LATTICE SPECTROSCOPY OF SU(2) WITH TWO ADJOINT FERMIONS

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Abstract

We present a numerical study of spectroscopic observables in the SU(2) gauge theory with two adjoint fermions. We compare our results using improved source and sink operators with previous determinations of masses that used point sources and sinks and investigate possible systematic effects in both cases. We discuss the finite volume effects on the spectrum, investigated by varying the size of the lattice and by changing the boundary conditions.

Introduction

Minimal walking technicolor (MWT), with gauge group $SU(2)$ and two flavours of adjoint Dirac fermions, is a candidate theory of electroweak symmetry breaking. The evidence accumulated so far for this theory favours a conformal or near-conformal scenario. However, more systematic studies need to be performed before the IR properties of the theory can be determined with confidence.

Lattice studies can identify conformal or near-conformal behaviour by studying the mass-dependence of the spectrum. The standard way to extract masses from lattice simulations is to look at the exponential decay of correlators of operators with the quantum numbers of interest. At finite time extent, there will be corrections due to excited states. The finite spatial extension of the lattice can also give sizeable corrections to the spectral masses. Below we systematically explore the effects of these corrections.

The simplest source and sink observables to study for mesons are fermion bilinears in which the two fermion fields are at the same lattice point (*point sources*). Experience from lattice QCD favours the use of *extended sources*, which prove to be affected by smaller systematic errors. Here we investigate whether this is also true for MWT.

Our computations were performed using the *HiRep* code and the *Chroma* suite of lattice software [1]. We used the Wilson gauge action, and the Wilson fermion formulation with the RHMC algorithm. The coupling was $\beta = 2.25$. Further details of some of these results have been published in [2].

Smearing effects

We systematically compared local, gaussian and wall-smeared sources on our ensembles. At our lightest masses, the wall-smeared sources have the largest overlap with the ground state, which is reflected in the flattest effective masses. In Fig. 1 we show the PS effective masses computed with the three methods.

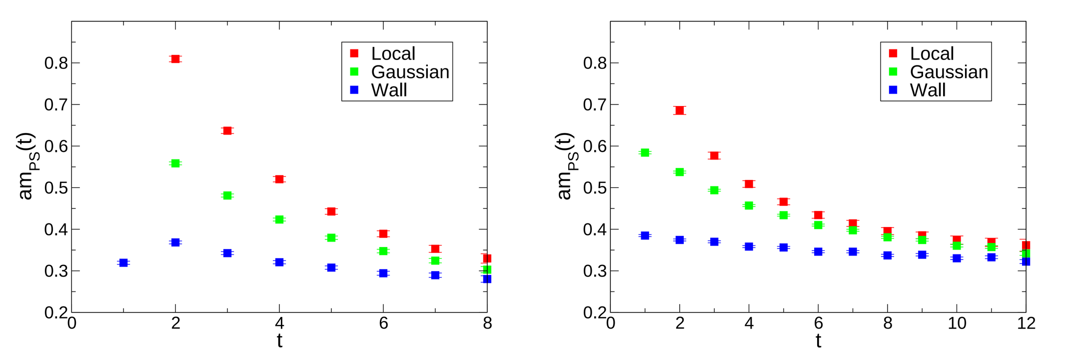

FIGURE 1: Comparison of the pseudoscalar mass from different smearings at $am_0 = -1.175$, on a 16×8^3 lattice (left) and a 24×12^3 lattice (right).

Smeared sources are more sensitive to the algorithm's autocorrelation time. We study this by grouping the N data into N/b blocks of length b and averaging over each block. A bootstrap analysis is then performed on the reduced dataset. When the block size is bigger than the autocorrelation we expect to see a plateau appearing in the standard deviation. We observe that the plateau starts at a block size corresponding to an integrated autocorrelation time of order 1. Our analysis of the autocorrelation is illustrated in Fig. 2 for the PS effective mass.

