Charmonium Potential at Non-Zero Temperature

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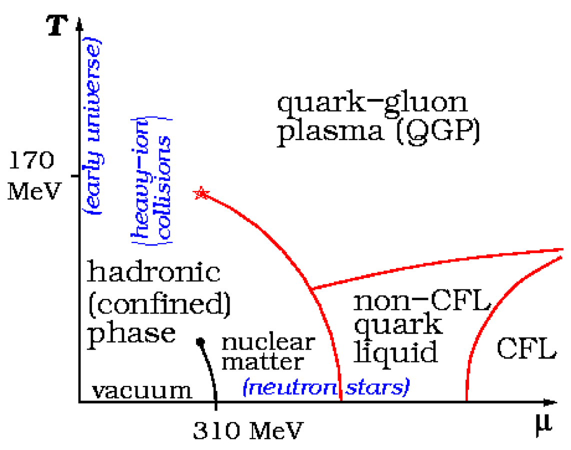
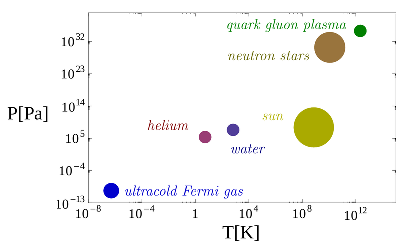
Introduction

To learn more about the early universe and structure of super-dense stars, the physics of matter at extremely high temperature and density must be understood. The strong interaction is dominant under these conditions. Hence the phase diagram of Quantum Chromodynamics (QCD) is investigated.

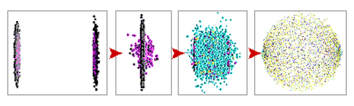


Quark Gluon Plasma



At extremely high temperature and density hadrons dissolve into a deconfined system of quarks and gluons, called a quark-gluon plasma (QGP).

Experimentally, QGP is investigated by orchestrating heavy ion collisions in particle accelerators.



Theoretically, a major tool for its study is lattice QCD (LQCD).

Lattice QCD

○ QCD formulated on a Euclidean space-time lattice.

○ Ambiguous infinite-dimensional functional integral that defines QCD becomes finite and well defined.

○ Evaluate integral using Monte Carlo techniques.

Due to the sign problem, the study of non-zero density LQCD is hampered. This makes non-zero temperature LQCD a more accessible means by which to study the QGP. The charmonium potential is of particular interest because J/ψ suppression is believed to be a signature of QGP [1], its calculation from first principles would also contribute to a better understanding of the the melting of meson states.

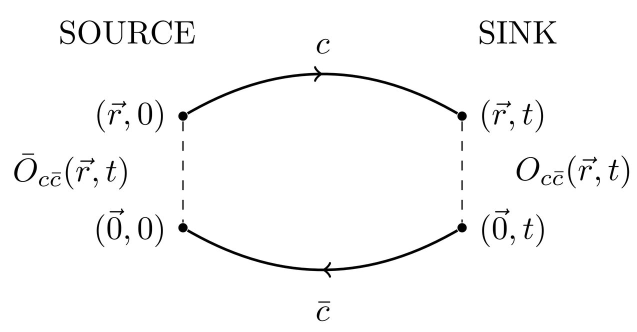
Method of Calculation

Assume charm quarks are heavy enough to be treated non-relativistically.

Solve Schrödinger equation to compute the potential.

$$V_{c\bar{c}}(\vec{r}) = \frac{1}{2\mu_{c\bar{c}}}\frac{\nabla^2\psi_{c\bar{c}}(\vec{r})}{\psi_{c\bar{c}}(\vec{r})} + E^{c\bar{c}}$$

To obtain the reduced mass, $\mu_{c\bar{c}}$, energy, $E^{c\bar{c}}$, and wavefunction, $\psi_{c\bar{c}}(\vec{r})$, of the c-$\bar{c}$ system analyse split-split correlation functions via methods i) and ii),



From analogy with statistical mechanics lattice spacing, a_τ, and size, N_t, in temporal direction are related to temperature by,

$$a_\tau N_t = \frac{1}{T}$$

Calculate $V_{c\bar{c}}(\vec{r})$ for different N_t to investigate non-zero temperature behaviour.

