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Nonequilibrium Quantum Meson Gas: Dimensional Reduction

1.-Scalar Quantum Field at Equilibrium: Dimensional Reduction (EDR)

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Quantum mesons, with microscopic dynamics described by unrenormalized relativistic Hermitian scalar quantum field operator $\phi(t, \mathbf{x})$ in $1+3$ dimensions: t: real time and $\mathbf{x} = (x_1, x_2, x_3)$. Unrenormalized mass parameter m. Unrenormalized quantum Hamiltonian operator. $H = \int d^3\mathbf{x} h(\Pi; \phi)$.

$h(\Pi; \phi) = 2^{-1} N[\Pi^2 + \sum_{i=1}^{3} (\partial\phi/\partial x_i)^2 + m^2\phi^2 + V(\phi)]$,

$\Pi = \partial\phi/\partial t$. N: normal product, $V(\phi) = g\phi^4/4!$. g. unrenormalized coupling constant. Ultraviolet cut-off: Λ.

Quantum meson gas: large statistical system Infinite number of degrees of freedom of the field $\rightarrow$ statistical behaviour (no "heat bath"). $\rightarrow$ quantum and thermal fluctuations

Scalar Quantum Field at Equilibrium: Dimensional Reduction (EDR)

Imaginary Time Formalism

Quantum gas at thermal equilibrium, at finite temperature $(K_B\beta_{eq})^{-1}$ (K_B = Boltzmann's constant) $\rightarrow$ equilibrium quantum density operator $\rho_{eq} = \exp(-\beta_{eq}H)$.

For given Λ, equilibrium generating functional $\rightarrow$ functional integral over field $\phi = \phi(\tau, \mathbf{x})$ with source $J_{eq} = J_{eq}(\tau, \mathbf{x})$ (τ = imaginary time):

$$Z_{eq}[J_{eq}] = \int [D\phi] \exp[-\int d^3\mathbf{x} \int_0^{\beta_{eq}} d\tau (s_{eq} - J_{eq}\phi)]$$

$$s_{eq} = \frac{1}{2}(\frac{\partial\phi}{\partial\tau})^2 + s_{eq,3}(m, g)$$

$$s_{eq,3}(m, g) = \frac{1}{2}\sum_{i=1}^{3}(\frac{\partial\phi}{\partial x_i})^2 + \frac{m^2}{2}\phi^2 + \frac{g\phi^4}{4!}$$

periodic boundary conditions: $\phi(\tau = 0, \mathbf{x}) = \phi(\tau = \beta_{eq}, \mathbf{x})$

$\Lambda \gg \beta_{eq}^{-1} > 0$ and $\Lambda \to +\infty \to$ Ultraviolet divergences $\to$ Perturbative renormalization.

$\to$ Renormalized field (ϕ_r), coupling constant ($g_{eq,r}$) and mass ($m_{eq,r} > 0$), mass counterterm δm_{eq}^2 and renormalization constants $Z_{eq,i}$, $i = 1, 3$. Use zero-temperature renormalization conditions.

After renormalization $\to$ high temperature (small β_{eq}) and large spatial scales.

Set $J_{eq}(\tau, \mathbf{x}) \simeq j_{eq}(\mathbf{x}) = j_{eq}$. Assume

$\int (d^3\mathbf{x}/(2\pi)^3) \exp[-i\mathbf{k}\mathbf{x}] j_{eq}(\mathbf{x})$ largest if $|\, \mathbf{k} \,| \ll \beta_{eq}^{-1}$, and negligible otherwise.

Series for approximate renormalized correlators resummed into renormalized (r) and dimensionally reduced (dr) equilibrium generating functional $Z_{eq,r,dr} = Z_{eq,r,dr}[j_{eq}]$

$$Z_{eq,r,dr} = \int [d\phi] \exp[-\beta_{eq} \int d^3\mathbf{x}(s_{eq,3}(m_{eq,r}, g_{eq,r,dr}) + \frac{\delta m_{eq,dr}^2}{2}\phi^2 - j_{eq}\phi)]$$

$\phi = \phi(\mathbf{x})$ (τ-independent).

$g_{eq,r,dr} = g_{eq,r} + \delta g_{eq,r}$

To one-loop order in $g_{eq,r}$,

$\delta g_{eq,r} \simeq -(3/16\pi^2) g_{eq,r}^2 (\ln(m_{eq,r}\beta_{eq}/4\pi) - \psi(1))$ ($\psi(1)$ constant and β_{eq}-independent).