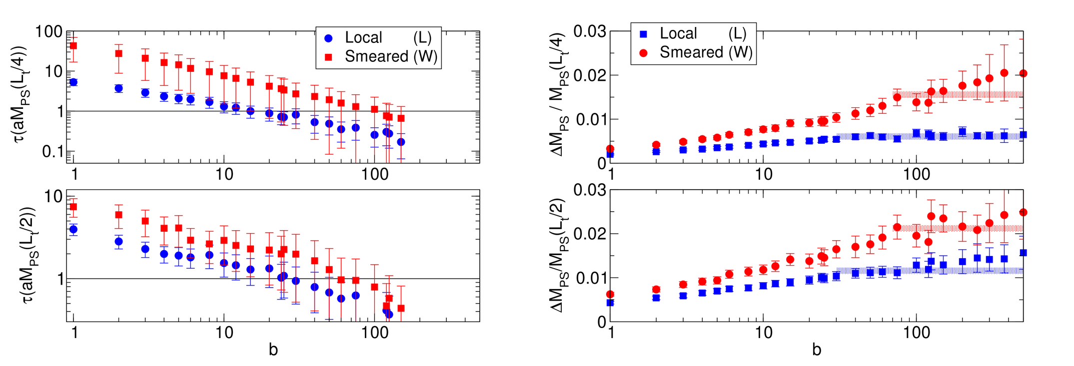

FIGURE 2: Autocorrelation analysis on a 24×12^3 lattice at $am_0 = -1.175$, for the PS effective mass in two temporal points. Left, integrated autocorrelation time; right, relative error. Plateaux of the relative error highlighted with faint rectangles.

We see that the measured autocorrelation times for the smeared results are larger than for the local results, and that the standard deviation increases up to a point where it appears to reach a plateau. The value of b where this plateau sets in is interpreted as the length in simulation time over which the data are uncorrelated. This picture is replicated across our ensembles. We have accounted for this by conducting our bootstrap analyses over appropriately reduced datasets. To quantify the flatness of the effective mass we define the ratio:

$$\frac{\Delta m_{PS}}{\Delta t} \equiv \left| \frac{m_{PS}(L_t/2 - \Delta t) - m_{PS}(L_t/2)}{\Delta t} \right| . \tag{1}$$

A value for $\Delta m_{PS}/\Delta t$ compatible with zero implies that the plateau in the effective mass is long at least Δt points. In Fig. 3, the quantity $\Delta m_{PS}/\Delta t$ is plotted for pseudoscalar effective masses.

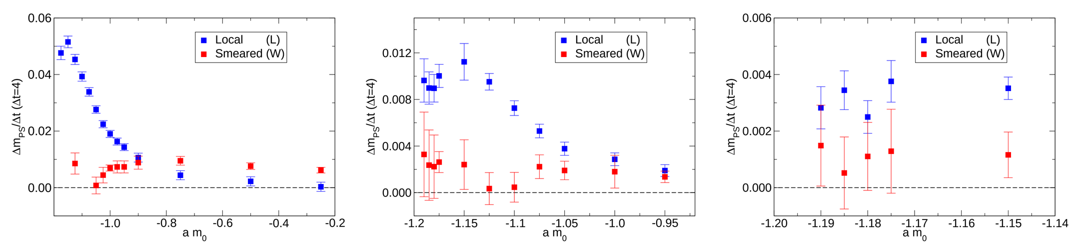

FIGURE 3: Incremental ratio $\Delta m_{PS}/\Delta t$ as a function of the bare mass.

One expects that at small masses the wave function of the pseudoscalar meson is more spread, hence the wall-smeared source should have a larger overlap with the ground state. whereas at large masses the local sources should work better. We see that this is indeed the case. The same analyses using the effective V meson mass and the effective PS decay constant produce very similar results.

Finite Volume Effects

We have calculated the PS and V masses, their ratio and PS decay constant on the 16×8^3 and 24×12^3 lattices for $am_0 = -1.05$, both from local and wall-smeared sources. We find that the volume dependence appears to strengthen with wall-smeared sources. To clarify this, we look at the effective PS mass in Fig. 4. By comparing the effective masses on the 24×12^3 and 64×8^3 lattices it is clear that the finite volume makes the pseudoscalar meson lighter. On the 16×8^3 lattice the mass estimated with the local sources is affected by two relatively large effects: the finite volume, which decreases the mass and the bad determination of the plateaux, which increases the mass. These two effects almost cancel each other. Therefore the finite volume effects are actually larger than estimated using just the local sources, and they are better estimated using wall-smeared sources at light enough masses. These conclusions are valid also for the vector meson mass and for the ratio M_V/M_{PS}.

