Temperature dependence of CP violation in the Standard Model

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CP violation in the Standard Model

- Resides in Yukawa couplings; requires at least three fermion families.
- For three families exactly one complex phase in the CKM matrix V.
- All CP-violating effects proportional to the *Jarlskog invariant*,
$$J = \left|\mathrm{Im}(V_{ij}V_{k\ell}V^*_{i\ell}V^*_{kj})\right| \approx 3\times 10^{-5}.$$
- No CP violation in case of horizontal degeneracy of quark masses.

(Cold) electroweak baryogenesis

- Current baryon asymmetry normalized to the CMB photon density:
$$n_B/n_\gamma \approx 6\times 10^{-10}.$$
- *Sakharov conditions* for baryon asymmetry generation in early Universe:
  - Baryon number violation.
  - C and CP violation.
  - Departure from thermal equilibrium.
- Problems with the "standard" electroweak baryogenesis scenario:
  - Particle physics lower bound on the Higgs mass implies a crossover electroweak phase transition $\Rightarrow$ not far enough off equilibrium.
  - Perturbatively, CP-violating effects suppressed by the *Jarlskog determinant* $J\Delta/v^{12}\approx 10^{-24}$, where $v\approx 246$ GeV is the Higgs expectation value and
$$\Delta = (m_u^2-m_c^2)(m_c^2-m_t^2)(m_t^2-m_u^2)(m_d^2-m_s^2)(m_s^2-m_b^2)(m_b^2-m_d^2).$$
- Cold electroweak baryogenesis scenario:
  - Satisfies the off-equilibrium condition by means of low-scale inflation [1].
  - Electroweak transition triggered by the Higgs coupling to the inflaton at the end of the inflation period, well below the electroweak scale.
  - Thanks to low temperature, infrared enhancement invalidates the naive perturbative estimate and allows for sizable CP violation effects [2, 3].

Results available in literature

- General strategy: integrate out quarks and simulate the resulting effective theory for Standard Model bosons numerically on the lattice.
- Use *derivative expansion* to identify the leading CP-violating operators.
- Smit [2] showed that there is no CP violation up to the *fourth order*; no CP violation is thus induced by the P-odd anomalous term in the action.
- Two independent calculations of CP-violating operators at *sixth order*:
  - [4] use *worldline formalism* and find CP-odd, P-odd (C-even) contributions.
  - [5] use *method of symbols* and find only CP-odd, P-even contributions; first CP-odd, P-odd contribution only appears at the next, eighth order [6].
- The two available calculations give qualitatively different results.
- Moreover, all previous calculations were restricted to zero temperature.
- Goal of our project: resolve the discrepancy and extend the results to nonzero temperature.

Method of (covariant) symbols

- Calculate $\mathrm{Tr}\log$ of the Dirac operator in background gauge and Higgs fields.
- Perform an expansion in number of external gauge legs and derivatives.
- *Method of symbols*: convenient way to calculate traces of differential operators. For a (matrix) function $M(x)$ and a (covariant) derivative D_x,
$$\mathrm{Tr}\, f(M(x),D_x) = \int_{x,p} \mathrm{tr}\left[f(M(x),D_x+\mathrm{i}p)\mathbb{1}\right].$$
- Loses manifest covariance due to appearance of "free" covariant derivatives.
- *Method of covariant symbols* [5] makes the expansion manifestly covariant already on the level of the integrand,
$$\mathrm{Tr}\, f(M(x),D_x) = \sum_{k=0}^{\infty}\frac{(-\mathrm{i})^k}{k!}\int_{x,p}\mathrm{tr}\left[(D_0\partial_0^p)^k f(\overline{M}(x),\overline{D}_x)\mathbb{1}\right],$$
$$\overline{M} = M + \mathrm{i}[D_\alpha,M]\frac{\partial}{\partial p_\alpha} - \frac{1}{2}[D_\alpha,[D_\beta,M]]\frac{\partial^2}{\partial p_\alpha\partial p_\beta}+\cdots,$$
$$\overline{D}_\mu = \mathrm{i}p_\mu + \frac{\mathrm{i}}{2}[D_\alpha,D_\mu]\frac{\partial}{\partial p_\alpha} - \frac{1}{3}[D_\alpha,[D_\beta,D_\mu]]\frac{\partial^2}{\partial p_\alpha\partial p_\beta}+\cdots.$$
- Apart from a rescaling factor, Higgs field appears in the result as $\varphi_\mu = \frac{1}{\phi}\partial_\mu\phi$.
- Charged weak boson fields appear in the result in covariant derivatives,
$$W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu \pm g' B_\mu W^\pm_\nu.$$

