Phase transition in the $U_L(3)\times U_R(3)$ meson model from large-n approximation

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1. The model, its phase structure

- The $U_L(n)\times U_R(n)$ symmetric model of complex scalar $n\times n$ matrix fields M represents an effective meson model without axial anomaly. The Lagrangian:

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}\,(\partial_\mu M^\dagger\partial^\mu M - m^2 M^\dagger M)\\ &- \frac{g_1}{n^2}\Big(\mathrm{Tr}\,(M^\dagger M)\Big)^2 - \frac{g_2}{n}\,\mathrm{Tr}\,\Big((M^\dagger M)^2\Big).\end{aligned}$$

- M can be decomposed to scalar (s^a) and pseudoscalar (π^a) parts: $M=(s^a+i\pi^a)T^a$.
- Previous RG-results for the phase structure:
  $\rightarrow n\geq 2$: no non-trivial fixed point of RG-flow for $d=3$, a first order thermal phase transition is expected [1]
  $\rightarrow n=2$: Functional RG established first order transition with $U_L(2)\times U_R(2)\rightarrow U_V(2)$ symmetry breaking pattern [2,3]
- Present investigation:
  $\rightarrow \mathcal{O}(n)$ accurate NLO-expression for the free energy density in a large-n approximation
  $\rightarrow$ determination of the finite T phase structure [4,5]
- Two-component symmetry breaking pattern:

$$<s^a>=\sqrt{2n}v_0\delta^{a0}+\sqrt{2n}\chi_8\delta^{a8}.$$

This allows for $U_L(n)\times U_R(n)\longrightarrow U_V(n-1)$ breaking.
$\longrightarrow$ pion-kaon multiplet splitting
$\longrightarrow$ different n-scalings for the two condensates!

2. Large-n approx. with auxiliary fields

- Introduction of three quadratic auxiliary fields x, y_1^a, y_2^a via Hubbard-Stratonovich transformation:

$$e^{iS[s^a,\pi^a]}=\int\mathcal{D}\{x,y_1^a,y_2^a\}e^{iS[s^a,\pi^a,x,y_1^a,y_2^a]},$$

with (d^{abc}, f^{abc} symm. and antisymm. structure constants)

$$y_1^a\rightarrow d^{abc}(s^bs^c+\pi^b\pi^c),\qquad y_2^a\rightarrow f^{abc}s^b\pi^c,$$
$$x\rightarrow(s^a)^2+(\pi^a)^2.$$

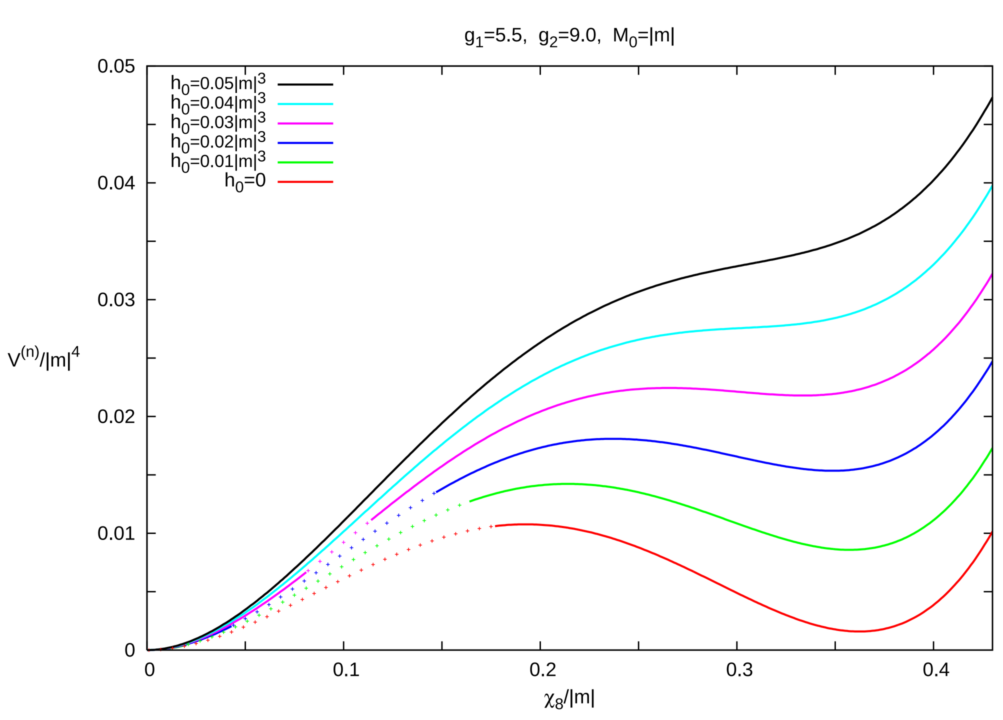
- Effect of neglecting heavy scalar excitations in controlled parts of the (g_1,g_2)-plane: only 1-loop contributions of the pseudoscalars (pions and kaons) are included in the eff. potential.