$Z_{eq}[J_{eq}] \rightarrow Z_{eq,r,dr}[j_{eq}]$, for small β_{eq} and large spatial scales (zero-mode approximation) $\rightarrow$ Equilibrium Dimensional Reduction (EDR)) in imaginary time formalism.

$$\delta m^2_{eq,dr} = \delta m^2_{eq,dr,1} + \sigma_{eq,1} + \delta m^2_{eq,dr,2}$$
$$\delta m^2_{eq,dr,1} = -\frac{g_{eq,r}}{2\beta_{eq}} \int_{|\mathbf{k}|\leq\Lambda} \frac{d^3\mathbf{k}}{(2\pi)^3 \omega(|\,\mathbf{k}\,|)^2}$$
$$\sigma_{eq,1} = \frac{g_{eq,r}}{2} \int \frac{d^3\mathbf{k}}{(2\pi)^3 \omega(|\,\mathbf{k}\,|)} \frac{1}{\exp \beta_{eq}\omega(|\,\mathbf{k}\,|) - 1}$$

$\omega(\mid \mathbf{k} \mid) = (m^2_{eq,r} + \mathbf{k}^2)^{1/2}$. $Z_{eq,r,dr}[j_{eq}]$: no ultraviolet divergences in coupling constant or field

$\delta m^2_{eq,dr,1}$ (one-loop order in $g_{eq,r}$) $\rightarrow$ linearly ultraviolet divergent. $\sigma_{eq,1}$ finite $\rightarrow$ (temperature-dependent) real self-energy, at order $g_{eq,r}$. $\delta m^2_{eq,dr,2}$ $\rightarrow$ higher orders in $g_{eq,r}$

Nonperturbatively, 4-dimensional ϕ^4 theory at equilibrium at zero temperature is trivial.

After EDR, renormalized $Z_{eq,r,dr}[j_{eq}]$ is in same class as non-trivial massive 3-dimensional ϕ^4 theory $\rightarrow$ phase transition at critical temperature T_c. Below T_c: interaction-free gas. Above T_c, gas has non-vanishing interactions.

In lattice computations: dimensional crossover in critical properties of ϕ^4 theory, as temperature increases

Real-Time Equilibrium Formalism

Full (unrenormalized) equilibrium real-time generating functional $Z_{eq}[J_+, J_-] \to n$-point unrenormalized real-time equilibrium field correlators (response of quantum gas to external perturbations) ($n \geq 2$). $J_+(t_+\mathbf{x})$ and $J_-(t_-\mathbf{x})$: two external sources.

$$Z_{eq}[J_+, J_-] = \exp[-i \int_{-\infty}^{+\infty} dt \int d^3\mathbf{x}(V(-i\frac{\delta}{\delta J_+}) - V(-i\frac{\delta}{\delta J_-}))] Z_{eq}^{(0)}[J_+, J_-]$$

$Z_{eq}^{(0)}[J_+, J_-]$: free (unrenormalized) equilibrium real-time generating functional, with standard free unrenormalized correlators

$$Z_{eq}^{(0)}[J_+, J_-] = C \exp\left[+\frac{1}{2} \int d^3\mathbf{x} d^3\mathbf{x}' s_{eq}^{(0)}\right]$$

Consistent with analytic continuation from real-time to imaginary-time one

For equilibrium, introduce alternative sources $J_c(t,\mathbf{x})$ and $J_\Delta(t,\mathbf{x})$, useful for high temperature:

$$J_c = \frac{J_+ + J_-}{2}, J_\Delta = J_+ - J_-$$

$$\to Z^{(0)}_{eq}[J_+, J_-] = Z^{(0)\prime}{}_{eq}[J_c, J_\Delta]$$

$$s^{(0)}_{eq} = \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dt' [J_\Delta(t,\mathbf{x}) \Delta^{(0)}_{\Delta,\Delta}(t,\mathbf{x};t',\mathbf{x}') \times J_\Delta(t',\mathbf{x}') + J_\Delta(t,\mathbf{x}) J_c(t',\mathbf{x}') (\Delta^{(0)}_{\Delta,c}(t,\mathbf{x};t',\mathbf{x}') + \Delta^{(0)}_{c,\Delta}(t',\mathbf{x}';t,\mathbf{x}))] \equiv s^{(0)\prime}_{eq}$$

$\Delta^{(0)}_{\Delta,c}$, $\Delta^{(0)}_{c,\Delta}$ and $\Delta^{(0)}_{\Delta,\Delta}$: retarded, advanced and correlated dynamical Green's functions

No contribution proportional to $J_c J_c$.