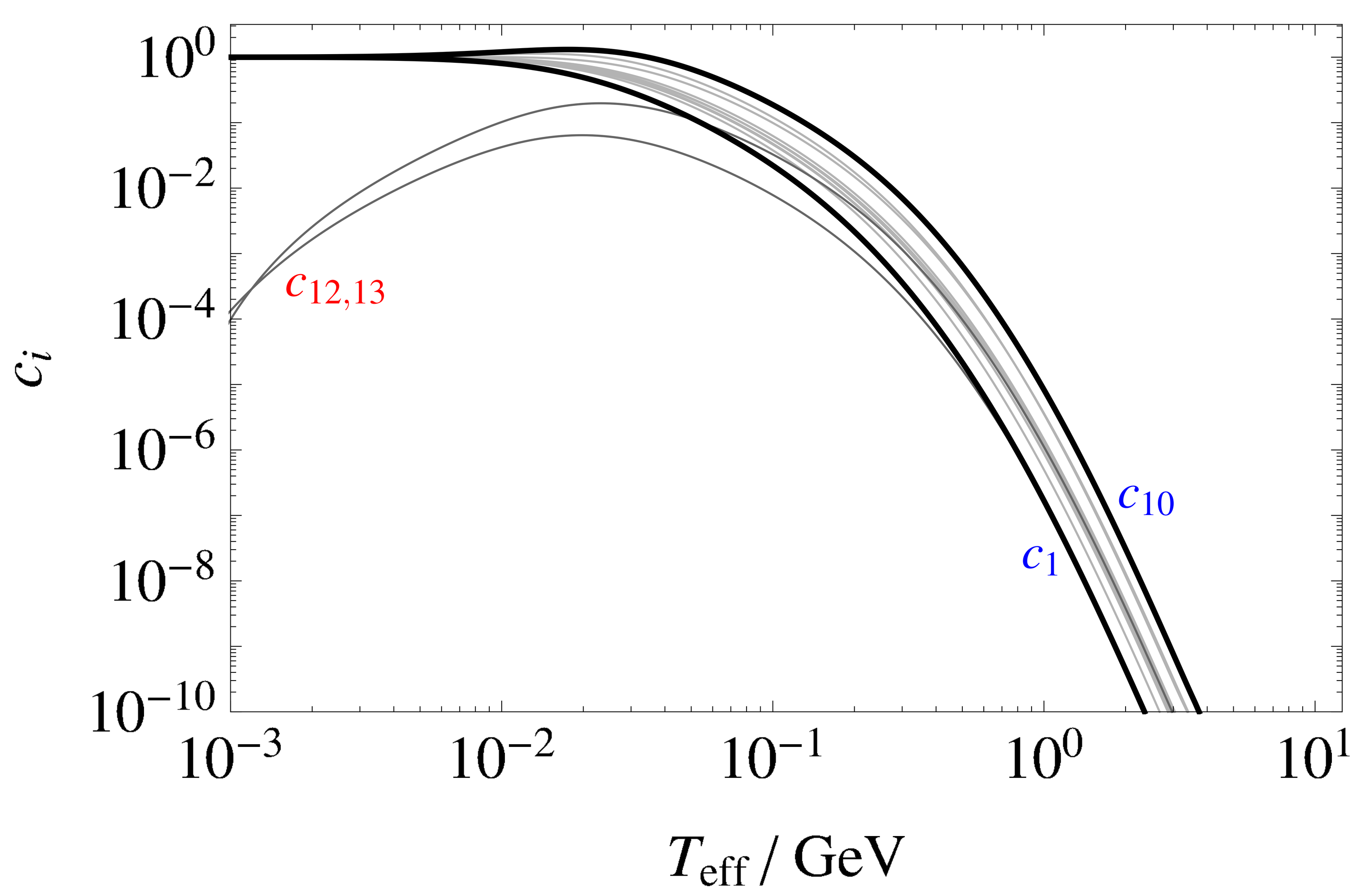
Effective action at nonzero temperature

- The Euclidean effective action for the Standard Model bosons acquires first CP-violating contributions at the sixth order of the derivative expansion.
- Our calculation [7] fully confirms the zero-temperature result of [5].
- Lorentz-invariant part of the result:

