Energy loss of a fast parton in an unstable QGP

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Motivation

When a highly energetic parton travels through the quark-gluon plasma (QGP), it losses its energy due to interactions with plasma constituents. This leads to the jet quenching observed in relativistic heavy-ion collisions. The problem of energy loss is rather well understood in the case of equilibrium plasma. However, the quark-gluon plasma produced in high-energy collisions reaches the state of local equilibrium only after a short but finite time interval. During this period the momentum distribution of plasma partons is anisotropic and such a plasma appears to be unstable, see the review [1]. We show here how to compute the energy loss of a parton in unstable plasma as an initial value problem. Our approach and preliminally result are described in [2].

General formula

We calculate the energy loss by treating the parton as an energetic classical particle with SU(N_C) color charge. Its motion across the quark-gluon plasma is described by the Wong equations

$$\frac{dx^\mu(\tau)}{d\tau} = u^\mu(\tau),$$
$$\frac{dp^\mu(\tau)}{d\tau} = gQ^a(\tau)F_a^{\mu\nu}(x(\tau))u_\nu(\tau),$$
$$\frac{dQ_a(\tau)}{d\tau} = -gf^{abc}u_\mu(\tau)A_b^\mu(x(\tau))Q_c(\tau),$$

where τ, $x^\mu(\tau)$, $u_\mu(\tau)$ and $p^\mu(\tau)$ are, respectively, the parton's proper time, its trajectory, four-velocity, and four-momentum; $F_a^{\mu\nu}$ and A_b^μ denote the chromodynamic field strength tensor and four potential, Q^a is the classical color charge of the parton; g is the coupling constant. We choose a gauge where the potential vanishes along the parton's trajectory:

$$u_\mu(\tau)A_a^\mu(x(\tau)) = 0.$$

The classical formula of the energy loss per unit time reads:

$$\frac{dE(t)}{dt} = \int d^3r\ \mathbf{E}_a(t,\mathbf{r})\mathbf{j}_a(t,\mathbf{r}).$$

We use the one-side Fourier transformation defined as:

$$f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} f(t,\mathbf{r}).$$

The linearized (Hard Loop) Yang-Mills equations take the form:

$$\nabla\cdot\mathbf{D}(t,\mathbf{r}) = \rho(t,\mathbf{r}), \qquad \nabla\cdot\mathbf{B}(t,\mathbf{r}) = 0,$$
$$\nabla\times\mathbf{E}(t,\mathbf{r}) = -\frac{\partial\mathbf{B}(t,\mathbf{r})}{\partial t}, \qquad \nabla\times\mathbf{B}(t,\mathbf{r}) = \mathbf{j}(t,\mathbf{r}) + \frac{\partial\mathbf{D}(t,\mathbf{r})}{\partial t}.$$

Applying the one-side Fourier transformation and using the Yang-Mills equations, we get the energy loss per unit time as

$$\frac{dE(t)}{dt} = gQ^a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi i}\int\frac{d^3k}{(2\pi)^3}e^{-i(\omega-\bar\omega)t}\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) \times\left[\frac{i\omega gQ^a v^j}{\omega-\bar\omega} + \varepsilon^{jkl}k^k B_{0a}^l(\mathbf{k}) - \omega D_{0a}^j(\mathbf{k})\right],$$

where $\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$ with $\varepsilon^{ij}(\omega,\mathbf{k})$ being the dielectric tensor. $\det[\Sigma(\omega,\mathbf{k})]=0$ is the dispersion equation which gives collective modes in the plasma. The energy-loss formula needs to be treated differently for stable and for unstable plasma.

Stable system

When the plasma is stable, all modes are damped, that is the poles of $\Sigma^{-1}(\omega,\mathbf{k})$ are located in the lower half-plane of the complex ω. Consequently, the contribution to the energy loss which corresponds to the poles of $\Sigma^{-1}(\omega,\mathbf{k})$, exponentially decays in time. The stationary contribution is given by the pole $\omega=\bar\omega=\mathbf{kv}$ which comes from the current $\mathbf{j}_a(\omega,\mathbf{k})$. We can also neglect the terms which include the initial values of the field. When we assume that the plasma is isotropic, the dielectric tensor can be expressed through its longitudinal and transverse components and in such a case the energy loss of a fast parton traversing the stable plasma is given as:

$$\frac{dE(t)}{dt} = -ig^2C_R\int\frac{d^3k}{(2\pi)^3}\frac{\bar\omega}{\mathbf{k}^2}\left[\frac{1}{\varepsilon_L(\bar\omega,\mathbf{k})} + \frac{\mathbf{k}^2v^2-\bar\omega^2}{\bar\omega^2\varepsilon_T(\bar\omega,\mathbf{k})-\mathbf{k}^2}\right],$$

which agrees with the textbook result [3].