$$\tilde{\Delta}^{(0)}_{\Delta,\Delta}(k) = -2\pi\delta(k^2 - m^2)n(|k^0|)$$
$$\tilde{\Delta}^{(0)}_{\Delta,c}(k) + \pi\epsilon(k^0)\delta(k^2 - m^2) = \tilde{\Delta}^{(0)}_{c,\Delta}(k) - \pi\epsilon(k^0)$$
$$\times\delta(k^2 - m^2) = -\frac{i}{2}[\frac{1}{k^2 - m^2 + i\epsilon} + \frac{1}{k^2 - m^2 - i\epsilon}]$$
$$n(|k^0|) = \frac{1}{2} + \frac{1}{\exp|k^0|\beta_{eq} - 1}$$

$$Z'_{eq}[J_c, J_\Delta] = \exp[-i\int_{-\infty}^{+\infty} dt \int d^3\mathbf{x}\frac{g}{4!}(\frac{\delta^3}{\delta J_c^3}\frac{\delta}{\delta J_\Delta} +$$
$$4\frac{\delta^3}{\delta J_\Delta^3}\frac{\delta}{\delta J_c})]Z^{(0)\prime}{}_{eq}[J_c, J_\Delta] = Z_{eq}[J_+, J_-]$$

$\rightarrow Z_{eq}[J_+, J_-] \rightarrow$ full unrenormalized real-time equilibrium correlators as power series in g: -three free correlators $\Delta^{(0)}_{\Delta,\Delta}$, $\Delta^{(0)}_{\Delta,c}$ and $\Delta^{(0)}_{c,\Delta}$, and

-two interactions $(\delta^3/\delta J_c^3)(\delta/\delta J_\Delta)$, $(\delta^3/\delta J_\Delta^3)(\delta/\delta J_c)$

Take $\Lambda \to \infty \to$ Renormalize using J_c and J_Δ (with zero temperature renormalization conditions). Renormalization required for:

i) two-point correlator with one external Δ-leg and one external c-leg needs mass ii) two four-point correlators with one external c-leg and three external Δ-legs and with one external Δ-leg and three external c-legs.

$\to$ renormalized functional $Z'_{eq,r}[J_c, J_\Delta]$

$m \to m_{eq,r}$, $g \to g_{eq,r}$ (plus renormalization constants and counterterms)

$\to$ Go to regime of high temperature (small β_{eq}) and large spatial and time scales.

Starting point: the renormalized functional $Z'_{eq,r}[J_c, J_\Delta]$

For $u = c, \Delta$, assume that:

$$J_u(x) = \int \frac{d^4k}{(2\pi)^4} \exp[-ikx] \tilde{J}_u(k)$$

takes on appreciable values when any k^μ ($\mu = 0,1,2,3$) is $\ll \beta_{eq}^{-1}$, (negligible otherwise).

Fourier transforms of $\Delta^{(0)}_{\Delta,c,r}$ and $\Delta^{(0)}_{c,\Delta,r}$ (with $m \to m_{eq,r}$) do not simplify in EDR.

$\Delta^{(0)}_{\Delta,\Delta,r}$ (with $m \to m_{eq,r}$) does simplify:

$$\tilde{\Delta}^{(0)}_{\Delta,\Delta,r} \simeq -\frac{2\pi\delta(k^2 - m^2_{eq,r})}{\beta_{eq} \mid k^0 \mid} \equiv \tilde{\Delta}^{(0)}_{\Delta,\Delta,r,dr}$$

$Z'_{eq,r}[J_c, J_\Delta] \to$

Effective dimensionally reduced renormalized equilibrium generating functional

$$Z'_{eq,r,dr}[J_c, J_\Delta] = \exp[-i\int_{-\infty}^{+\infty} dt \int d^3\mathbf{x}(4\frac{g_{eq,r,dr}}{4!}\frac{\delta^3}{\delta J^3_\Delta} \times \frac{\delta}{\delta J_c} + \frac{\delta m^2_{eq,dr}}{2}\frac{\delta}{\delta J_\Delta}\frac{\delta}{\delta J_c})] Z^{(0)\prime}{}_{eq,r,dr}[J_c, J_\Delta]$$

$$Z^{(0)\prime}_{eq,r,dr}[J_c, J_\Delta] = C \exp[+\frac{1}{2}\int d^3\mathbf{x} d^3\mathbf{x}' s^{(0)}_{eq,r,dr}{}']$$

$$s^{(0)}_{eq,r,dr}{}' = \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dt' [J_\Delta(t,\mathbf{x}) \times$$
$$\Delta^{(0)}_{\Delta,\Delta,r,dr}(t,\mathbf{x};t',\mathbf{x}') J_\Delta(t',\mathbf{x}') + J_\Delta(t,\mathbf{x}) J_c(t',\mathbf{x}') \times$$
$$(\Delta^{(0)}_{\Delta,c,r}(t,\mathbf{x};t',\mathbf{x}') + \Delta^{(0)}_{c,\Delta,r}(t',\mathbf{x}';t,\mathbf{x}))]$$

$g_{eq,r,dr} = g_{eq,r} + \delta g_{eq,r}$

$g_{eq,r,dr}$ is finite.