i) Conventional LQCD Hadron Spectroscopy

Express the correlation function of two operators as the sum over the eigenstates of the Hamiltonian operator labelled by n,

$$\begin{aligned} C(\vec{r},t) &= \langle O_{c\bar{c}}(\vec{r},t)\bar{O}_{c\bar{c}}(\vec{r},0)\rangle \\ &= \sum_n \langle 0|\hat{O}_{c\bar{c}}(\vec{r},t)|n\rangle\langle n|\hat{\bar{O}}_{c\bar{c}}(\vec{r},0)|0\rangle e^{-tE_n^{c\bar{c}}}, \end{aligned}$$

Define an effective mass,

$$m_{\text{eff}}(t+1) = \ln\left[\frac{C(t)}{C(t+1)}\right].$$

Perform a two parameter fit of $C(\vec{r},t)$ in time interval where m_{eff} plateaus, here $C(\vec{r},t) = Z(\vec{r})e^{-tE_0}$, since the ground state dominates.

○ $Z(\vec{r}) = \psi^{src}_{c\bar{c}}(\vec{r})\psi^{snk}_{c\bar{c}}(\vec{r}) \implies \psi_{c\bar{c}}(\vec{r}) = \sqrt{Z(\vec{r})}$

○ $E_0^{c\bar{c}} = m_0^{c\bar{c}}$ for zero-momentum state.

ii) Spectral Functions

At zero momentum correlation functions and spectral functions are related by,

$$C(t) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega)e^{-\omega t}.$$

Using this relation can extract $\rho(\omega)$ using Maximum Entropy Method.

Peaks in $\rho(\omega)$ at ω correspond to states with energy $\omega = E_n^{c\bar{c}}$.

Area under peak is proportional to wavefunction, $\psi_{c\bar{c}}(\vec{r})$.

Lattice Simulation Details

A Symanzik-improved, two plaquette action [2] generated two flavour, dynamical, anisotropic gauge configurations for this study [3]:

N_s	N_t	N_{cfg}	$T(MeV)$	T/T_c
12	80	250	92	0.42
12	32	1000	230	1.05
12	28	1000	263	1.20
12	24	500	306	1.40
12	20	1000	368	1.68
12	16	1000	459	2.09

$a_\tau^{-1} = 7.35 \pm 0.03$ GeV
$a_s = 0.162$ fm
$a_s/a_\tau = 6$
$m_\pi/m_\rho = 0.54$ [4]

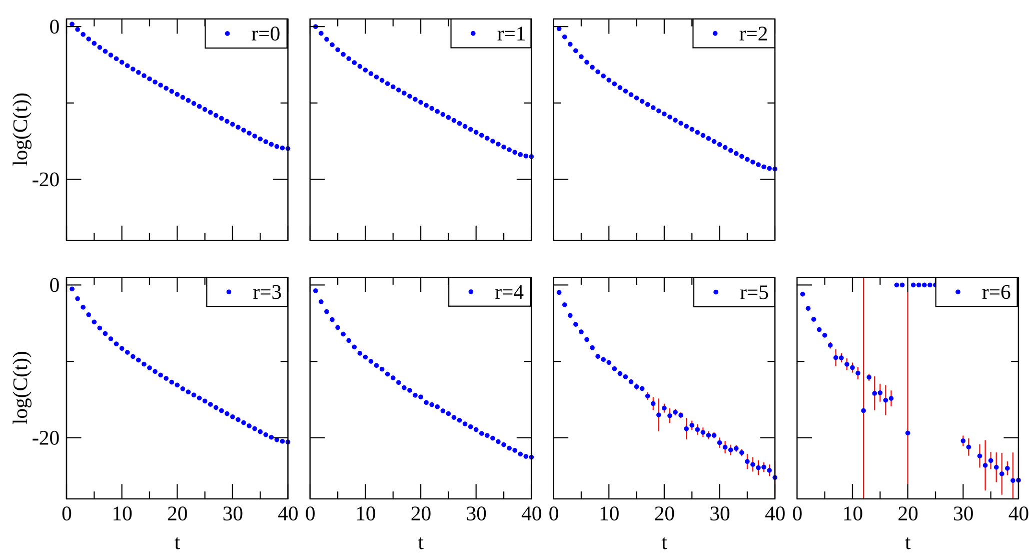
Chroma [5] is used to apply an anisotropic clover fermion action [6] to obtain the correlation functions of several different operators:

Operator	O_h Rep.	Lowest J^{PC}	Channel Name	Particle Name
1	A_1	0^{++}	a_0	${}^3P_0(\chi_{c0})$
γ_5	A_1	0^{-+}	π	${}^1S_0(\eta_c)$
γ_i	T_1	1^{--}	ρ	${}^3S_0(J/\psi)$
$\gamma_5\gamma_i$	T_1	1^{++}	a_1	${}^3P_1(\chi_{c1})$
$\gamma_i\gamma_j$	T_1	1^{+-}	b_1	${}^1P_1(h_c)$
$\gamma_i\nabla_i$	A_1	0^{++}	$\rho\times\nabla\_A_1$	${}^3P_0(\chi_{c0})$
$\epsilon_{ijk}\gamma_j\nabla_k$	E	1^{++}	$\rho\times\nabla\_T_1$	${}^3P_1(\chi_{c1})$
$s_{ijk}\gamma_j\nabla_k$	T_2	2^{++}	$\rho\times\nabla\_T_1$	${}^3P_2(\chi_{c2})$

Preliminary Results

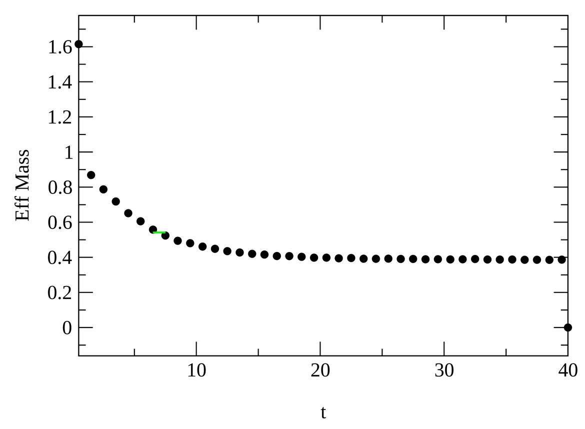
Spectroscopy:

The $N_t = 80$ π-channel correlation functions,

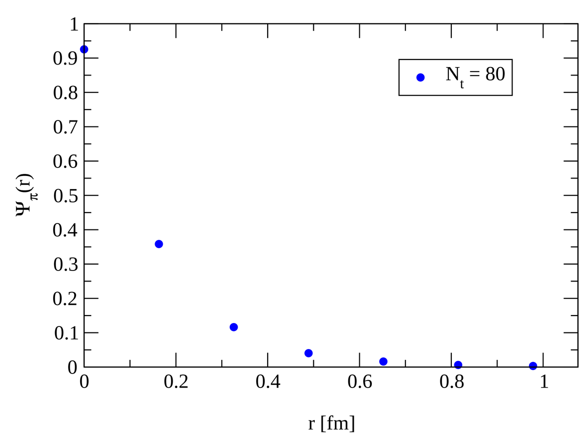


become noisier as $\vec{r}$ increases - a generic observation.

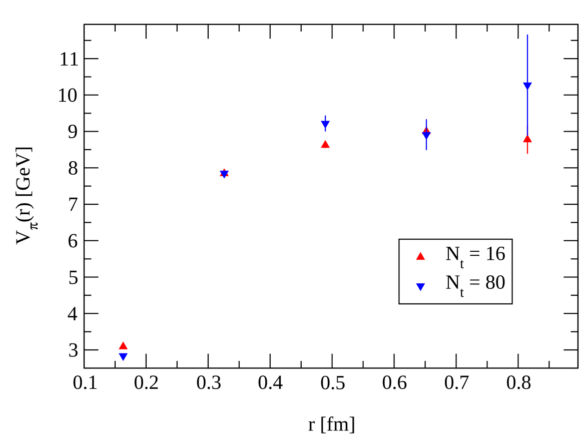
Corresponding effective mass plot for $\vec{r} = 1a_s$,