$$\begin{aligned}V=&\,n^2\Big[(m^2-ix-y_1^0)v_0^2+\frac{1}{2}(x^2+(y_1^0)^2)\Big]\\ &+n\Big[(m^2-ix-iy_1^0)\chi_8^2+\frac{1}{2}(y_1^8)^2-i\sqrt{2g_2}y_1^8\chi_8(2v_0-\chi_8)\Big]\\ &-\frac{i}{2}\Big[(n^2-2n)\int_p\ln(-p^2+M_\pi^2)+2n\int_p\ln(-p^2+M_K^2)\Big].\end{aligned}$$

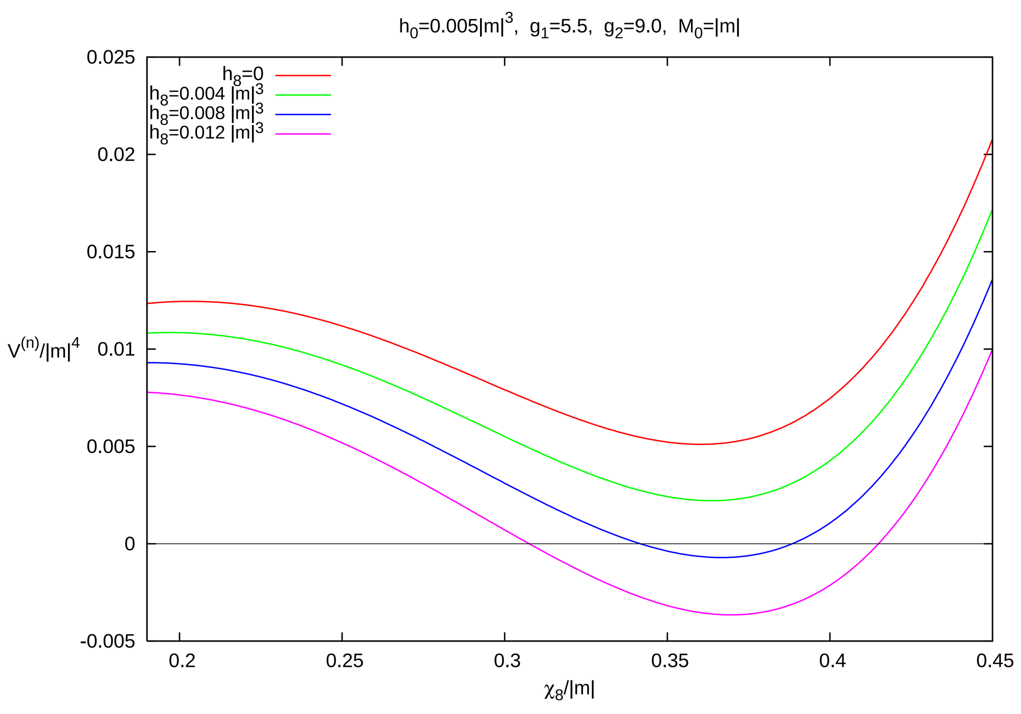
- Renormalization done in an MS-like scheme at scale $M_0=|m|$.

3. Nontrivial minima at $T=0$

- v_0 condensate is determined from the $\mathcal{O}(n^2)$ LO-part of V.
- χ_8 condensate can be obtained by minimizing the $\mathcal{O}(n)$-part $n\cdot V^{(n)}$ in the background of v_0.
- Nontrivial ($\chi_8\neq 0$) metastable minima; depth tuned by diagonal explicit symmetry breaking ($\Leftrightarrow \mathcal{L}_{h_0}=h_0s^0$).



- Explicit breaking in the "8" direction ($\Leftrightarrow \mathcal{L}_{h_8}=h_8s^8$) can drive the nontrivial minimum to be the true ground state.