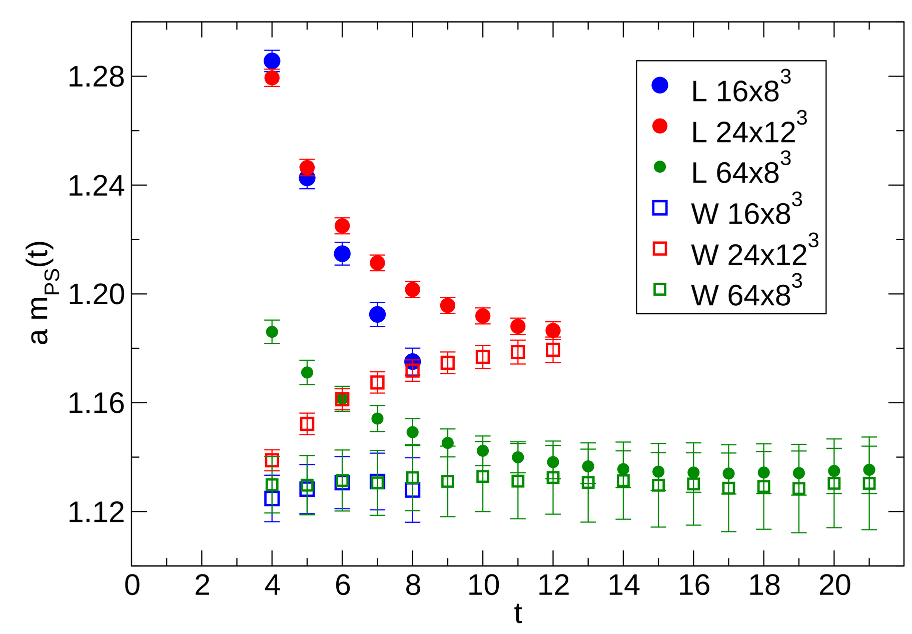

FIGURE 4: Effective PS mass on different volumes for $am_0 = -1.05$ (L=local, W=wall).

Having understood the effects of smearing and inadequate plateaux, we have extended our calculations to lighter masses and larger volumes. We show the effective PS mass for $am_0 = -1.15$ in Fig. 6. We see that for $L_s \geq 24$ there is good agreement. Furthermore, for $L_t = 80$ there is a very long plateau which agrees with the plateaux for $L_t = 64$.

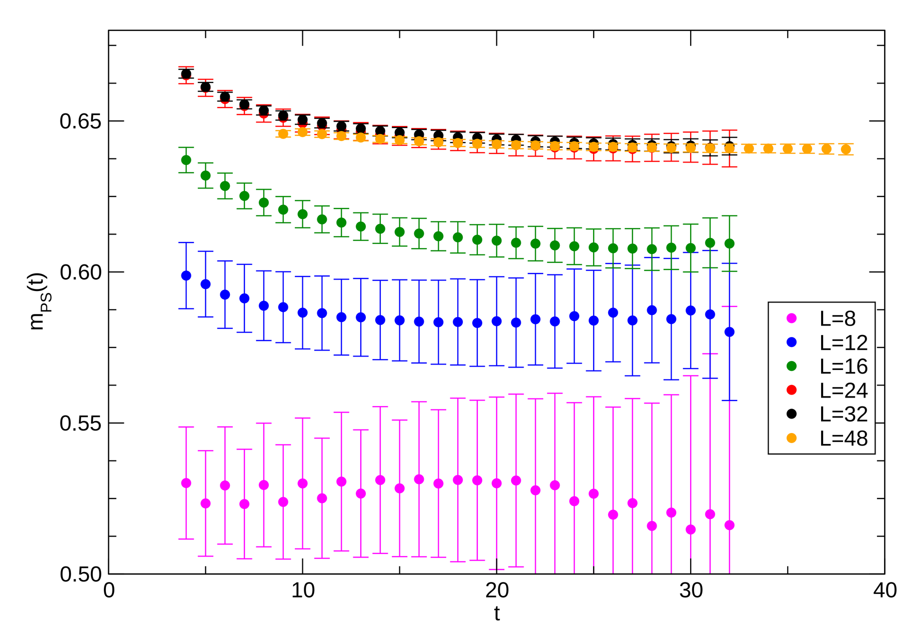

FIGURE 5: Effective PS mass on different volumes for $am_0 = -1.15$.