Same $\delta m_{eq,dr}$ and $\delta g_{eq,r}$ as in EDR

$Z'_{eq,r,dr}[J_c, J_\Delta]$ is super-renormalizable.

$Z'_{eq,r,dr}[J_c, J_\Delta] \rightarrow$ valuable hints for nonequilibrium gas and its dimensional reduction (NEDR)

Scalar Quantum Field Off-Equilibrium

Classical lagrangian density:

$$l_{cl}(\phi) \ \equiv \ \frac{1}{2}[(\partial^{\mu}\phi)(\partial_{\mu}\phi) - m^2\phi^2] - V(\phi)$$

Density operator for $t > t_0$:

$$\rho(t, t_0) = U(t, t_o)\rho_{in}U^{+}(t, t_0)$$

$$U(t, t_o) = \exp[-iH(t - t_0)]$$

For statistical scalar field system, possible initial nonequilibrium ρ_{in} at t_0, including self-interactions:

$$\rho_{in} = \exp\left[-\beta_{eq}\int d^3\mathbf{x}[\lambda(\mathbf{x})_1 h_{in}(\Pi;\phi) + \lambda(\mathbf{x})_2\phi(\mathbf{x})]\right]$$

$h_{in}(\Pi;\phi) = h(\Pi;\phi)$ with $m \to m_{in}$ and $g \to g_{in}$. $\lambda_s = \lambda(\mathbf{x})_s$, $s = 1, 2$ are given data with spatial variations (new length scales) $\rightarrow$ spatial inhomogeneities in the nonequilibrium initial state

Suppose: i) $\lambda(\mathbf{x})_s$, $s = 1, 2$ finite and continuous for any $\mathbf{x}$ and slowly-varying in length scales typical of renormalized ϕ^4 theory (for zero temperature renormalization conditions)

ii) $\lambda(\mathbf{x})_1 > 0$ for any $\mathbf{x}$ and approaches $+1$, as $|\mathbf{x}| \rightarrow \infty$ along any direction,

iii) $\lambda(\mathbf{x})_2$ is small and approaches zero, as $|\mathbf{x}| \rightarrow \infty$.

$\beta_{eq}^{-1}(> 0)$ interpreted as equilibrium temperature at large distances.

If $\lambda(\mathbf{x})_2 \equiv 0$, $\rightarrow$ $\lambda(\mathbf{x})_1 \neq 1$ $\rightarrow$ local equilibrium at short scales, global nonequilibrium at intermediate scales and thermal equilibrium at very large distances.

If $\lambda(\mathbf{x})_2 \neq 0 \rightarrow$ nonequilibrium initial state (breaking symmetry $\phi \rightarrow -\phi$).

$|\chi\rangle =$ generic eigenstate of $\phi \rightarrow$

$$\langle\chi|\rho(T,t_0)|\chi\rangle = \int[d\phi_2]\int[d\phi_1]\langle\chi|U(T,t_0)|\phi_2\rangle \times \langle\phi_2|\rho_{in}|\phi_1\rangle\langle\phi_1|U(T,t_0)^+|\chi\rangle$$

$\langle\chi|U(T,t_0)|\phi_2\rangle$ and $\langle\phi_1|U(T,t_0)^+|\chi\rangle \rightarrow$ (real-time) functional integrals (T:large time).

Two external sources, $J_+(t_+)$ and $J_-(t_-)$ associated to $\langle\chi|U(T,t_0)|\phi_2\rangle$ and $\langle\phi_1|U(T,t_0)^+|\chi\rangle$.

External source, $J_{in}(\tau,\mathbf{x})$ for ρ_{in}

Unrenormalized nonequilibrium generating functional:

$$Z[J_+,J_-,J_{in}] = \int[d\chi]\int[d\phi_2]\int[d\phi_1] I_+ I_{in} I_-$$

$$I_\pm = \int[D\phi_\pm]\,\text{exp}\left[\pm i\int d^3\mathbf{x}\int_{t_0}^{T} dt_\pm (l_{cl}(\phi_\pm) + J_\pm\phi_\pm)\right]$$

$I_+ = I_+[\chi, \phi_2; J_+]$ with boundary conditions: $\phi_+(t_0, \mathbf{x}) = \phi_2(t_0, \mathbf{x})$ and $\phi_+(T, \mathbf{x}) = \chi(\mathbf{x})$

$I_- = I_-[\phi_1, \chi; J_-]$ with boundary conditions: $\phi_-(t_0, \mathbf{x}) = \phi_1(t_0, \mathbf{x})$ and $\phi_-(T, \mathbf{x}) = \chi(\mathbf{x})$.