4. Application of the potential at $n=3$

- Elimination of the auxiliary fields simplifies the solution.
- Gap equations for the masses as functions of v_0 and χ_8:

$$\begin{aligned}M_\pi^2 &= m^2+(g_1+g_2)\Big[2v_0^2+T_{M_\pi}+\frac{2}{n}(\chi_8+T_{M_K}-T_{M_\pi})\Big]\\ &+\frac{2g_1}{n}\Big[2v_0\chi_8-\chi_8^2-\frac{1}{2}(T_{M_K}-T_{M_\pi})\Big],\\ M_K^2 &= M_\pi^2-2g_2\Big[\chi_8(v_0-\chi_8)-\frac{1}{4}(T_{M_K}-T_{M_\pi})\Big]\\ &-\frac{2g_1}{n}\Big[2v_0\chi_8-\chi_8^2-\frac{1}{2}(T_{M_K}-T_{M_\pi})\Big].\end{aligned}$$

- Equations of state for the condensates v_0 and χ_8:

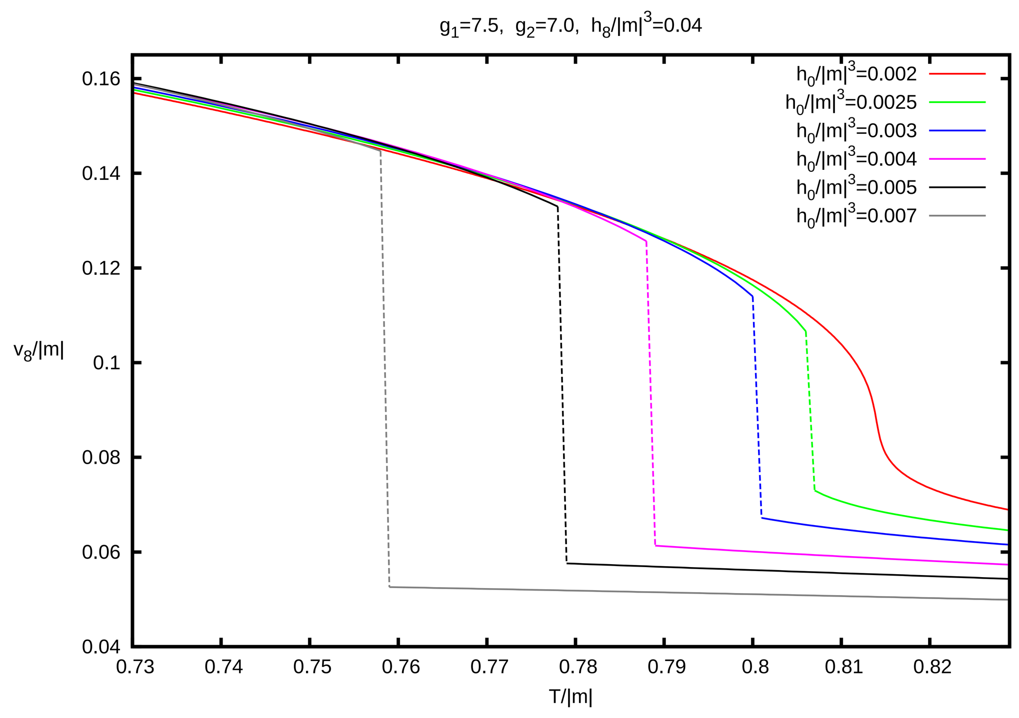
$$\begin{aligned}0 &= h_8-M^2\chi_8-2g_2(v_0-\chi_8)\Big[\chi_8(2v_0-\chi_8)\\ &-\frac{1}{2}(T_{M_K}-T_{M_\pi})-g_2\chi_8T_{M_K}\Big],\\ 0 &= h_0-M^2v_0-\frac{2g_2}{n}\chi_8\Big(2v_0\chi_8-\chi_8^2-\frac{1}{2}(T_{M_K}-T_{M_\pi})\Big).\end{aligned}$$

$\rightarrow$ Mass: $M^2\equiv M_\pi^2-2g_1\left(2v_0\chi_8-\chi_8^2-(T_{M_K}-T_{M_\pi})/2\right)/n$,
$\rightarrow$ Tadpole integral: $T_\mu=\int_p i/(p^2-\mu^2)$.

