The thermal photon production rate at NLO

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Introduction and motivation

The thermal photon production rate is a *transport coefficient*, yielding the rate of emission of photons from an equilibrated QGP.

Field-theoretical definition

$$\frac{d\Gamma}{d^3k} = \frac{e^2}{(2\pi)^3 2k}\int d^4X e^{-iK\cdot X}\langle J^\mu(0)J_\mu(X)\rangle, \quad k^0=k,$$

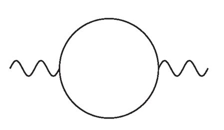
i.e. a Wightman current-current correlator for an *on-shell hard photon*, $k\sim T$.

Complete perturbative leading-order ($\alpha_{\mathrm{EM}}\alpha_s$) evaluation in [1]. The NLO correction is of relative size g, i.e. $\alpha_{\mathrm{EM}}g^3$.

The currently-existing NLO calculations of transport coefficients [2,3] show a pattern of very large $\mathcal{O}(g)$ corrections.

An NLO calculation of the photon rate can:
- show if this pattern is confirmed and help understand if some resummations/reorganizations could help
- improve phenomenological analyses and comparisons with current RHIC and future LHC data, when summed with prompt photons from the initial hard collision and photons from the final hadronic phase.

The leading-order calculation



$= 0$ (kinematics)

⇒ **two loop diagrams** at the top of the poster. The scaling of the momentum P ⇒ regions in *Scales and regions*.

Two regions contribute to the leading-order rate:
1. **2→2 region**: 2→2 processes, decomposes in the soft and hard regions, a divergence canceling between them [4]
2. **the collinear region**: 2→3 and 3→2 processes. Clearly separated from 2→2 at LO

The end-result is

$$(2\pi)^3\frac{d\Gamma}{d^3k}\bigg|_{\mathrm{LO}} = \mathcal{A}(k)\left[\log\frac{T}{m_\infty}+C_{2\to2}(k)+C_{\mathrm{coll}}(k)\right]$$

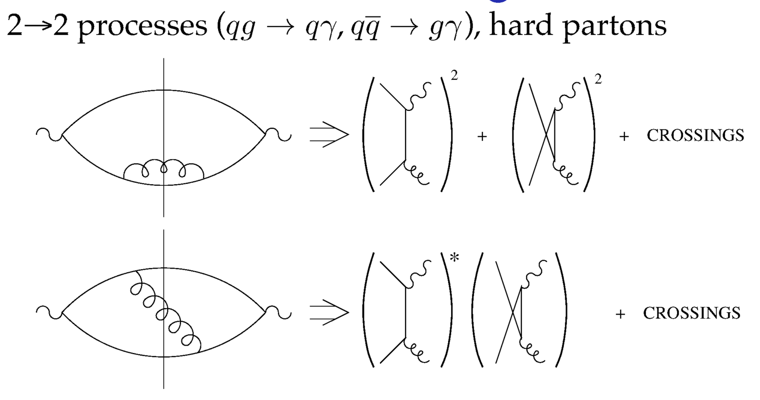
$$\mathcal{A}(k)=\alpha_{\mathrm{EM}}\,g^2\,C_FT^2\frac{n_{\mathrm{F}}(k)}{2k}\sum_f Q_f^2d_f$$

Structure of the NLO calculation

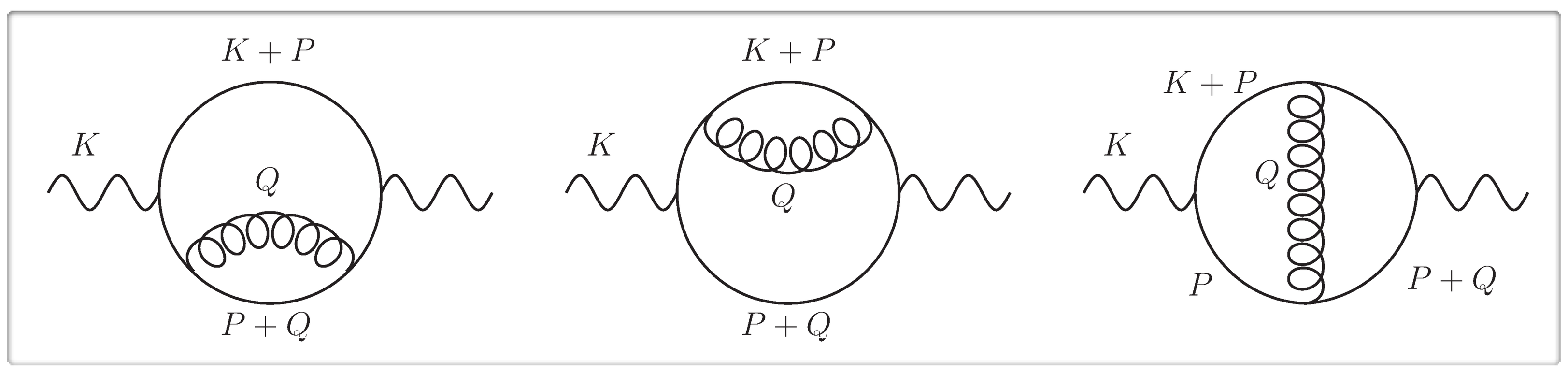
At NLO:
- the soft and collinear regions receive $\mathcal{O}(g)$ corrections
- *new region*, the semi-soft region ⇒ interpolates between the three vertices of the triangle.

