Baryon Asymmetry from First Principles without Boltzmann or Kadanoff-Baym

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Abstract

We present a formalism that allows the computation of the baryon asymmetry of the universe from first principles of statistical physics and quantum field theory that is applicable to certain types of beyond the Standard Model physics (such as the neutrino Minimal Standard Model – νMSM) and does not require the solution of Boltzmann or Kadanoff-Baym equations. The formalism works if a thermal bath of Standard Model particles is very weakly coupled to a new sector (sterile neutrinos in the νMSM case) that is out-of-equilibrium. The key point that allows a computation without kinetic equations is that the number of particles in this new sector produced during the relevant cosmological period remains small. In such a case, it is possible to expand the formal solution of the von Neumann equation perturbatively and obtain a master formula for the lepton asymmetry expressed in terms of non-equilibrium Wightman functions. These correlation functions can then be evaluated perturbatively; the validity of the perturbative expansion depends on the parameters of the model considered. We illustrate the use of the formalism with a toy model (containing only two active and two sterile neutrinos) and with the more realistic νMSM.

Introduction and Objectives

- Experimental fact: There exists an asymmetry between the number of baryon n_b and anti-baryons $n_{\bar b}$ [1]:
$$\eta = (n_b - n_{\bar b})/n_\gamma = (6.1 \pm 0.2) \times 10^{-10}, \tag{1}$$
where n_γ is the number density of photons
- Difficult problem: System has to be out-of-equilibrium in order to produce a baryon asymmetry (third Sakharov condition)
- Popular technique: Kinetic equations
  - Pros: Simple
  - Cons: Not fully quantum mechanical, relies on assumptions
- Better technique: Kadanoff-Baym equations
  - Pros: Fully quantum mechanical
  - Cons: Very hard to solve, gauge invariance is delicate when truncating
- Why so hard in QFT? Because of the need to resum secular terms
- Our objective: Develop a simple formalism that allows the computation of the asymmetry from first principles of quantum field theory and statistical physics that is applicable when secular terms remain small

Lagrangian

A generic Lagrangian for leptogenesis-type scenarios containing A active and B sterile neutrinos can be written as (α runs from 1 to A and I from 1 to B):
$$\mathcal{L}_{AB} = \mathcal{L}_{SM} + \bar N_I i\partial\!\!\!/ N_I - \frac{M_{IJ}}{2}\bar N_I^c N_J - \underbrace{F_{\alpha I}\bar L_\alpha N_I \tilde\Phi}_{\bar L_\alpha J_\alpha} + \text{h.c.}, \tag{2}$$
where L is the active lepton doublet, N the sterile neutrino singlet, Φ the Higgs doublet, M_{IJ} are Majorana masses and $F_{\alpha I}$ are Yukawa couplings.

Derivation of the Master Formula for the Asymmetry [2]

- What we want to solve: Lepton asymmetry at time t for a particular flavor α:
$$\Delta_\alpha(t,\vec x) = \mathrm{Tr}\left[\hat\rho(t) L_\alpha^\dagger(t,\vec x) L_\alpha(t,\vec x)\right], \tag{3}$$
where $\hat\rho(t)$ is the appropriate density operator satisfying the initial condition:
$$\hat\rho(0) = \hat\rho_S \otimes \hat\rho_{SM}. \tag{4}$$
$\hat\rho_S$ is the density operator for sterile neutrinos and $\hat\rho_{SM} = e^{-\hat H_{SM}/T}$ is the equilibrium density operator for SM particles (we take $\hat\rho_S = |0\rangle\langle 0|$ in the following).
- Time evolution of ρ_S: Given by the von Neumann equation:
$$i\frac{d\hat\rho(t)}{dt} = \left[\hat H_I(t), \hat\rho(t)\right], \tag{5}$$
where $\hat H_I(t)$ is the interaction Hamiltonian. An iterative solution can be found:
$$\hat\rho(t) = \hat\rho(0) - i\int_0^t dt' \left[\hat H_I(t'), \hat\rho(0)\right] - \int_0^t dt' \int_0^{t'} dt'' \left[\hat H_I(t'), \left[\hat H_I(t''), \hat\rho(t'')\right]\right]. \tag{6}$$
which is also a perturbative solution if "$\hat H_I t \ll 1$" (i.e. secular terms remain small).
- Master formula for the asymmetry (at $O(H_I^2)$): Plug Eq. (6) in Eq. (3):
$$\Delta_\alpha(t',\vec x) = \int_0^{t'} dt \int_0^t dt_1 \int d^3y \int d^3y_1\, 4\Big[\mathrm{Im}(F_{\beta I}F^*_{\gamma J})\,\mathrm{Im}(\langle \tilde J_I \bar{\tilde J}_J\rangle \langle \nu_\alpha^\dagger \nu_\gamma\rangle\langle \nu_\alpha \bar\nu_\beta\rangle) - \mathrm{Im}(F^*_{\gamma J}F_{\beta I})\,\mathrm{Im}(\langle \bar{\tilde J}_J \tilde J_I\rangle\langle \nu_\gamma \nu_\alpha^\dagger\rangle\langle\nu_\alpha\bar\nu_\beta\rangle)\Big]. \tag{7}$$
- Remarks:
  - Only assumption is perturbation theory is valid (secular terms remain small)
  - Perturbative in the Yukawas F, exact in SM couplings
  - Asymmetry expressed in terms of equilibrium Wightman functions
  - Neatly separates CP violating effects from thermal effects
  - Asymmetry at $O(H_I^2)$ is automatically zero, first nonzero term is $O(H_I^4)$