$\langle \phi_2 | \rho_{in} | \phi_1 \rangle \rightarrow$ functional integral for I_{in}:

$$I_{in} = \int [D\phi_{in}] \exp \left[- \int d^3\mathbf{x} \int_0^{\beta_{eq}} (s_{in} - J_{in}\phi_{in}) \right]$$
$$s_{in} = \frac{1}{2\lambda(\mathbf{x})_1} (\frac{\partial \phi_{in}}{\partial \tau})^2 + s_{in}^{(0)}(m_{in}) + s_{in}^{(V)}(g_{in})$$
$$s_{in}^{(0)}(m_{in}) = \lambda(\mathbf{x})_1 [\frac{1}{2} \sum_{i=1}^{3} (\frac{\partial \phi_{in}}{\partial x_i})^2 + \frac{m_{in}^2}{2} \phi_{in}^2]$$
$$s_{in}^{(V)}(g_{in}) = \lambda(\mathbf{x})_1 V_{in}(\phi_{in}) + \lambda(\mathbf{x})_2 \phi_{in}$$

Boundary conditions : $\phi_{in}(\tau = 0, \mathbf{x}) = \phi_1(t_0, \mathbf{x})$ and $\phi_{in}(\tau = \beta_{eq}, \mathbf{x}) = \phi_2(t_0, \mathbf{x})$. .

$Z[J_+, J_-, J_{in}] \to$ (unrenormalized) nonequilibrium (T-independent) two-point correlators

For nonequilibrium, introduce same alternative sources J_c and J_Δ as for equilibrium

$$Z[J_+, J_-, J_{in}] \equiv Z'[J_c, J_\Delta, J_{in}] \to$$

$$Z'[J_c, J_\Delta, J_{in}] = \int [d\chi] \int [d\phi'_c] \int [d\phi'_\Delta] \int [D\phi_c] \int [D\phi_\Delta]$$
$$\times I_{in} \exp i \int_{t_0}^{T} dt[(-m^2\phi_c - \partial^\mu \partial_\mu \phi_c)\phi_\Delta$$
$$-V(\phi_c + \frac{\phi_\Delta}{2}) + V(\phi_c - \frac{\phi_\Delta}{2}) + J_c\phi_\Delta + J_\Delta\phi_c]$$

$\phi_1 = \phi'_c - 2^{-1}\phi'_\Delta$, $\phi_2 = \phi'_c + 2^{-1}\phi'_\Delta$, $\phi_c = 2^{-1}(\phi_+ + \phi_-)$, $\phi_\Delta = \phi_+ - \phi_-$. Functional integrals over ϕ_c and ϕ_Δ with boundary conditions: $\phi_c(t_0) = \phi'_c$, $\phi_c(T) = \chi$, $\phi_\Delta(t_0) = \phi'_\Delta$ and $\phi_\Delta(T) = 0$.

For $V = 0$ and $\lambda_2 = 0$, $Z'[J_c, J_\Delta, J_{in}] = Z'^{(0)}[J_c, J_\Delta, J_{in}]$.

$$Z'^{(0)}[J_c, J_\Delta, J_{in}] = C \exp\left[+\frac{1}{2}\int d^3\mathbf{x} d^3\mathbf{x}' s_{ne}^{(0)\prime}\right]$$

$$s_{ne}^{(0)\prime} = \int_{t_0}^{T} dt \int_{t_0}^{T} dt' [J_\Delta(t,\mathbf{x}) \Delta^{(0)}_{\Delta,\Delta}(t,\mathbf{x};t',\mathbf{x}')$$
$$\times J_\Delta(t',\mathbf{x}') + J_\Delta(t,\mathbf{x}) J_c(t',\mathbf{x}') (\Delta^{(0)}_{\Delta,c}(t,\mathbf{x};t',\mathbf{x}') +$$
$$\Delta^{(0)}_{c,\Delta}(t',\mathbf{x}';t,\mathbf{x}))] + \int_{t_0}^{T} dt \int_0^{\beta_{eq}} d\tau J_\Delta(t,\mathbf{x}) J_{in}(\tau,\mathbf{x}')$$
$$\times (\Delta^{(0)}_{\Delta,in}(t,\mathbf{x};\tau,\mathbf{x}') + \Delta^{(0)}_{in,\Delta}(\tau,\mathbf{x}';t,\mathbf{x})) + \int_0^{\beta_{eq}} d\tau \times$$
$$\int_0^{\beta_{eq}} d\tau' J_{in}(\tau,\mathbf{x}) \Delta^{(0)}_{in,in}(\tau,\mathbf{x};\tau',\mathbf{x}')) J_{in}(\tau',\mathbf{x}')$$

No contributions proportional to $J_c J_c$ and $J_c J_{in}$.