The hard region

2→2 processes ($qg\to q\gamma$, $q\bar{q}\to g\gamma$), hard partons

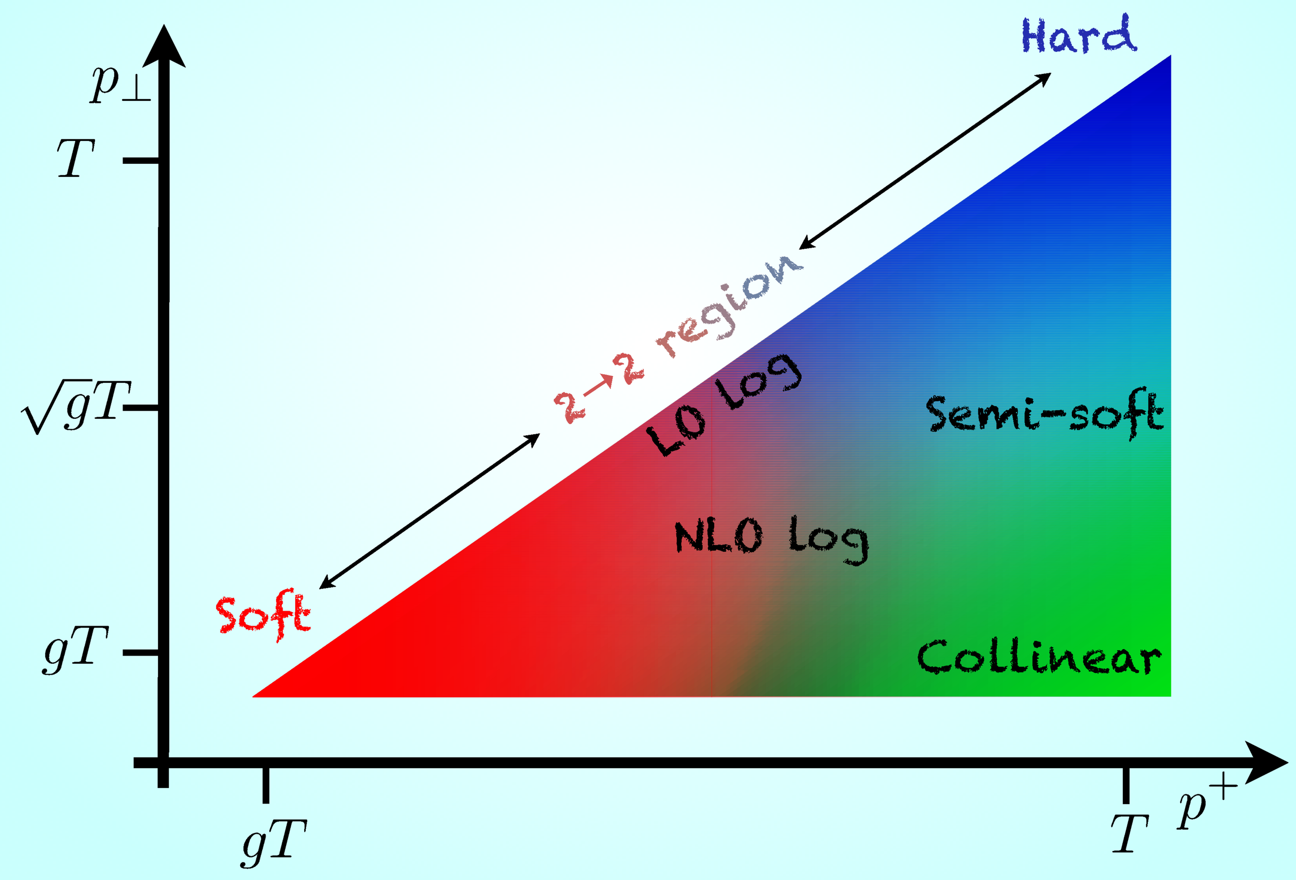


It contributes at LO.
IR divergence (*LO log*) is removed by HTL resummation in the soft region.
Corrections are $\mathcal{O}(g^2)$ ⇒ NNLO.



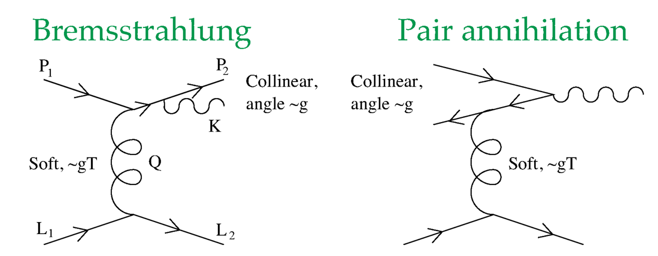
Scales and regions

In light-cone coordinates $p^+\equiv p_0+p_z$ and $p_\perp$ are the variables we plot. Scaling of $p^-\equiv p_0-p_z$ deducted from momentum conservation.



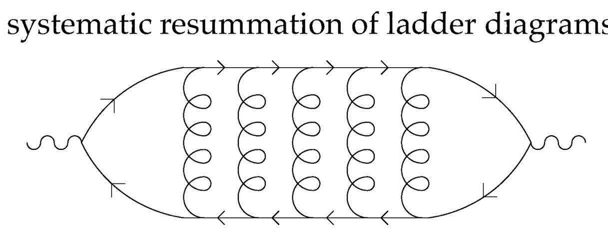
The collinear region

2→3 and 3→2 process also contribute to LO! [5] A soft scattering kicks a hard fermion slightly off-shell



collinearity ⇒ small perpendicular relative momenta ⇒ long photon formation time. *Interference* between multiple soft scattering becomes relevant and unsuppressed ⇒ *Landau-Pomeranchuk-Migdal (LPM) effect*

The collinear rate is obtained by a systematic resummation of ladder diagrams [1]