Unstable system

When the plasma is unstable, there are modes which exponentially grown in time. The poles of $\Sigma^{-1}(\omega,\mathbf{k})$ corresponding to the instabilities are located in the upper half-plane of complex ω. Expressing the initial values $\mathbf{D}_0$ and $\mathbf{B}_0$ through the current $\mathbf{j}$, we obtain

$$\frac{dE(t)}{dt} = ig^2C_Rv^iv^l\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi i}\int\frac{d^3k}{(2\pi)^3}e^{-i(\omega-\bar\omega)t}\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) \times\left[\frac{\omega\delta^{jl}}{\omega-\bar\omega} - \left(k^jk^k-\mathbf{k}^2\delta^{jk}\right)\left(\Sigma^{-1}\right)^{kl}(\bar\omega,\mathbf{k}) + \omega\bar\omega\varepsilon^{jk}(\bar\omega,\mathbf{k})\left(\Sigma^{-1}\right)^{kl}(\bar\omega,\mathbf{k})\right].$$

Two-stream system

First we consider the two-stream system and we assume that the chromodynamic field is dominated by the longitudinal chromoelectric field, that is $\mathbf{B}(\omega,\mathbf{k})=0$ and $\mathbf{E}(\omega,\mathbf{k})=\mathbf{k}(\mathbf{k}\cdot\mathbf{E}(\omega,\mathbf{k}))/\mathbf{k}^2$. This gives

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \varepsilon_L(\omega,\mathbf{k})\frac{k^ik^j}{\mathbf{k}^2}, \qquad \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{k^ik^j}{\mathbf{k}^2}, \qquad \varepsilon_L(\omega,\mathbf{k}) = 1 + \frac{g^2}{2\mathbf{k}^2}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0^+}\mathbf{k}\cdot\frac{\partial\mathbf{f}(\mathbf{p})}{\partial\mathbf{p}}.$$

The distribution function of the two stream system is chosen to be:

$$f(\mathbf{p}) = (2\pi)^3 n\left[\delta^{(3)}(\mathbf{p}-\mathbf{q}) + \delta^{(3)}(\mathbf{p}+\mathbf{q})\right].$$

The energy loss per unit lenght for the two-stream system simplifies to:

$$\frac{dE(t)}{dt} = g^2C_R\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\frac{e^{-i(\omega+\omega')t}}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{\bar\omega}{\mathbf{k}^2}\left[\frac{\omega}{\omega-\bar\omega} + \frac{\bar\omega}{\omega}\right].$$

This equation gives a non-zero energy loss in the vacuum limit when $\varepsilon_L\to 1$ which should be subtracted.

Prolate system

When the momentum distribution is elongated along the beam, the system is called PROLATE. The distribution function of extremely prolate system can be written as: $f(\mathbf{p}) = \delta\left(\mathbf{p}^2-(\mathbf{p}\cdot\mathbf{n})^2\right)$, where $\mathbf{n}$ is the unit vector along which the distribution is elongated.

Since the matrix Σ is symetric and it depends only on two vectors $\mathbf{n}$ and $\mathbf{k}$, it can be decomposed as: $\Sigma = aA + bB + cC + dD$, where the tensors are defined as:

$$A^{ij}(\mathbf{k}) = \delta^{ij} - \frac{k^ik^j}{\mathbf{k}^2}, \quad B^{ij}(\mathbf{k}) = \frac{k^ik^j}{\mathbf{k}^2}, \quad C^{ij}(\mathbf{k},\mathbf{n}) = \frac{n_T^in_T^j}{\mathbf{n}_T^2}, \quad D^{ij}(\mathbf{k},\mathbf{n}) = k^in_T^j + k^jn_T^i,$$

with the following vector $\mathbf{n}_T$

$$n_T^i = \left(\delta^{ij} - \frac{k^ik^j}{\mathbf{k}^2}\right)n^j$$

The coefficients a, b, c, d are found from the equations: $k^i\Sigma^{ij}k^j = \mathbf{k}^2 b$, $n_T^i\Sigma^{ij}n_T^j = \mathbf{n}_T^2(a+c)$, $n_T^i\Sigma^{ij}k^j = \mathbf{n}_T^2\mathbf{k}^2 d$, $\mathrm{Tr}\Sigma = 2a+b+c$.