Unrenormalized free field correlators for J_c and J_Δ:

$$\Delta^{(0)}_{in,in}(\tau,\mathbf{x};\tau',\mathbf{x}') = \sum_\gamma f_\gamma(\mathbf{x}) f^*_\gamma(\mathbf{x}') \times$$

$$\frac{\cosh\gamma_0(\beta_{eq}/2-|\tau-\tau'|)}{2\gamma_0\sinh(\beta_{eq}\gamma_0/2)}$$

$$\Delta^{(0)}_{in,\Delta}(\tau,\mathbf{x};x') = \sum_\gamma \int \frac{d^3\mathbf{k} f^*_\gamma(\mathbf{x})}{2(2\pi)^3}\exp(-i\mathbf{k}\mathbf{x}') \times$$
$$\int d^3\mathbf{x}'' f_\gamma(\mathbf{x}'')\exp(i\mathbf{k}\mathbf{x}'') \times$$
$$[\frac{f_-(\gamma_0,1,\tau)\sin\omega(|\ \mathbf{k}\ |)(t'-t_0)}{\omega(|\ \mathbf{k}\ |)\lambda(\mathbf{x}'')_1} + \tag{1}$$
$$\frac{if_+(\gamma_0,1,\tau)\cos\omega(|\ \mathbf{k}\ |)(t'-t_0)}{f_0(\gamma_0,1)}]$$

$$\Delta^{(0)}_{\Delta,in}(x;\tau,\mathbf{x}) = \Delta^{(0)}_{in,\Delta}(\tau,\mathbf{x};x) \tag{2}$$

$$\Delta^{(0)}_{\Delta,\Delta}(x;x') = -\sum_\gamma \int d^3\mathbf{x}'' \int d^3\mathbf{x}''' f_\gamma(\mathbf{x}'') f^*_\gamma(\mathbf{x}''') \times$$
$$\int \frac{d^3\mathbf{k}}{(2\pi)^3}\int \frac{d^3\mathbf{k}'}{(2\pi)^3} \times$$
$$\exp[i(-\mathbf{k}\mathbf{x}-\mathbf{k}'\mathbf{x}'+\mathbf{k}\mathbf{x}''+\mathbf{k}'\mathbf{x}''')]\frac{\cosh(\gamma_0\beta_{eq}/2)}{2\gamma_0\sinh(\beta_{eq}\gamma_0/2)} \times$$
$$[\cos\omega(|\ \mathbf{k}\ |)(t-t_0)\cos\omega(|\ \mathbf{k}'\ |)(t'-t_0) +$$
$$\frac{\gamma_0^2\sin\omega(|\ \mathbf{k}\ |)(t-t_0)\sin\omega(|\ \mathbf{k}'\ |)(t'-t_0)}{\omega(|\ \mathbf{k}\ |)\omega(|\ \mathbf{k}'\ |)\lambda(\mathbf{x}'')_1\lambda(\mathbf{x}''')_1}]$$

$$\Delta^{(0)}_{\Delta,c}(x;x') = \int \frac{d^4k}{(2\pi)^4}\exp[-ik(x-x')] \times$$
$$[-\frac{i}{2}[\frac{1}{k^2-m^2+i\epsilon}+\frac{1}{k^2-m^2-i\epsilon}] -$$

$$\frac{\epsilon(k^0)}{2}2\pi\delta(k^2-m^2)]$$

$\Delta^{(0)}_{c,\Delta} = \Delta^{(0)}_{\Delta,c}$, with $\epsilon(k^0) \to -\epsilon(k^0)$. $f_\gamma(\mathbf{x})$, associated to spatial inhomogeneity $\lambda(\mathbf{x})_1$ in initial condition (γ_0^2, eigenvalue) fulfills:

$$L\frac{f_\gamma(\mathbf{x})}{[\lambda(\mathbf{x})_1]^{1/2}} = \gamma_0^2\frac{f_\gamma(\mathbf{x})}{[\lambda(\mathbf{x})_1]^{1/2}}$$

$$L\varphi = -\lambda(\mathbf{x})_1^{1/2}\sum_{k=1}^{3}\frac{\partial}{\partial x_k}[\lambda(\mathbf{x})_1\frac{\partial}{\partial x_k}(\lambda(\mathbf{x})_1^{1/2}\varphi)] + \lambda(\mathbf{x})_1^2 m_{in}^2\varphi$$

