kaon decay $K \rightarrow \pi e^+ e^-$
- Flavor changing neutral current
  $\rightarrow$ forbidden in Standard Model at “tree-level”, i.e.

$\Rightarrow$ no Standard Model process
lattice QCD for rare Kaon decays

- experiments at NA62 & LHCb (CERN), J-PARC (Koto) to measure rare Kaon decays

\[ K \rightarrow \pi e^+e^- \quad K \rightarrow \pi \nu\bar{\nu} \]

- Standard Model prediction requires calculation of hadronic amplitudes
  \rightarrow Lattice QCD for long distance contributions, e.g.

\[ e^+ + e^- \rightarrow K^+ + \pi^+ + W^+ + \rightarrow RBC/UKQCD developed method, currently only ones to calculate this \]

- RBC/UKQCD exploratory study (small lattice, unphysical quark-masses)

- influenced NA62 to look into \( K \rightarrow \pi \ell^+\ell^- \)

- work in progress: study the optimal setup for physical point calculation,
  \rightarrow see poster by F. Ó hÓgáin
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- Standard Model very successful model to describe elementary particles and their interactions, but many open questions
- low-energy precision test of the standard model
  \[ \rightarrow \text{QCD: first principles calculations using Monte Carlo (Lattice QCD)} \]
- hadronic contribution to the magnetic moment of the muon (including electro-magnetic effects)
- broad flavour physics program
  - leptonic meson decays (including electro-magnetic effects)
  - rare Kaon decays
  - \( \ldots \)
- QED corrections to \( K \rightarrow \ell \nu \), rare \( K \) decays currently using \( 60\% \) of Tesseract project dp008