Another way to investigate finite volume effects is to vary the boundary conditions. For sufficiently large lattice we should see no difference between observables calculated with periodic and twisted boundary conditions. We illustrate this for the PS mass in Fig. 6, where we see that the results for different boundary conditions appear to be heading towards convergence for $L_s \geq 24$, in agreement with our results for the effective mass in Fig. 5 above.

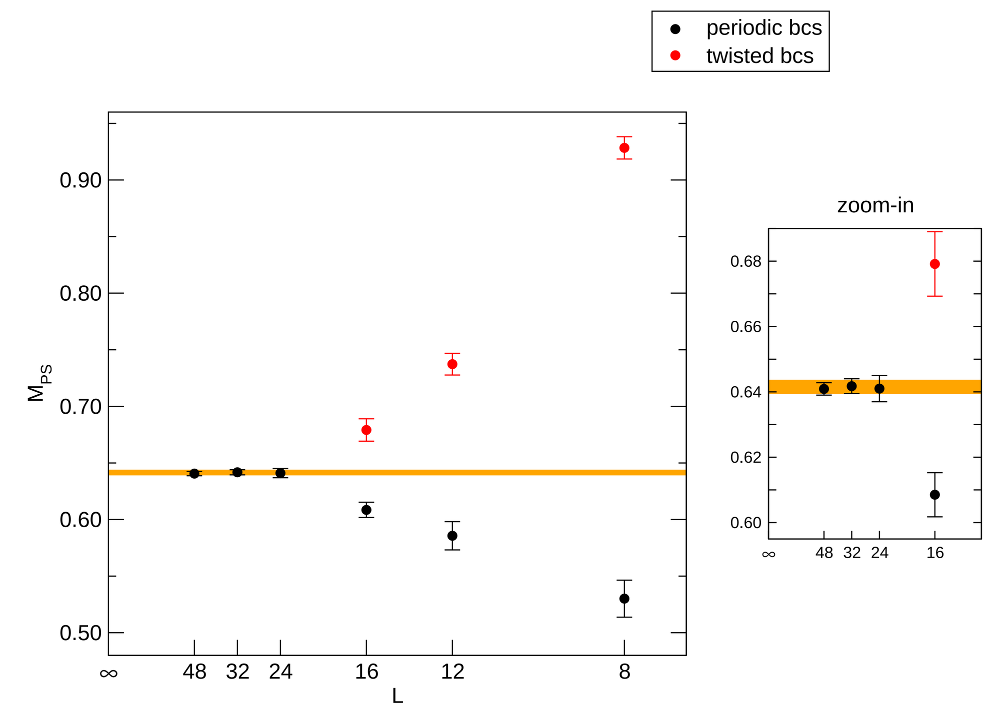

FIGURE 6: PS mass for periodic and twisted boundary conditions for various lattice sizes at $am_0 = -1.15$.

We can use our analysis of the finite-volume effects to estimate the size of the remaining errors. As an example we plot M_V/M_{PS} in Fig. 7. We see we have enough points with sufficiently small errors to confirm the existence of a plateau, despite larger errors on some points.

