$\mathcal{N} = 4$ super Yang-Mills plasma

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1 Introduction

The AdS/CFT duality of the five-dimensional gravity in the anti de Sitter geometry and the conformal field theories offers a unique tool to study strongly coupled theory such as $\mathcal{N} = 4$ super Yang-Mills. Some intriguing features of strongly coupled systems have been revealed but relevance of the results for non-supersymmetric systems, which are of our actual interest, remains an open issue. In particular, one asks how properties of the $\mathcal{N} = 4$ super Yang-Mills plasma (SYMP) are related to those of usual quark-gluon plasma (QGP) studied experimentally in relativistic heavy-ion collisions. While such a comparison is, in general, a difficult problem, a systematic comparative analysis can be performed in the domain of weak coupling where perturbative methods are applicable. Our rather detailed comparison is presented in [1] and here we show our main results.

2 Collective excitations

Dispersion equations

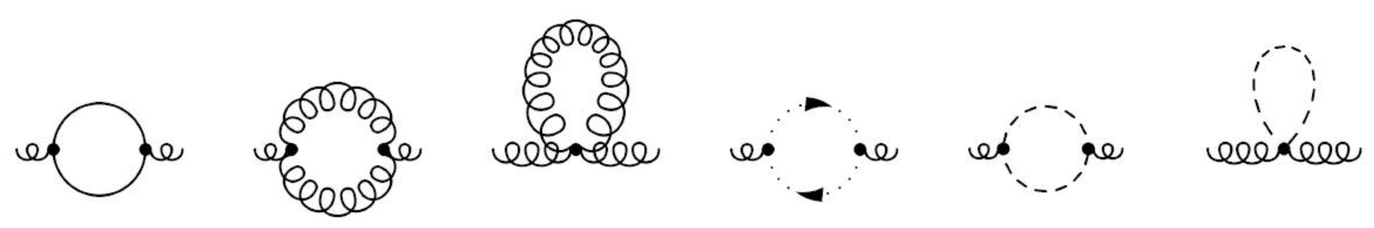
The collective excitations of $\mathcal{N} = 4$ super Yang-Mills plasma are determined by the dispersion equations which for gluons, fermions and scalars read

$$\det\Big[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)\Big] = 0, \qquad \det\Big[\not k - \Sigma(k)\Big] = 0, \qquad k^2 + P(k) = 0,$$

where $\Pi^{\mu\nu}(k)$, $\Sigma(k)$, and $P(k)$ are the retarded self-energies of gluons, fermions and scalars. The self-energies were computed perturbatively at one-loop level via Keldysh-Schwinger formalism [2]. Since collective modes were looked for, the Hard Loop Approximation was applied.

Gluon collective excitations

The contributions to the polarization tensor are given by the diagrams



which in Hard Loop Approximation give

$$\Pi^{\mu\nu}_{ab}(k) = g^2 N_c \delta_{ab} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{k^2 p^\mu p^\nu - (k^\mu p^\nu + p^\mu k^\nu - g^{\mu\nu}(k\cdot p))(k\cdot p)}{(k\cdot p + i0^+)^2} \tag{1}$$

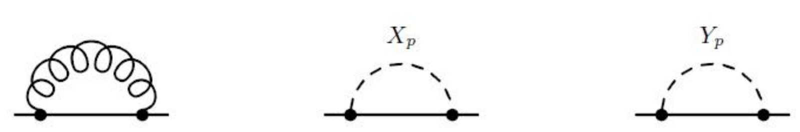
where

$$f(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + 8n_f(\mathbf{p}) + 6n_s(\mathbf{p}) \tag{2}$$

is the effective distribution function of plasma constituents. The polarization tensor (1) has exactly the same form as in the quark-gluon plasma. So are the gluon collective excitations.

Fermion collective excitations

The contributions to the fermion self-energy are given by the diagrams



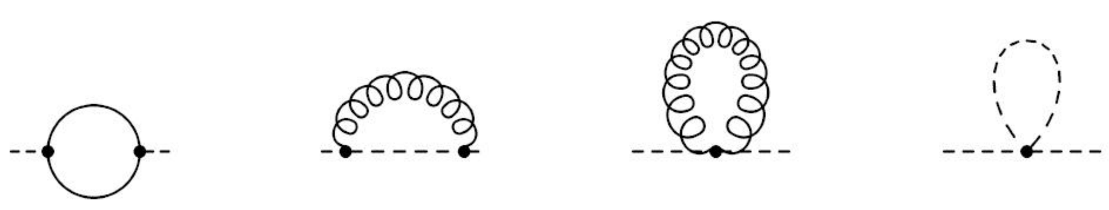
which give the fermion self-energy as

$$\Sigma^{ij}_{ab}(k) = \frac{g^2}{2} N_c \delta_{ab} \delta^{ij} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{\not p}{k\cdot p + i0^+} \tag{3}$$

which, as the polarization tensor (1), depends on the effective distribution function (2). The fermion self-energy (3) has the same structure as in the usual QCD plasma of quarks and gluons. So are the fermion collective excitations.