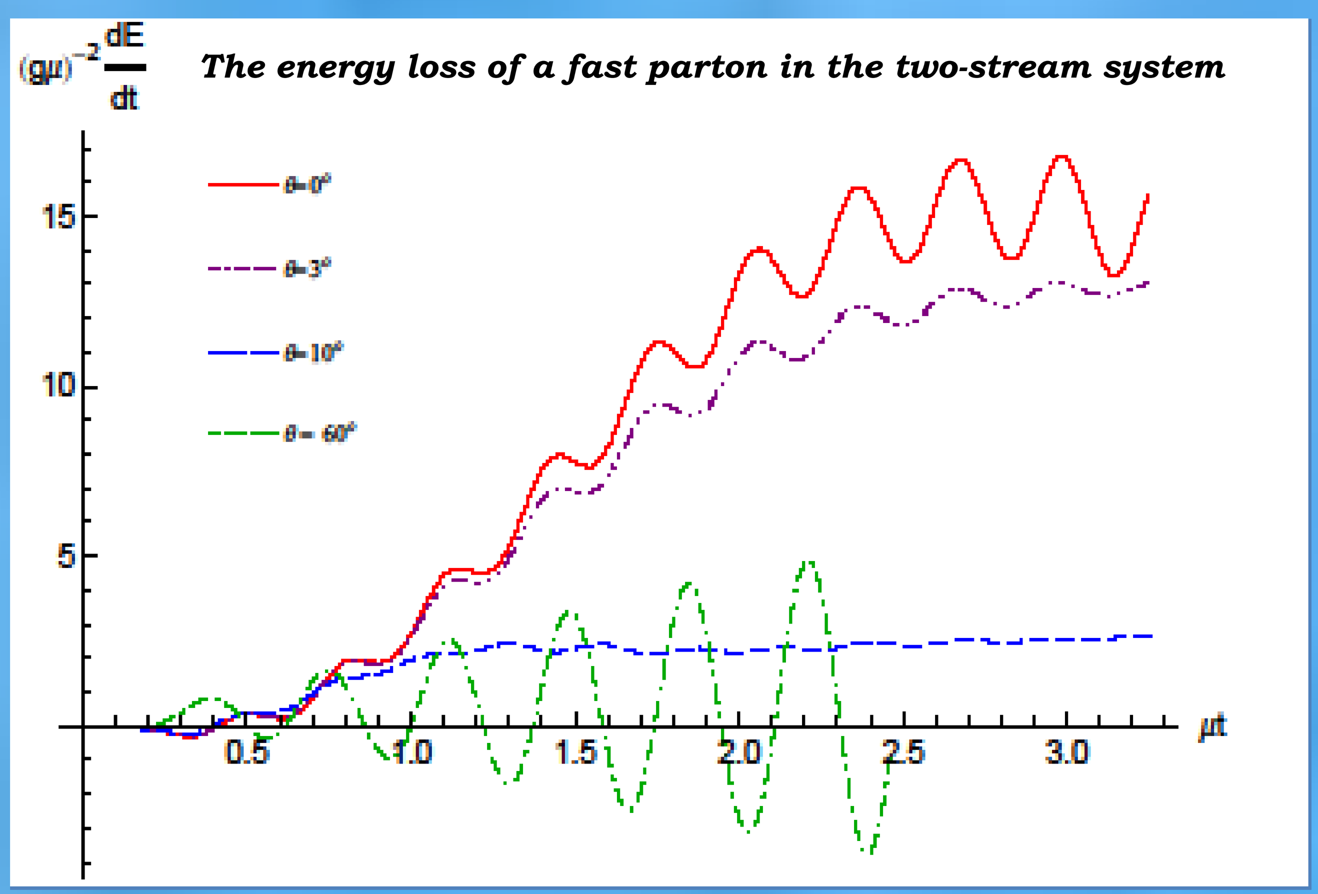
The inverse matrix equals:

$$\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) = \frac{1}{a}A^{ij} + \frac{-a(a+c)B^{ij} + \left(-d^2\mathbf{k}^2\mathbf{n}_T^2 + bc\right)C^{ij} + adD^{ij}}{a\left(d^2\mathbf{k}^2\mathbf{n}_T^2 - b(a+c)\right)}.$$