Standard functional techniques: $Z'[J_c, J_\Delta, J_{in}]$ in terms of $Z^{(0)}[J_c, J_\Delta, J_{in}]$. Take $\Lambda \to \infty$

Renormalization in $Z'[J_c, J_\Delta, J_{in}]$ using J_c, J_Δ and J_{in} (with zero temperature renormalization conditions) $\to$ renormalized nonequilibrium generating functional $Z'_r[J_c, J_\Delta, J_{in}]$: renormalized nonequilibrium free correlators, with $m \to m_r$ and so on with $m_{in,r}$, g_r, $g_{in,r}$

Nonequilibrium Dimensional Reduction

High temperature (small β_{eq}) and large distance regime for $Z'_r[J_c, J_\Delta, J_{in}]$.

Small β_{eq}, m_r and $m_{in,r} \ll \beta_{eq}^{-1}$

Same approximations regarding J_c and J_Δ as for EDR.

Small β_{eq} approximation in renormalized nonequilibrium free correlators $\Delta^{(0)}_{\Delta,\Delta,r}$, $\Delta^{(0)}_{\Delta,in,r}$, $\Delta^{(0)}_{in,\Delta,r}$ and $\Delta^{(0)}_{in,in,r}$

$\Delta^{(0)}_{\Delta,c,r}$ and $\Delta^{(0)}_{c,\Delta,r}$ remain unaltered.

$J_{in}(\tau, \mathbf{x}) \simeq j_{in}(\mathbf{x})$, $\int_0^{\beta_{eq}} d\tau \simeq \beta_{eq}$

renormalized I_{in} in NEDR:

$$I_{in,r,dr} = I_{in,r,dr}(\phi_2, \phi_1) = \langle \phi_2 | \rho_{in,r,dr} | \phi_1 \rangle \times$$
$$\exp\left\{ \frac{\beta_{eq}}{2} \int d^3\mathbf{x} j_{in}(\mathbf{x}) \left[\phi_1(t_0, \mathbf{x}) + \phi_2(t_0, \mathbf{x}) \right] \right\}$$
$$\langle \phi_2 | \rho_{in,r,dr} | \phi_1 \rangle \simeq \exp[-\beta_{eq} \int d^3\mathbf{x} (s^{(0)}_{in,r}(m_{in,r}) +$$
$$s^{(V)}_{in}(g_{in,r,dr}) + \frac{\lambda(\mathbf{x})_1 \delta m^2_{in,dr}}{2} \phi_{in}(t_0, \mathbf{x})^2 +$$
$$\frac{(\phi_2(t_0, \mathbf{x}) - \phi_1(t_0, \mathbf{x}))^2}{2\lambda(\mathbf{x})_1 \beta^2_{eq}})]$$

$s^{(0)}_{in,r}(m_{in,r})$ and $s^{(V)}_{in}(g_{in,r,dr})$. $\phi_{in}(\tau, \mathbf{x})$, m_{in}, g_{in} replaced by $\phi_{in}(t_0, \mathbf{x}) = 2^{-1}(\phi_1(t_0, \mathbf{x}) + \phi_2(t_0, \mathbf{x}))$, $m_{in,r}$, $g_{in,r,dr}$.

Leading contributions resummed into new renormalized dimensionally reduced nonequilibrium generating functional $Z'_{r,dr}[J_c, J_\Delta, j_{in}]$:

$$Z'_{r,dr}[J_c, J_\Delta, j_{in}] = \exp[-i \int_{t_0}^{T} dt \int d^3\mathbf{x} (\frac{g_{r,dr}}{4!} 4 \frac{\delta^3}{\delta J^3_\Delta} \times$$
$$\frac{\delta}{\delta J_c} + \delta m^2_{dr} \frac{\delta}{\delta J_\Delta} \frac{\delta}{\delta J_c})] U_{in,r,dr} Z^{(0)\prime}{}_{r,dr}[J_c, J_\Delta, j_{in}]$$