$$(2\pi)^3\frac{d\Gamma}{d^3k}\bigg|_{\mathrm{coll}} = \mathcal{A}(k)\int dp_z\frac{p_z^2+(p_z+k)^2}{p_z^2(p_z+k)^2}\frac{n_{\mathrm{F}}(k+p)(1-n_{\mathrm{F}}(p))}{g^2C_FT^2n_{\mathrm{F}}(k)}\int\frac{d^2p_\perp}{(2\pi)^2}2\mathrm{Re}\left(\mathbf{p}_\perp\cdot\mathbf{f}(\mathbf{p}_\perp;p_z,k)\right)$$

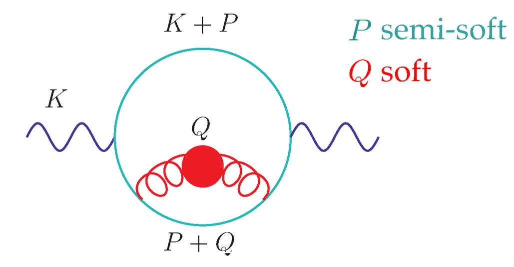
$$2\mathbf{p}_\perp = i\frac{k(p_\perp^2+m_\infty^2)}{2p_z(p_z+k)}\mathbf{f}(\mathbf{p}_\perp)+\int\frac{d^2q_\perp}{(2\pi)^2}\mathcal{C}(q_\perp)\left(\mathbf{f}(\mathbf{p}_\perp)-\mathbf{f}(\mathbf{p}_\perp+\mathbf{q}_\perp)\right)$$

m_∞ ⇒ fermion asymptotic mass, $C(q_\perp)$ transverse collision rate (ladder rung)

At NLO three $\mathcal{O}(g)$ corrections:
1. $m_\infty = C_Fg^2T^2/4+\mathcal{O}(g)$. $\mathcal{O}(g)$ computed in [6]
2. $C(q_\perp)$ receives $\mathcal{O}(g)$ corrections: one-loop soft ladder rungs above. Computed in [3]
3. $p_z\sim gT$, $p_z+k\sim gT$: constant p_z integrand ⇒ cancels against UV linear soft NLO