$$\Gamma_{\mathrm{eff}} = -\frac{\mathrm{i}}{2}N_c J G_{\mathrm{F}}\kappa_{\mathrm{CP}}\int \mathrm{d}^4x\left(\frac{v}{\phi}\right)^2(O_0+O_1+O_2),$$
$$\begin{aligned}
O_0 =& -\frac{c_1}{3}(W^+)^2W^-_{\mu\mu}W^-_{\nu\nu} + \frac{5c_2}{3}(W^+)^2W^-_{\mu\nu}W^-_{\mu\nu} - \frac{c_1}{3}(W^+)^2W^-_{\mu\nu}W^-_{\nu\mu} + \\
&+\frac{4c_3}{3}W^+_\mu W^+_\nu W^-_{\mu\alpha}W^-_{\alpha\nu} - \frac{2c_1}{3}W^+_\mu W^+_\nu W^-_{\mu\alpha}W^-_{\nu\alpha} - 2c_4 W^+_\mu W^+_\nu W^-_{\alpha\mu}W^-_{\alpha\nu} + \\
&+\frac{4c_3}{3}W^+_\mu W^+_\nu W^-_{\mu\nu}W^-_{\alpha\alpha} - \mathrm{c.c.},\\
O_1 =& \frac{8}{3}(Z_\mu+\varphi_\mu)\big[c_5(W^+)^2W^-_\mu W^-_{\nu\nu} - c_6(W^+)^2W^-_\nu W^-_{\mu\nu} - c_6(W^+)^2W^-_\nu W^-_{\nu\mu} - \\
&- c_3(W^+\cdot W^-)W^+_\mu W^-_{\nu\nu} + c_7(W^+\cdot W^-)W^+_\nu W^-_{\mu\nu} + c_7 W^+_\mu W^+_\nu W^-_\alpha W^-_{\alpha\nu} - \\
&- c_{12}(W^+\cdot W^-)W^+_\nu W^-_{\nu\mu} - c_{12}W^+_\mu W^+_\nu W^-_\alpha W^-_{\nu\alpha} + c_{13}W^-_\mu W^+_\nu W^+_\alpha W^-_{\nu\alpha}\big] - \mathrm{c.c.},\\
O_2 =& 4(Z_\mu Z_\nu + \varphi_\mu\varphi_\nu)\big[c_8(W^+)^2W^-_\mu W^-_\nu - c_8(W^-)^2W^+_\mu W^+_\nu\big] - \\
&-\frac{16}{3}(Z\cdot\varphi)\big[c_9(W^+\cdot W^-)^2 - 2c_6(W^+)^2(W^-)^2\big] + \\
&+\frac{4}{3}(Z_\mu\varphi_\nu + Z_\nu\varphi_\mu)\big[c_{10}(W^+)^2W^-_\mu W^-_\nu + c_{10}(W^-)^2W^+_\mu W^+_\nu - \\
&-2c_{11}(W^+\cdot W^-)(W^+_\mu W^-_\nu + W^+_\nu W^-_\mu)\big],
\end{aligned}$$
$$\kappa_{\mathrm{CP}} = \frac{\Delta}{G_{\mathrm{F}}}\int\frac{\mathrm{d}^4p}{(2\pi)^4}(p^2)^3\prod_{f=1}^{6}\frac{1}{(p^2+m_f^2)^2} \approx 3\times 10^2.$$

Temperature dependence of the couplings

- The couplings only depend on $T_{\mathrm{eff}} \equiv Tv/\phi$.
- Contributions from regions with $\phi \ll v$ are thus suppressed.



Conclusions and outlook

- Using the previous zero-temperature result leads to baryon asymmetry *four orders of magnitude* larger than the observed value [3].
- The steep dependence of the couplings on temperature constrains the applicability of the cold electroweak baryogenesis scenario to $T \lesssim 1$ GeV.
- Within the cold electroweak baryogenesis scenario, Standard Model still seems capable to generate sufficient baryon asymmetry in the early Universe!
- Follow-up work is currently under way.

References

[1] J. García-Bellido *et al.*, *Phys. Rev.* **D60** (1999) 123504.
[2] J. Smit, *JHEP* **09** (2004) 067.
[3] A. Tranberg *et al.*, *Phys. Lett.* **B690** (2010) 207.
[4] A. Hernandez *et al.*, *Nucl. Phys.* **B812** (2009) 290.
[5] C. García-Recio and L. L. Salcedo, *JHEP* **07** (2009) 015.
[6] L. L. Salcedo, *Phys. Lett.* **B700** (2011) 331.
[7] T. Brauner *et al.*, *Phys. Rev. Lett.* **108** (2012) 041601.