Scalar collective excitations

Finally, the diagrams



contribute to the scalar self-energy which equals

$$P^{AB}_{ab}(k) = -2g^2 N_c \delta_{ab} \delta^{AB} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p}. \tag{4}$$

It depends, as Π and Σ, only on the effective distribution function (2). The scalar dispersion relation is like that of massive relativistic particle *i.e.* $E_p = \pm\sqrt{m^2_{\rm eff} + \mathbf{p}^2}$.

Effective action

Since the self-energy of a given field is the second functional derivative of the action with respect to the field, we can find the Hard Loop effective actions of $\mathcal{N} = 4$ super Yang-Mills as

$$\mathcal{L}_2^{(A^a_\mu)}(x) = \frac{1}{2}\int d^4y \; A^a_\mu(x)\Pi^{\mu\nu}_{ab}(x-y)A^b_\nu(y), \tag{5}$$

$$\mathcal{L}_2^{(\Psi^a_i)}(x) = \int d^4y \; \bar\Psi^a_i(x)\Sigma^{ij}_{ab}(x-y)\Psi^b_j(y), \tag{6}$$

$$\mathcal{L}_2^{(\Phi^a_A)}(x) = \int d^4y \; \Phi^a_A(x)P^{AB}_{ab}(x-y)\Phi^b_B(y). \tag{7}$$

The form of the above effective actions appear to be unique. Consequently, the properties of the plasma, which are controlled by the Hard Loop dynamics, are unique as well.

3 Collisional processes

Since temperature (T) is the only dimensional parameter which characterizes the equilibrium plasma of massless constituents, the transport coefficients are all expressed through powers of T and only numerical coefficients can differ for different systems. In the paper [3] we considered the collisional energy loss and momentum broadening of a particle traversing the equilibrium plasma. The dimensional argument does not work here because the two quantities depend not only on the plasma temperature but on the energy of test particle as well.

We computed the energy loss and momentum broadening due to the processes, like the Compton scattering on selectrons, the matrix element of which is independent of momentum transfer. In the limit of high energy of test particle, the two quantities were found as

$$\frac{dE}{dx} = -\frac{e^4}{2^5 3\pi}T^2, \qquad \hat q = \frac{e^4\zeta(3)}{12\pi^3}T^3. \tag{8}$$

In QED and QCD plasma the energy loss and momentum broadening appear to be strongly dominated by the Coulomb-like interactions *i.e.* the contributions from the matrix elements squared of elementary processes, which grow as t^{-2} for $t \to 0$. Nevertheless the two quantities (up to the logarithmic terms) are similar to those corresponding to the momentum independent matrix elements. The table explains why the similarity occurs.

		Contact $\lvert\mathcal{M}\rvert^2 \sim e^4$	**Coulomb** $\lvert\mathcal{M}\rvert^2 \sim e^4\frac{s^2}{t^2}$
energy change in single collision	ΔE	$\sim E$	$\sim e^2 T$
cross section	σ	$\sim \frac{e^4}{ET}$	$\sim \frac{e^2}{T^2}$
density	ρ	$\sim T^3$	$\sim T^3$
inverse mean free path	$\lambda^{-1} = \sigma\rho$	$\sim \frac{e^4T^2}{E}$	$\sim e^2 T$
energy loss	$\frac{dE}{dx} \sim \frac{\Delta E}{\lambda}$	$\sim e^4T^2$	$\sim e^4T^2$

The estimate shows that the two interactions corresponding to very different differential cross sections lead to very similar energy losses.

4 Conclusion

The dynamics of QCD is obviously rather different than that of $\mathcal{N} = 4$ super Yang-Mills theory. Nevertheless the two plasmas in the weak coupling regime are surprisingly similar at the leading order. The form of gluon collective excitations is identical and the same is true for the fermion (quark) modes. The scalar modes in SYMP are as of massive relativistic particle. The energy loss and momentum broadening of a highly energetic test particle are rather similar in the two plasma systems. The differences mostly come from different numbers of degrees of freedom in both plasmas.

[1] A. Czajka and St. Mrówczyński, Phys. Rev. D, in print
[2] A. Czajka and St. Mrówczyński, Phys. Rev. D **83**, 045001 (2011).
[3] A. Czajka and St. Mrówczyński, Phys. Rev. D **84**, 105020 (2011)

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