SU(2) with one Dirac flavour in the adjoint representation

Ed Bennett, Biagio Lucini (Swansea University), Agostino Patella (CERN)

ABSTRACT: *We present an initial investigation into the SU(2) gauge theory with a single adjoint Dirac flavour on the lattice. By a change of basis, we show that this can be reformulated as a theory with two adjoint Majorana flavours, which provides a more convenient framework to discuss the meson mass spectrum. Initial results for the spectroscopy of the theory are shown, including meson and glueball masses.*

Theory in question

- SU(2) with 1 Dirac fermion in the adjoint representation
- Can be reexpressed as SU(2) with 2 Majorana flavous in the adjoint representation
- May provide a route to lattice SUSY
- Probes the end of the conformal window
- Also of interest from a technicolor perspective

Majorana fields

- Fields described by Majorana spinors
- Spinors obey the Majorana constraint:

$$\psi_{\mathrm{MC}} \equiv C\overline{\psi}_{\mathrm{M}}^{\mathsf{T}} = \psi_{\mathrm{M}}$$

  i.e. invariant under charge conjugation
- Easier to simulate even number of adjoint Majoranas on the lattice

Supersymmetry

- Introduces boson-fermion symmetry
- For supersymmetric YM theory, a new fermion field is needed to partner the gauge bosons
- New field is in the adjoint representation
- Matching degrees of freedom requires a Majorana field

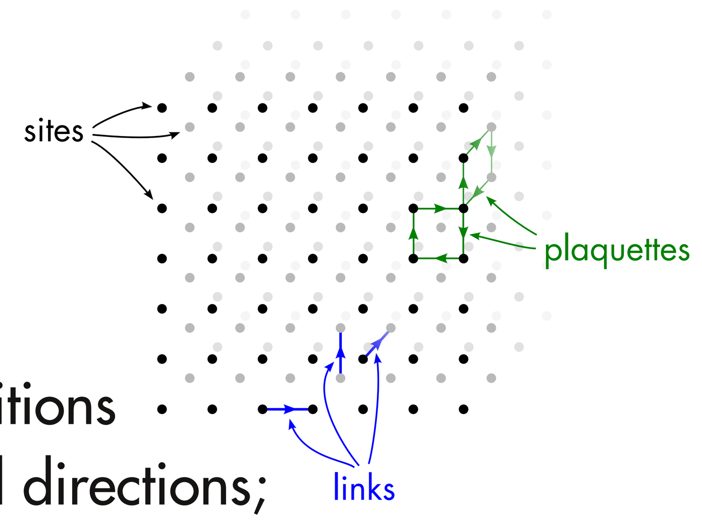
Conformal Window

- A region of parameter space at intermediate N_f where theories are conformal
- 2 Dirac/4 Majorana is conformal
- 1 Dirac/2 Majorana predicted to be confining
- 1.5 Dirac/3 Majorana predicted to be borderline

Technicolor

- Alternative explanation of EWSB to the elementary Higgs
- Introduces techniquarks, transforming under a new gauge symmetry
- Electroweak symmetry now broken by $\langle\overline{\psi}\psi\rangle$
- Extended technicolor (adding additional gauge bosons) needed to give mass to standard model fermions
- Walking technicolor mitigates flavour-changing neutral currents
- Walking conjectured to be found immediately below the lower end of the conformal window

Lattice formulation



- Discretised finite Euclidean (3+1)-dimensional spacetime
  - (Anti)periodic boundary conditions
  - L sites in spatial directions; T in temporal direction
- Fermion fields ψ defined on lattice sites
- Gauge fields U_μ on links between sites

Decomposition to Majoranas

$$\psi = \psi_{\mathrm{M}+} + i\psi_{\mathrm{M}-}$$

$$\psi_{\mathrm{M}+} = \frac{\psi + C\overline{\psi}^{\mathsf{T}}}{2}, \quad \psi_{\mathrm{M}-} = \frac{\psi - C\overline{\psi}^{\mathsf{T}}}{2i}$$

- This can be verified by imposing the Majorana constraint
- The terms of the action can be shown to be:

$$\overline{\psi}\psi = \overline{\psi}_{\mathrm{M}+}\psi_{\mathrm{M}+} + \overline{\psi}_{\mathrm{M}-}\psi_{\mathrm{M}-}$$

$$\overline{\psi}\not{\partial}\psi = \overline{\psi}_{\mathrm{M}+}\not{\partial}\psi_{\mathrm{M}+} + \overline{\psi}_{\mathrm{M}-}\not{\partial}\psi_{\mathrm{M}-}$$

  i.e. the two-Majorana action is identical to the one-Dirac one
- Hadron operators split into two channels:

$$\overline{\psi}_{\mathrm{M}+}\Gamma\psi_{\mathrm{M}-} = \begin{cases} \frac{1}{2i}\overline{\psi}\Gamma\psi & \Gamma = \gamma_\mu, \gamma_0, \gamma_0\gamma_5 \\ \frac{1}{4i}\left(\psi^{\mathsf{T}}C\Gamma\psi - \overline{\psi}\Gamma C\overline{\psi}^{\mathsf{T}}\right) & \Gamma = \mathbb{1}, \gamma_5\gamma_\mu, \gamma_5 \end{cases}$$

