Pressure of massless hot scalar fields in the boundary effective theory framework

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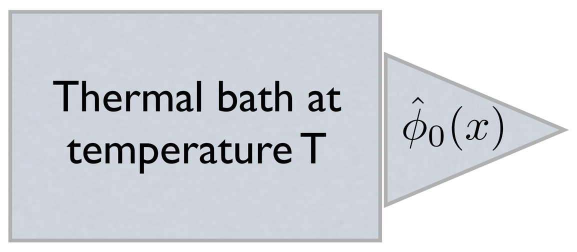
Features of BET

In a recent paper, we have proposed an alternative approach to thermal field theories, denoted by boundary effective theory (BET). The central idea of the method is to respect the double integral structure of the partition function in the functional integral formalism,

$$Z = \int [D\phi_0(\boldsymbol{x})]\, \rho[\beta;\phi_0,\phi_0]\ ,$$

where

$$\rho[\beta;\phi_0,\phi_0] = \int_{\phi(0,\boldsymbol{x})=\phi(\beta,\boldsymbol{x})=\phi_0(\boldsymbol{x})} [D\phi(\tau,\boldsymbol{x})] \quad e^{-S[\phi]}$$



$$\hat{\phi}\,|\phi_0(\boldsymbol{x})\rangle = \phi_0(\boldsymbol{x})|\phi_0(\boldsymbol{x})\rangle \qquad \boxed{\rho_\beta[\phi_0(\boldsymbol{x}),\phi_0(\boldsymbol{x})] = e^{-S_d[\phi_0(\boldsymbol{x})]}}$$

Any thermal observable can be constructed by integrating the appropriate functional of $\phi_0(\boldsymbol{x})$ over the fields $\phi_0(\boldsymbol{x})$ weighted by the corresponding diagonal element of the density matrix.

To each dynamical configuration $\phi(\tau,\boldsymbol{x})$ there corresponds a static configuration

$$\phi_0(\boldsymbol{x}) = \phi(0,\boldsymbol{x}) = \phi(\beta,\boldsymbol{x}).$$

We say that $\phi_0(\boldsymbol{x})$ is the (time) boundary value of $\phi(\tau,\boldsymbol{x})$.

Natural separation of infrared modes

The field $\phi_0(\boldsymbol{x})$ has still another remarkable property: it is the zero (static) component of the dynamical field $\phi(\tau,\boldsymbol{x})$:

$$\boxed{\phi(\tau,\boldsymbol{x}) = \phi_0(\boldsymbol{x}) + \eta(\tau,\boldsymbol{x})}$$

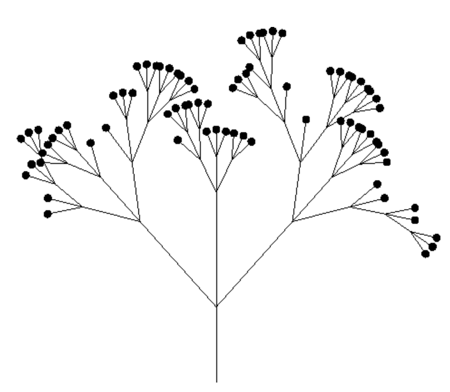
Therefore, the effective theory encoded in $\rho[\beta;\phi_0,\phi_0]$ contains all the infrared physics, and the double integral structure of Z naturally separates the potentially divergent modes.