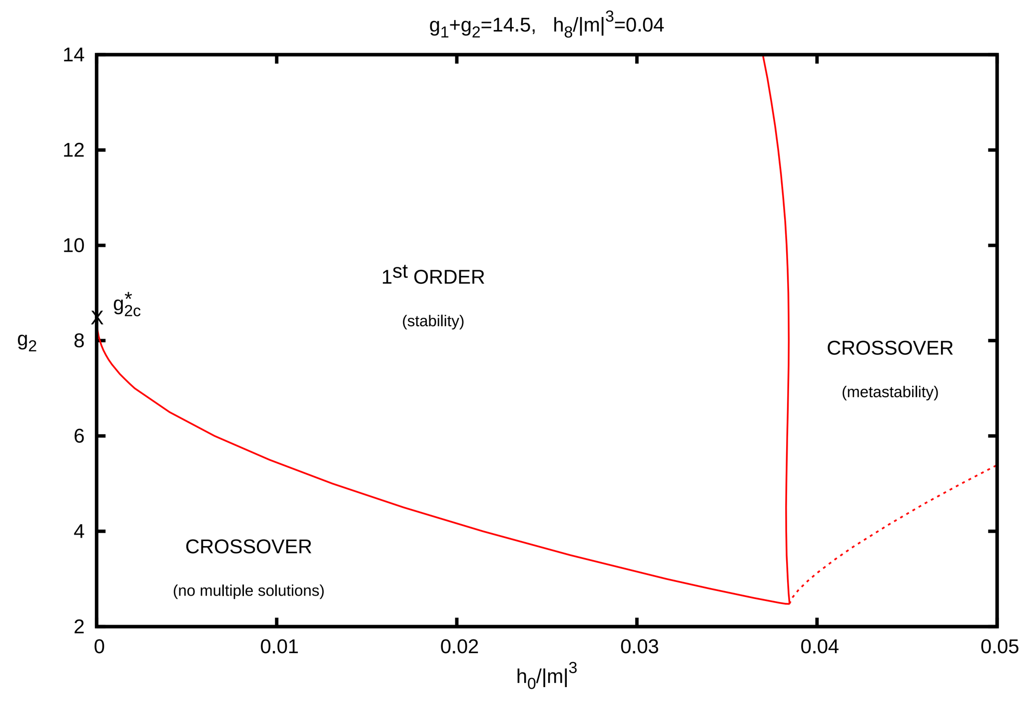
- $1/n$-expanded solution:
  $\longrightarrow M^2=M_{00}^2+M_1^2/n,\quad v_0=v_{00}+v_{01}/n.$
- Put $n=3$ and substitute the sums into V!

5. Finite temperature investigation

- Strongly correlated evaporation of the two condensates suggests the possibility of a single, unique transition.
- Step 1: vary g_2 and h_0 upon g_1+g_2 and h_8 held fixed.



- Edge of the region of first order transitions located by vanishing $v_8=\chi_8/\sqrt{3}$ discontinuity. Accompanying singularity of dv_0/dh_0 demonstrates the v_0-v_8 coupling.