$$U_{in,r,dr} = \exp[-\beta_{eq} \int d^3\mathbf{x}(\lambda_1 \frac{g_{in,r,dr}}{4!} \frac{\delta^4}{\delta j_{in}^4} +$$
$$\lambda_1 \frac{\delta m_{in,dr}^2}{2} \frac{\delta^2}{\delta j_{in}^2} + \lambda_2 \frac{\delta}{\delta j_{in}})]$$
$$Z^{(0)\prime}{}_{r,dr}[J_c, J_\Delta, j_{in}] = C \exp\left[\int \frac{d^3\mathbf{x} d^3\mathbf{x}'}{2} s^{(0)}_{ne,r,dr}{}'\right]$$
$$s^{(0)}_{ne,r,dr}{}' = \int_{t_0}^T dt \int_{t_0}^T dt' [J_\Delta(t,\mathbf{x}) \Delta^{(0)}_{\Delta,\Delta,r,dr}(t,\mathbf{x};t',\mathbf{x}')$$
$$\times J_\Delta(t',\mathbf{x}') + J_\Delta(t,\mathbf{x}) J_c(t',\mathbf{x}')(\Delta^{(0)}_{\Delta,c,r}(t,\mathbf{x};t',\mathbf{x}') +$$
$$\Delta^{(0)}_{c,\Delta,r}(t',\mathbf{x}';t,\mathbf{x}))] + \int_{t_0}^T dt \beta_{eq} J_\Delta(t,\mathbf{x}) j_{in}(\mathbf{x}')$$
$$\times(\Delta^{(0)}_{\Delta,in,r,dr}(t,\mathbf{x};\mathbf{x}') + \Delta^{(0)}_{in,\Delta,r,dr}(\mathbf{x}';t,\mathbf{x})) +$$
$$\beta_{eq}^2 j_{in}(\mathbf{x}) \Delta^{(0)}_{in,in,r,dr}(\mathbf{x};\mathbf{x}')) j_{in}(\mathbf{x}')$$

$Z^{(0)\prime}{}_{r,dr}[J_c, J_\Delta, j_{in}]$ is the NEDR free renormalized functional. $\mathbf{x}$-dependent mass renormalization counterterms $\delta m_{in,dr}^2$ and δm_{dr}^2

$Z'_{r,dr}[J_c, J_\Delta, j_{in}]$: super-renormalizable . Neither $g_{r,dr}$ and $g_{in,r,dr}$ nor fields involved in $Z'_{r,dr}[J_c, J_\Delta, j_{in}]$ require infinite renormalizations.

. $g_{in,r,dr}$ and $g_{r,dr}$ are effective off-equilibrium coupling constants,

$g_{in,r,dr} = g_{in,r} + \delta g_{in,r}$ and $g_{r,dr} = g_r + \delta g_r$. $\delta g_{in,r}$ and δg_r are ultraviolet finite, β_{eq}-dependent and position-dependent corrections. Studied at one-loop orders in $g_{in,r}$ and g_r.

Recast $Z'_{r,dr}[J_c, J_\Delta, j_{in}]$ into nonperturbative form:

$$Z'_{r,dr}[J_c, J_\Delta, j_{in}] = \int [d\chi]\rho_{dr}(\chi; T, t_0; J_c, J_\Delta, j_{in})$$

$$\rho_{dr}(\chi; T, t_0; J_c, J_\Delta, j_{in}) = \int [d\phi'_c][d\phi'_\Delta] \int [D\phi_c][D\phi_\Delta] \times$$

$$I_{in,r,dr}(\phi'_c + 2^{-1}\phi'_\Delta, \phi'_c - 2^{-1}\phi'_\Delta) \exp i \int_{t_0}^{T} dt \int d^3\mathbf{x} \times$$

$$[(-m_r^2\phi_c - \partial^\mu\partial_\mu\phi_c - \frac{\partial V(\phi_c)}{\partial\phi_c})\phi_\Delta - \delta m_{dr}^2\phi_c\phi_\Delta +$$

$$J_c\phi_\Delta + J_\Delta\phi_c]$$

$\phi_1 = \phi'_c - 2^{-1}\phi'_\Delta$, $\phi_2 = \phi'_c + 2^{-1}\phi'_\Delta$, $\phi_c = 2^{-1}(\phi_+ + \phi_-)$, $\phi_\Delta = \phi_+ - \phi_-$.

Consistency checked using Gaussian functional integrations.

Conclusions

NEDR generalizes nontrivially EDR in the imaginary-time formalism and in real-time one .

$Z'_{r,dr}[J_c, J_\Delta, j_{in}]$ for nonequilibrium fields is the main result $\rightarrow$ dynamics of nonequilibrium ϕ^4 theory in $(1+3)$-dimensional Minkowski space in NEDR regime

Structure $(-m_r^2\phi_c - \partial^\mu\partial_\mu\phi_c - \frac{\partial V(\phi_c)}{\partial\phi_c})$ characterizes classical field dynamics (with renormalization effects included).

Presentation based upon

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Many references to previous works by other authors on finite temperature field theory at equilibrium (in imaginary time and real time formalisms), EDR in imaginary time formalism and various aspects of nonequilibrium field theory are given in those references.