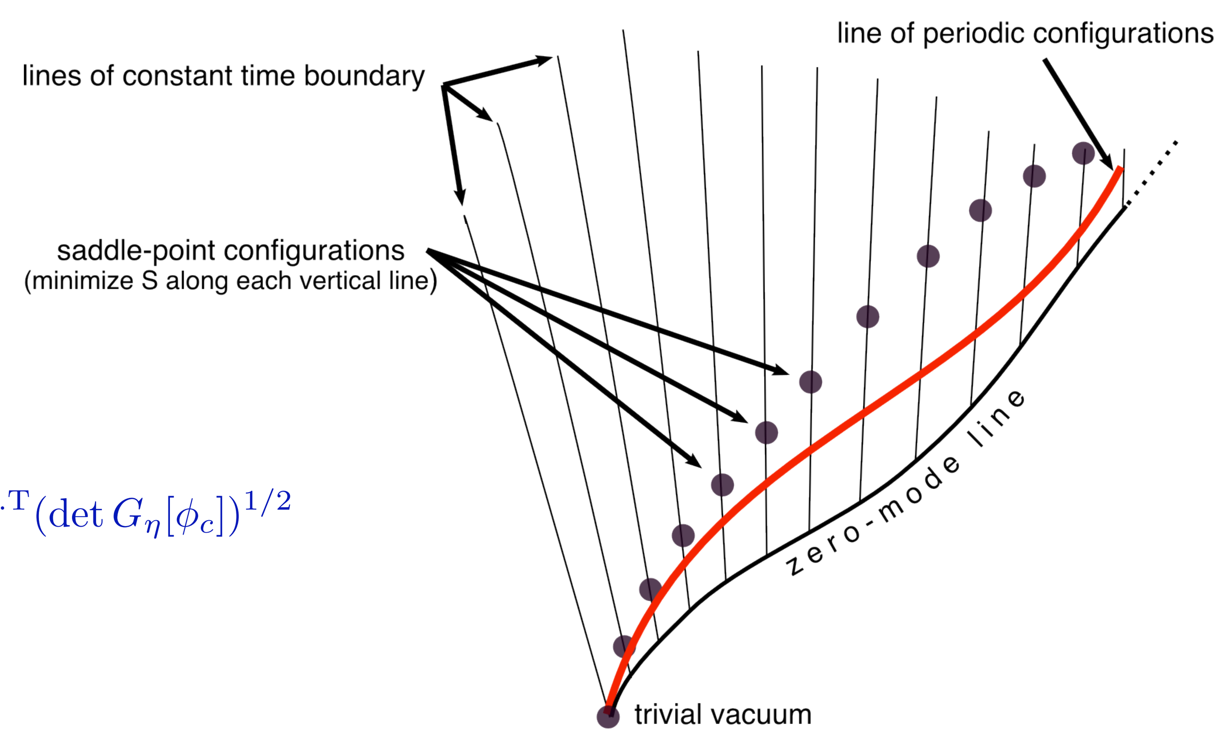
Saddle-point approximation

$$Z_R = \int [D\phi_0(\boldsymbol{x})] \int_{\eta(0,\boldsymbol{x})=\eta(\beta,\boldsymbol{x})=0} [D\eta(\tau,\boldsymbol{x})] \quad e^{-S[\phi_c+\eta]\,+\,\mathrm{C.T}}$$

In the vicinity of ϕ_c, the action is approximately quadratic,

$$S[\phi_c+\eta] \;=\; S[\phi_c] + \frac{1}{2}\int (d^4x)_E\; \eta(x)\left[\Box_E + m_0^2 + U''(\phi_c(x))\right]\eta(x) \;+\; \mathcal{O}(\eta^3)\ .$$





$$Z_R[\beta] \;\approx\; \int [\mathcal{D}\phi_0(\boldsymbol{x})]\; e^{-S[\phi_c]\,+\,\mathrm{C.T}} (\det G_\eta[\phi_c])^{1/2}$$