- Correlators of hadron operators then become

$$\left\langle (\overline{\psi}_{\mathrm{M}+}\Gamma\psi_{\mathrm{M}-})^{\dagger}(x)\overline{\psi}_{\mathrm{M}+}\Gamma\psi_{\mathrm{M}-}(0)\right\rangle =$$

$$\begin{cases} \frac{1}{4}\left[\mathrm{tr}(\overline{\Gamma}D^{-1}(x;x))\mathrm{tr}(\Gamma D^{-1}(0;0)) - \mathrm{tr}(\overline{\Gamma}D^{-1}(x;0)\Gamma D^{-1}(0;x))\right] & \gamma_\mu \ldots \\ -\frac{1}{4}\mathrm{tr}\overline{\Gamma}D^{-1}(x;0)\Gamma D^{-1}(0;x) & \gamma_5 \ldots \end{cases}$$

  - Same forms as singlet and triplet of the one-flavour Dirac theory

Reexpression Consequences

- Code (and configurations) for Dirac fields can be used for Majorana fields with minimal modification
- The channel including the γ_μ is not considered here since singlet techniques are not fully controlled
- γ_5 state is expected to be most accessible

Quantum Numbers

- Parity of Dirac fields maps to a combination of charge conjugation and flavour rotation of the corresponding Weyl fields
- Thus states with γ_5 and γ_μ are not pseudoscalar and vector but rather scalar and axial vector

Results

- The HiRep code[1, 2] was used at compute facilities in Swansea and Liverpool.
- Noise ruled out some meson channels, others were deliberately not considered due to singlet techniques not being fully controlled.
- Exploring closer to the chiral limit proved prohibitively computationally expensive.

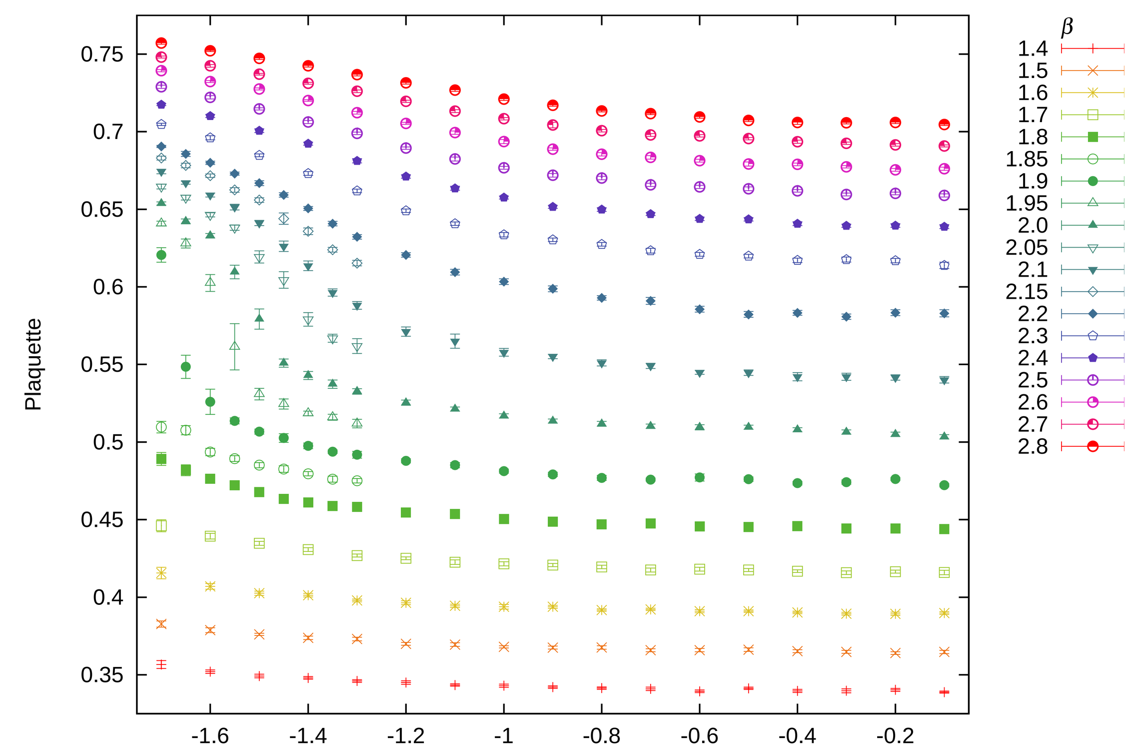


Fig. 1: Average plaquette on a 4^4 lattice; $1.4 \leq \beta \leq 2.8$, $-1.7 \leq m \leq -0.1$; used as a parameter sweep to identify the bulk phase and region of interest