$\psi_{c\bar{c}}(\vec{r})$ can be successfully extracted,

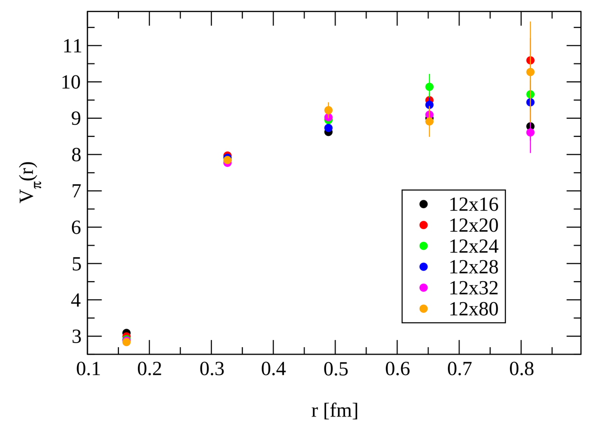


Comparing potential obtained for $N_t = 16$ and $N_t = 80$,



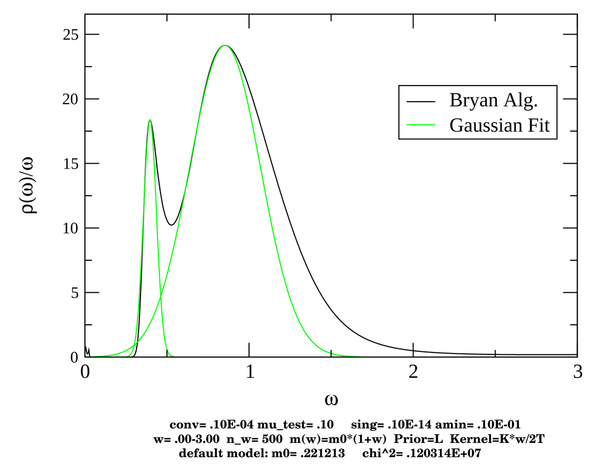
see evidence that potential flattens for higher temperature in agreement with deconfinement.

However, data points for greater $\vec{r}$ have larger errors - as expected from noisier correlation functions. This makes temperature dependence of potential difficult to discern when results for all N_t are plotted.



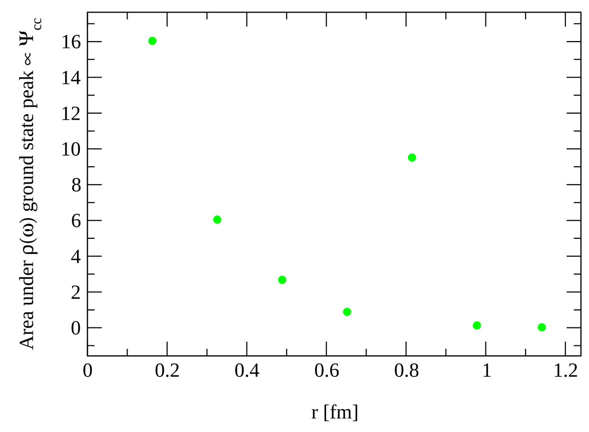
Spectral Functions:

$N_t = 32$ π-channel spectral function for $\vec{r} = 1a_s$,



the energy of the ground state indicated by the centre of the first peak is similar to that indicated by the $N_t = 80$ effective mass plot.

As a first step plot area under spectral function ground state peak against $\vec{r}$, which is proportional to $\psi_{c\bar{c}}(\vec{r})$,



spurious point $\approx \vec{r} = 0.8$ where peak could not be resolved. Once this problem is overcome will proceed to calculate potential.

Summary

Charmonium wavefunction using method i) and ii) and potential using method i) can be successfully extracted. Potential using method ii) will follow after more analysis.

Observe a temperature dependence of the charmonium potential in agreement with deconfinement but need to increase reliability of this claim.

Will increase statistics on 12^3 lattices and move to 32^3 lattices to achieve this.

Acknowledgements

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References

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[2] C. Morningstar and M. J. Peardon, "The glueball spectrum from novel improved actions," *Nucl.Phys.Proc.Suppl.* **83** (2000) 887–889, `arXiv:hep-lat/9911003v1`.

[3] M. Oktay and J. Skullerud, "Momentum-dependence of charmonium spectral functions from lattice QCD," `arXiv:hep-lat/1005.1209v1`.

[4] G. Aarts, C. Allton, M. B. Oktay, M. Peardon, and J.-I. Skullerud, "Charmonium at high temperature in two-flavor QCD," *Phys. Rev.* **D76** (2007) 094513, `arXiv:hep-lat/0705.2198`.

[5] **SciDAC** Collaboration, R. G. Edwards and B. Joo, "The Chroma software system for lattice QCD," *Nucl. Phys. Proc. Suppl.* **140** (2005) 832, `arXiv:hep-lat/0409003`.

[6] R. G. Edwards, B. Joo, and H.-W. Lin, "Tuning for Three-flavors of Anisotropic Clover Fermions with Stout-link Smearing," *Phys. Rev.* **D78** (2008) 054501, `arXiv:hep-lat/0803.3960`.