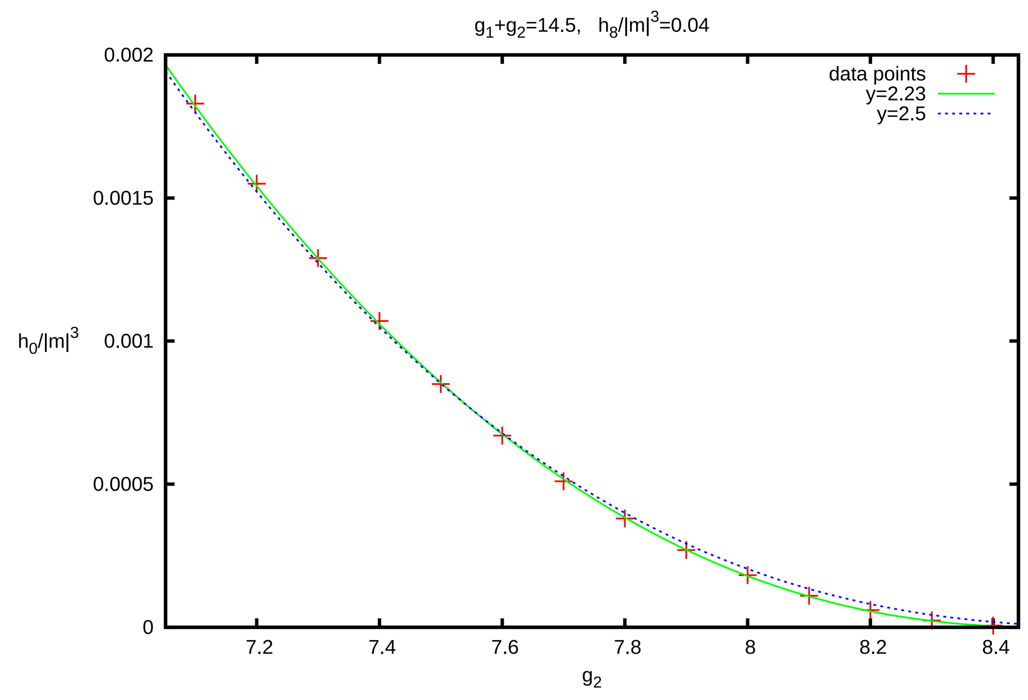


- Dashed line separates regions with three resp. two minima of the effective potential at $T=0$. A nontrivial $\chi_8\neq 0$ minimum exists as a stable or metastable state only in the upper part.
- Transition of v_8 (at $T=T_{8c}$) is continuous on the $h_0=0$ axis for $g_2=g_{2c}^*$, and so is the transition of v_0 on the whole axis.

6. Tricriticality conjecture

- Alternative scenarios for symmetry restoration:
  A: $v_0(T_{8c})\neq 0\quad\Rightarrow\quad U_V(2)\rightarrow U_V(3)\rightarrow U_L(3)\times U_R(3)$
  B: $v_0(T_{8c})=0\quad\Rightarrow\quad U_V(2)\rightarrow U_L(3)\times U_R(3)$
- In case B, the set of $g_2\in(0,g_{2c}^*)$ is a critical line of the $U_V(3)\rightarrow U_L(3)\times U_R(3)$ restoration
  $\Rightarrow$ transition continuous in large-n approximation
- Expected crossing of two critical lines at g_{2c}^*: tricritical point.
- Scaling law [6]: $\quad g_{2c}-g_{2c}^*=\text{const.}\times h_{0c}^{1/y},\quad y=5/2.$
- $g_2>g_{2c}^*$, $\quad h_0=0$: the approximate symmetry restoration proceeds via a first order transition.

7. Tricriticality test I



- A high quality fit of the inverse scaling form

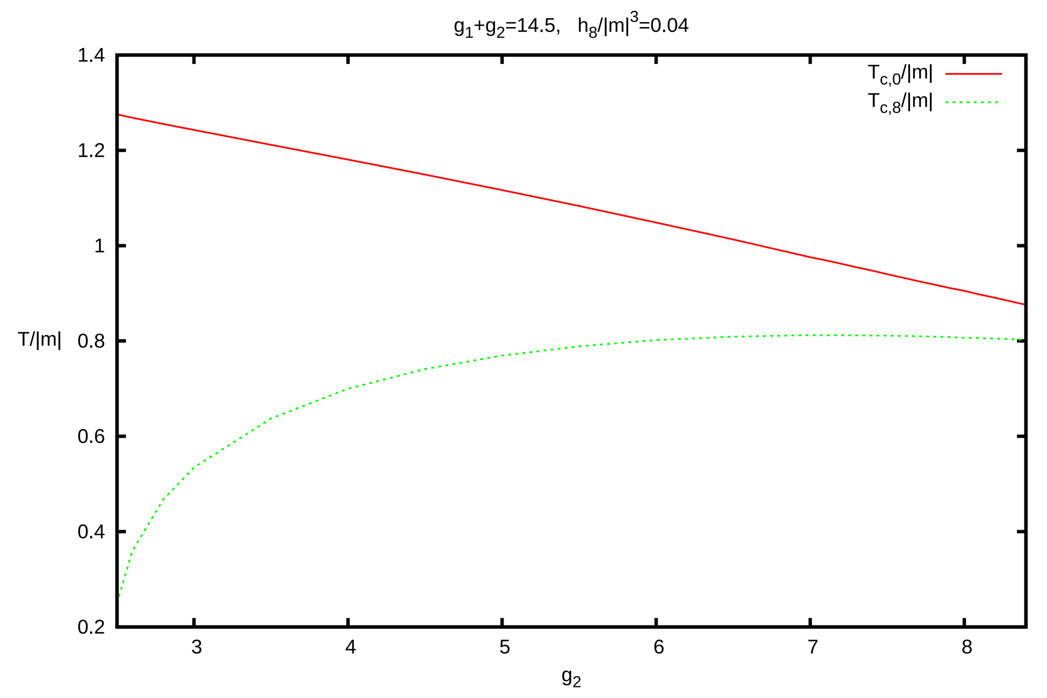
$$\begin{aligned}h_{0,c}&=a(g_2-b)^y, & a&=(8.80\pm0.25)\cdot10^{-4},\\ b&=8.49\pm0.01, & y&=2.23\pm0.02.\end{aligned}$$

Scaling exponent is close to $y=5/2$.