Example 1: Toy Model [2]

- Model: $A = 2$, $B = 2$, neglect $\mathcal{L}_{SM}$, Higgs non-dynamical
- Two ways to compute the asymmetry:
  - Exactly: Lagrangian quadratic in the fields, similar to a neutrino oscillation problem
  - Perturbatively: Using Eq. (7) (or, more precisely, its $O(H_I^4)$ version)
- Validity of perturbation theory:
  - Solutions are oscillatory (no damping) with different frequencies (exact vs perturbative masses) $\rightarrow$ Buildup of a phase differenc over time
  - Secular phase differences are small when:
$$|\omega_{ij,\pm} - \tilde\omega_{ij,\pm}|\, t \ll 1 \quad\Rightarrow\quad t \ll \frac{2|\vec p|}{\left|(m_i^2 \pm m_j^2) - (\tilde m_i^2 \pm \tilde m_j^2)\right|}, \tag{8}$$
where ω ($\tilde\omega$) and m ($\tilde m$) are the exact (perturbative) frequencies and masses.

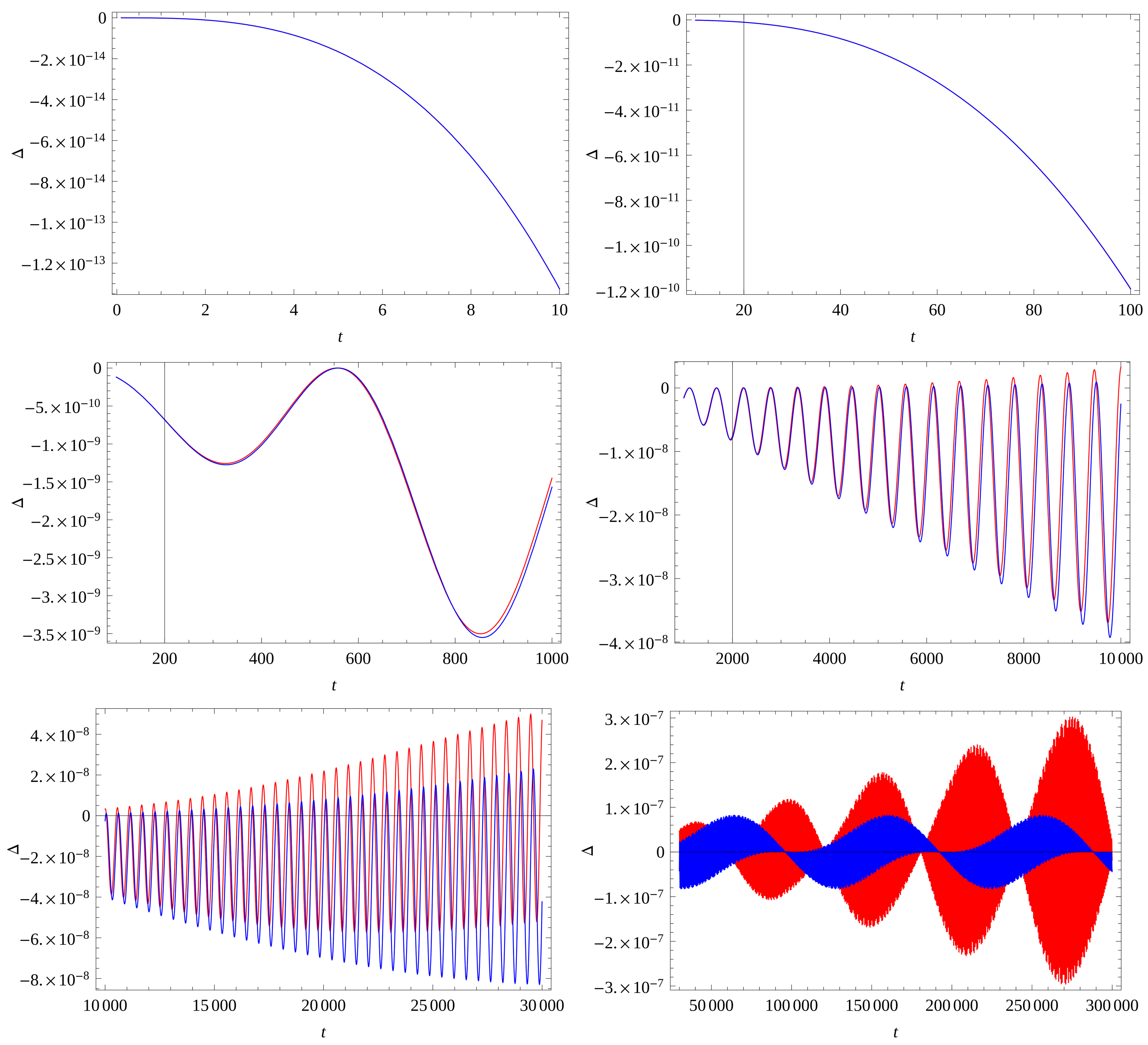
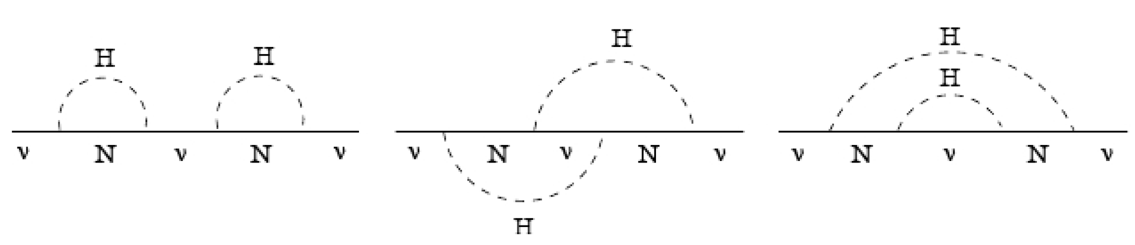


Figure : Plots of active lepton asymmetry production (for one flavor) per unit of phase space as a function of time (expressed in GeV^{-1}). The red and blue lines correspond to the exact and perturbative results, respectively. We note that the exact and perturbative results agree very well up to $t \approx 6000 \approx t_s/3$, where we start to see small discrepancies that grow larger with time.

Example 2: νMSM (in progress)

- Model: $A = 3$, $B = 2$ plus full Standard Model interactions
- Outline of the baryogenesis scenario:
  - Start with a thermal distribution of SM particles and no sterile neutrinos
  - A lepton asymmetry is produced via coherent and resonant oscillations of active-sterile neutrinos (sterile neutrinos must be nearly degenerate in mass)
  - Lepton asymmetry is converted into a baryon asymmetry via sphaleron processes
- Validity of perturbation theory: When number of sterile neutrino produced is small
$$n_{\rm sterile} \sim \int_0^{t_{\rm sph}} \Gamma_{\rm sterile}(t')dt' \overset{\text{power laws}}{\sim} \left.\frac{\Gamma_{\rm sterile}}{H}\right|_{t_{\rm sph}} \sim \frac{f^2 T}{T^2/M_{\rm Pl}} \sim 0.1, \tag{9}$$
where we have used $T = T_{\rm sph} \sim 100$ GeV (temperature at which sphalerons become inefficient) and $f = 10^{-9}$ (a possible Yukawa coupling in the νMSM)
- Typical diagrams to include (with resummed lines):



Conclusion

Applicable to other models (e.g. weak washout leptogenesis)?

References

[1] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. **180**, 330 (2009).
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