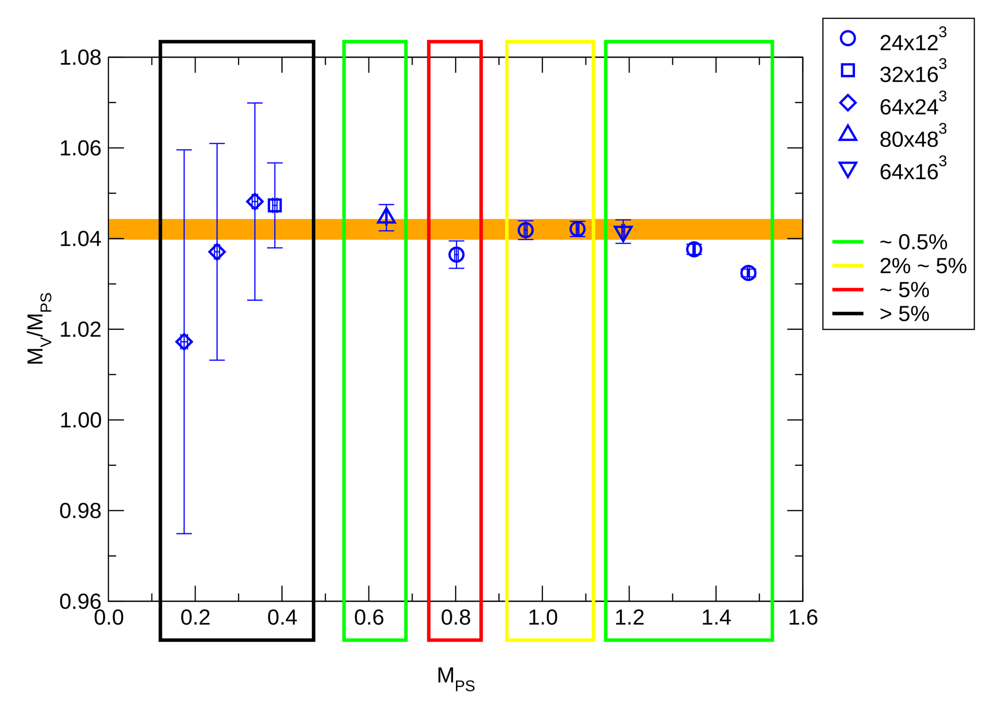

FIGURE 7: Ratio of M_V to M_{PS}, with an estimate of the remaining finite-volume errors.

Finally, we have investigated finite volume effects in the gluonic sector. These appear to be larger than for mesons. We plot the string tension and glueball masses in Fig. 8. Despite the larger pseudoscalar mass, it appears we need L_s up to 32 to control the 2^{++} glueball mass.

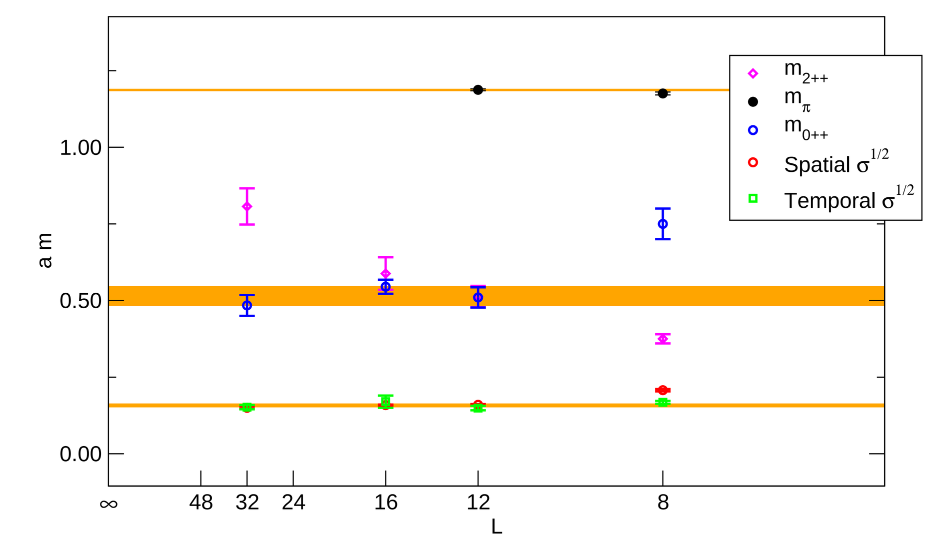

FIGURE 8: Finite volume effects for gluonic observables (string tension and glueball masses) and m_{PS} at $am_0 = -1.05$.

Summary

We have investigated the effects of both smearing and finite volumes on determinations of the spectrum of minimal walking technicolor. We have found wall-smearing leads to better plateaux, at the cost of larger autocorrelation times. Finite volume effects can be underestimated if only local sources are used. In general, larger volumes (compared to experience from QCD) are needed to control finite volume effects, especially for gluonic observables. Nevertheless, our conclusions regarding the near-conformal dynamics of this theory are robust.

References

[1] **SciDAC Collaboration, LHPC Collaboration, UKQCD Collaboration** Collaboration, R. G. Edwards and B. Joo, *Nucl.Phys.Proc.Suppl.* **140** (2005) 832, [hep-lat/0409003].

[2] F. Bursa, L. Del Debbio, D. Henty, E. Kerrane, B. Lucini, A. Patella, C. Pica and T. Pickup *et al.*, Phys. Rev. D **84** (2011) 034506 [arXiv:1104.4301 [hep-lat]].