The 1-loop effective action

Fluctuations of the boundary field produce a new determinant. Gathering all the 1-loop contributions, we obtain:

$$\boxed{\beta\,\Gamma[\phi] = S[\phi_c] + \frac{1}{2}\,\mathrm{Tr}\log\left(\Delta_F^{-1} + U''(\phi_c)\right)}$$

where Δ_F is the usual thermal propagator and

$$\frac{\beta}{(2\pi)^3\delta(\boldsymbol{p}_1+\boldsymbol{p}_2)}\,\Gamma_R^{(2)}(\boldsymbol{p}_1,\boldsymbol{p}_2;\mu) = 2|\boldsymbol{p}_1|\,\tanh\frac{\beta|\boldsymbol{p}_1|}{2} + \beta\, m^2(|\boldsymbol{p}_1|;\beta)\ ,$$

$$\beta\, m^2(k;\beta) = \frac{\lambda}{24\beta}\,\frac{\tanh \beta k/2}{\beta k}\left(1 + \frac{\beta k}{\sinh\beta k}\right)\ .$$

The 1-loop effective potential

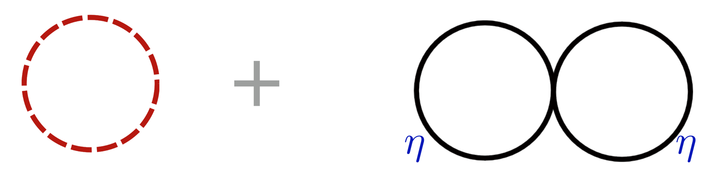
For a detailed discussion of the effective potential for scalar theories with single and double-well interactions, visit the poster by Daniel Kroff.

The pressure to lowest order in BET

Performing the quadratic integration over ϕ_0, one obtains the saddle-point approximation for Z,

$$P_{sp}(\beta) = \frac{\pi^2}{90\beta^4} - \frac{1}{2\beta}\lim_{\Lambda\to\infty}\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\,\log\left(1 + \beta m^2(k;\beta)\,\frac{\coth\beta k/2}{2k}\right)$$

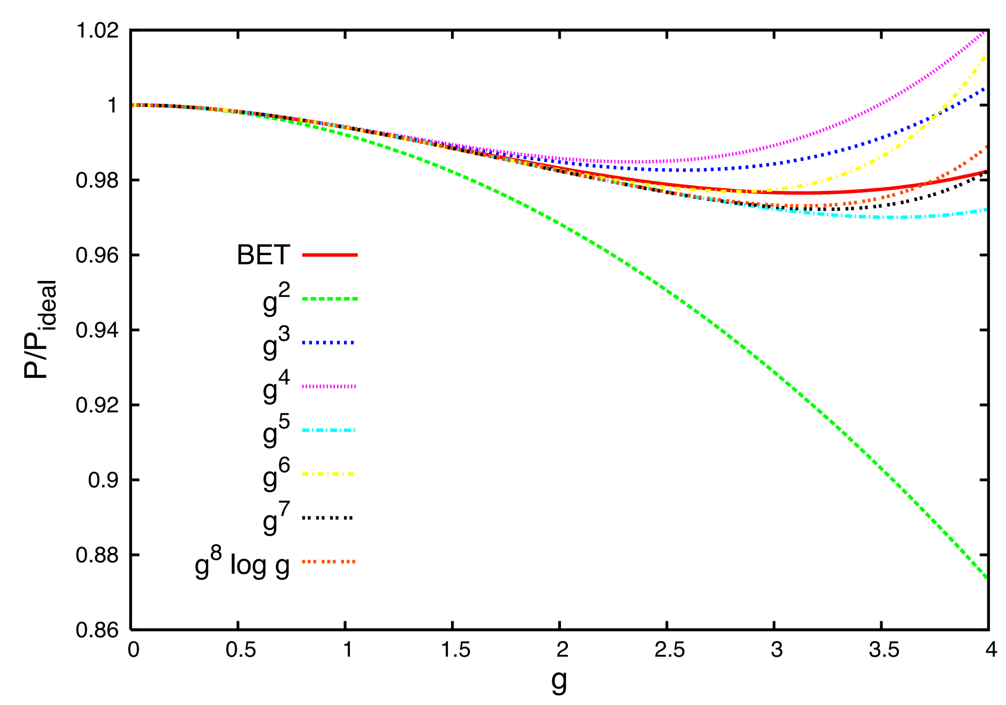
The second term on the r.h.s. of the previous equation can be identified with a series of daisy-diagrams, where the petal is given by $\beta\, m^2$. As one can easily check, the $\mathcal{O}(\lambda)$ term in P_{sp} is UV divergent. The 2-loop diagram carries the divergence. Such a spurious divergence is removed when we consistently add the remaining 2-loop corrections to the saddle-point approximation for the integration over η.