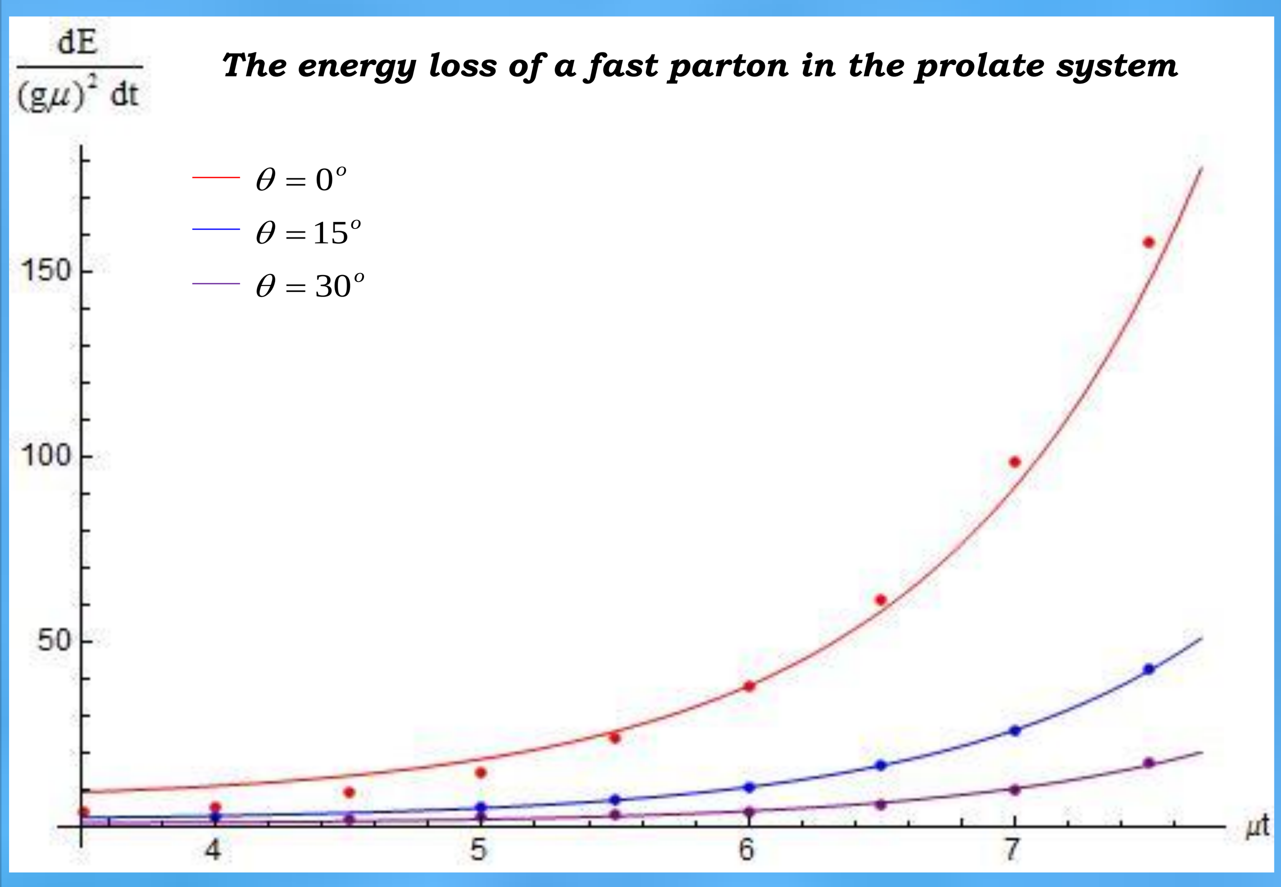
The poles of the inverse matrix sigma give the dispersion equations, which determine the locations of singularities of the integrand of the energy loss in the prolate system. The dispersion equation can be written as:

$$a\left(d^2\mathbf{k}^2\mathbf{n}_T^2 - b(a+c)\right) = -\frac{\omega^2\left(\omega^2-\omega_1^2(\mathbf{k})\right)\left(\omega^2-\omega_2^2(\mathbf{k})\right)\left(\omega^2-\omega_-^2(\mathbf{k})\right)\left(\omega^2-\omega_+^2(\mathbf{k})\right)}{\left(\omega^2-(\mathbf{k}\cdot\mathbf{n})^2\right)^2} = 0.$$

Results



The parton energy loss per unit length as a function of time for several angles between the parton's velocity $\mathbf{v}$ and stream velocity $\mathbf{u}$.



The parton energy loss per unit length as a function of time for several angles between the parton's velocity $\mathbf{v}$ and the vector $\mathbf{n}$.

Conclusions

- We have developed a formalism where the energy loss of a fast parton in a plasma medium is found as a solution of initial value problem.
- In case of stable plasma, we reproduce correctly the standard energy-loss formula.
- In case of unstable plasma the energy loss per unit length is not constant, as in an equilibrium plasma, but it exhibits a strong time and directional dependence.

References

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