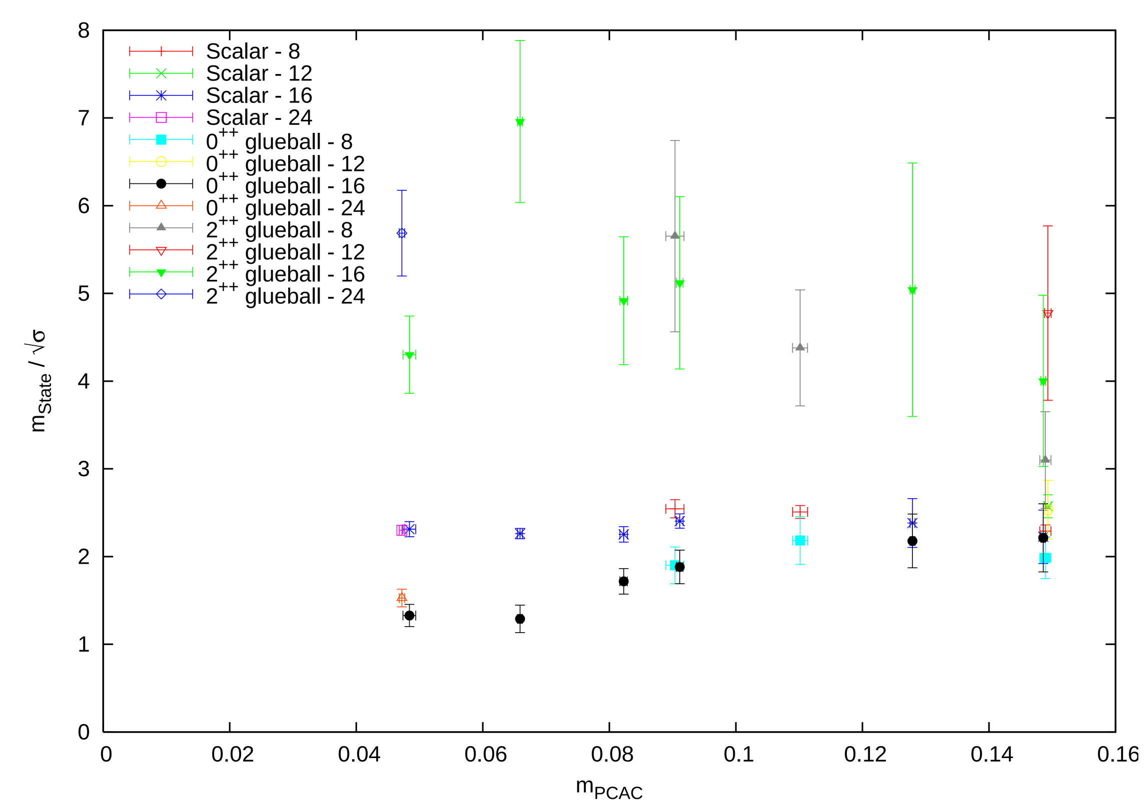


Fig. 2: Scalar meson and 0^{++} and 2^{++} glueball masses; $\beta = 2.05$, $-1.7 \leq m_{\mathrm{bare}} \leq -0.1$. Normalised by the fundamental string tension. Lattice sizes: 16×8^3, 24×12^3, 32×16^3, 48×24^3

Conclusions

- Exploratory study
- First quantitative investigation of this theory
- Scalar meson and 0^{++} glueball nearly degenerate

Ongoing & Future Work

- Study of finite size effects
- Investigation of topological charge and susceptibility
- Exploration of other channels with improved statistics
- Use of Schrödinger functional techniques

References

1: L. Del Debbio, A. Patella, C. Pica, *Higher representations on the lattice: Numerical simulations. SU(2) with adjoint fermions*, Phys. Rev. D81 (2010) 094503

2: L. Del Debbio, B. Lucini, A. Patella, C. Pica, A. Rago, *Conformal versus confining scenario in SU(2) with adjoint fermions*, Phys. Rev. D80 (2009) 074507