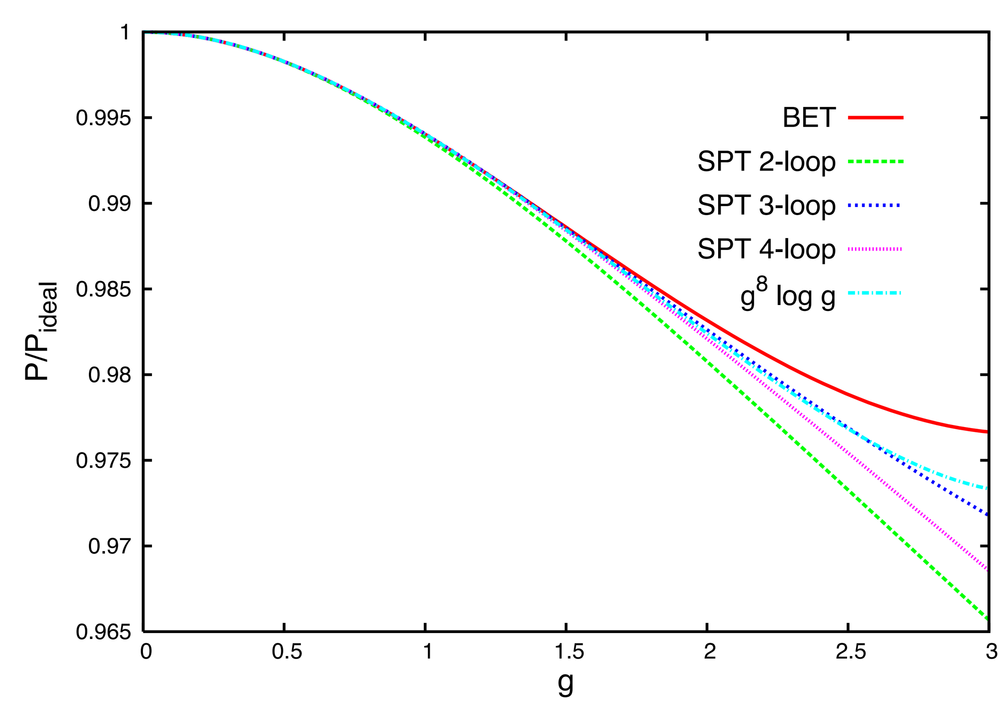
Finally, the renormalized pressure is given by

$$\boxed{P_{BET}(\beta) = \frac{\pi^2}{90\beta^4} - \frac{\lambda}{1152\beta^4} - \frac{1}{2\beta}\lim_{\Lambda\to\infty}\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\left\{\log\left(1 + \frac{\beta m^2(k;\beta)}{2}\,\frac{\coth\beta k/2}{k}\right) - \frac{\beta m^2(k;\beta)}{2}\,\frac{\coth\beta k/2}{k}\right\}\ .}$$

Results and comparison with weak-coupling and SPT



Pressure normalized by P_{ideal} as a function of the coupling constant $g = \sqrt{\lambda}$ in BET and weak-coupling formalisms (Ref. [6]). The renormalization scale is $\mu = 2\pi T$.



Comparison of the normalized pressure obtained using BET and SPT calculations at two, three, and four loops reported in Ref. [5]. The renormalization scale is $\mu = 2\pi T$.

We are working on the first correction to P_{BET} and the interesting application to the theory with Yukawa and $\lambda\phi^4$ interactions. The challenging extension of BET to gauge theories is under investigation.

Acknowledgements

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