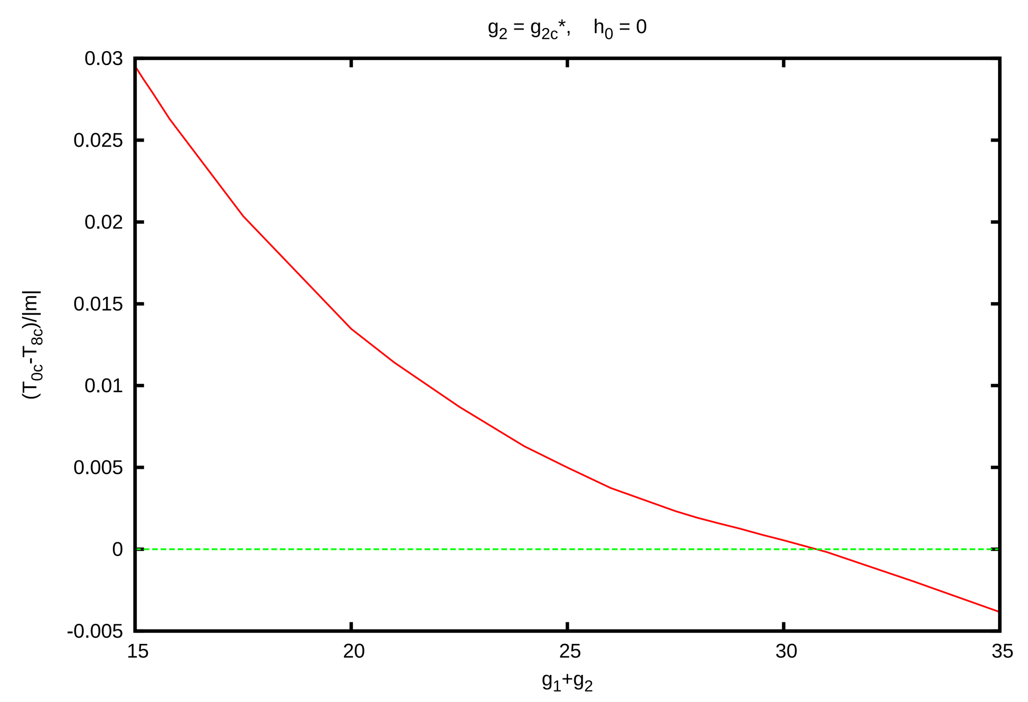
- Fixing the exponent to $y=2.5$, we still get a reasonable fit:

$$\begin{aligned}h_{0,c}&=a(g_2-b)^{2.5}, & a&=(6.03\pm0.09)\cdot10^{-4},\\ b&=8.49\pm0.01.\end{aligned}$$

8. Tricriticality test II



- Step 2: Crossing critical temperatures, coinciding transitions?
- Minimum distance of $T_{0c}-T_{8c}$ on the second order line with increasing h_8 is always at $g_{2c}^*=g_1+g_2$.
- Increasing g_1+g_2 to ≈ 30.8, at $h_8\approx 0.038m^3$ the transition temperatures are equal. For $g_1+g_2\gtrsim 30.8$ the v_0-transition precedes the evaporation of v_8.



- $U_L(3)\times U_R(3)$ symmetry restored in a single step from $U_V(2)$.

9. Conclusions & References

- Finite T phase transitions of $U_L(3)\times U_R(3)$ symmetric linear sigma model can be investigated with large-n technique.
- Upon neglecting heavy scalar fluctuations regions are found where the symmery breaking pattern $U_L(3)\times U_R(3)\rightarrow U_V(2)$ is realized.
- Exploration of the 4-dimensional parameter space g_1,g_2,h_0,h_8 reveals a region of single step (unique) transitions, while a most common $U_L(3)\times U_R(3)\rightarrow U_V(3)\rightarrow U_V(2)$ is conjectured.
- A tricritical point is conjectured at the crossing of the $U_V(3)\rightarrow U_L(3)\times U_R(3)$ and $U_V(2)\rightarrow U_V(3)$ critical lines.

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