Numerical solution both at LO and NLO by Fourier-transforming in impact-parameter space

The semi-soft region



Two kinematical regions ⇒ different processes
1. Timelike Q ⇒ 2→2 processes with massive (plasmon) gluon
2. Spacelike Q ⇒ 2→3 and 3→2 processes: wider-angle bremsstrahlung and pair annihilation, no LPM interference

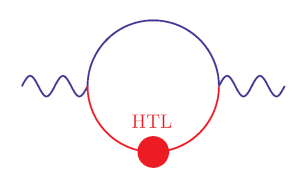
Gluon integration ⇒ gauge-invariant matrix element ⇒ we generalize the AGZ sum rule [7].

Limits and divergences:
- ↑ $p_\perp\to\infty$: subtract the appropriate hard limit
- ↓ $p_\perp\to0$: subtract the appropriate collinear limit
- ↙ $p_\perp\to0\wedge p^+\to0$: IR log, combines with UV soft log (NLO log)

Final integral over p^+ only known numerically, gives $C_{\mathrm{semisoft}}(k)$ in the final result

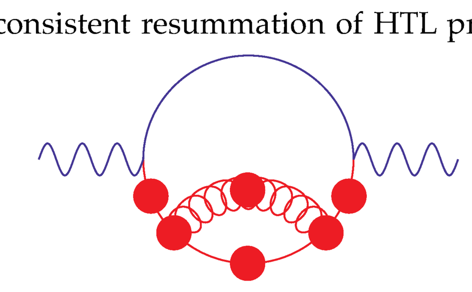
The soft region

At **leading order**



⇒ removes IR divergence of the hard region.

At NLO consistent resummation of HTL propagators and vertices



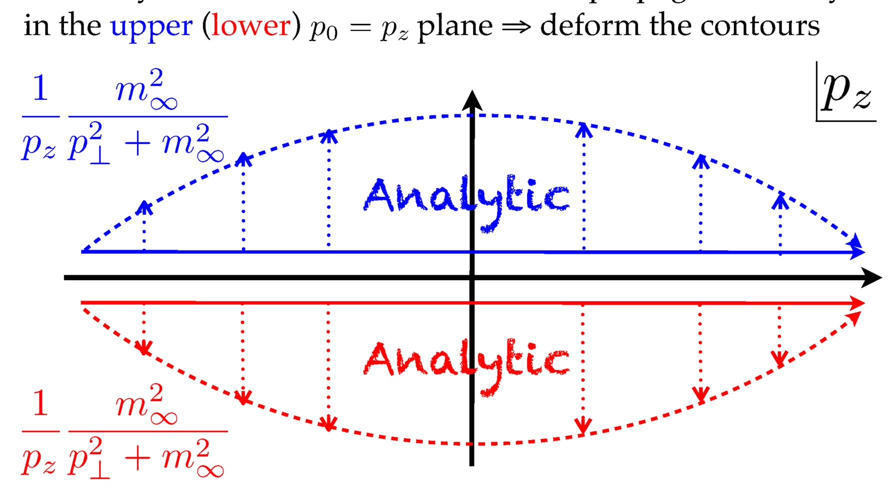
A daunting task! Unless there's a...

Fermionic sum rule

The LO soft diagram above ⇒ proportional to

$$\int\frac{dp_z}{2\pi}\left\{\left(1-\frac{p_z}{p}\right)\left[S_R^+(p_z,p)-S_A^+(p_z,p)\right]+\left(1+\frac{p_z}{p}\right)\left[S_R^-(p_z,p)-S_A^-(p_z,p)\right]\right\}$$

Causality ⇒ retarded (advanced) HTL propagators analytic in the upper (lower) $p_0=p_z$ plane ⇒ deform the contours



Dramatic simplification along the arcs ⇒ a *simple analytical result* (also obtained in [8]).

The same arguments at **NLO** ⇒ tremendous reduction of the complexity of the calculation:
- HTL vertices do not contribute on the arcs at infinity.
- The NLO analytical result is UV divergent
  - → $p_z\to\infty$: linear, cancels small p_z collinear NLO
  - ↗ $p_\perp\to\infty$: log, combines with semisoft IR (NLO log)

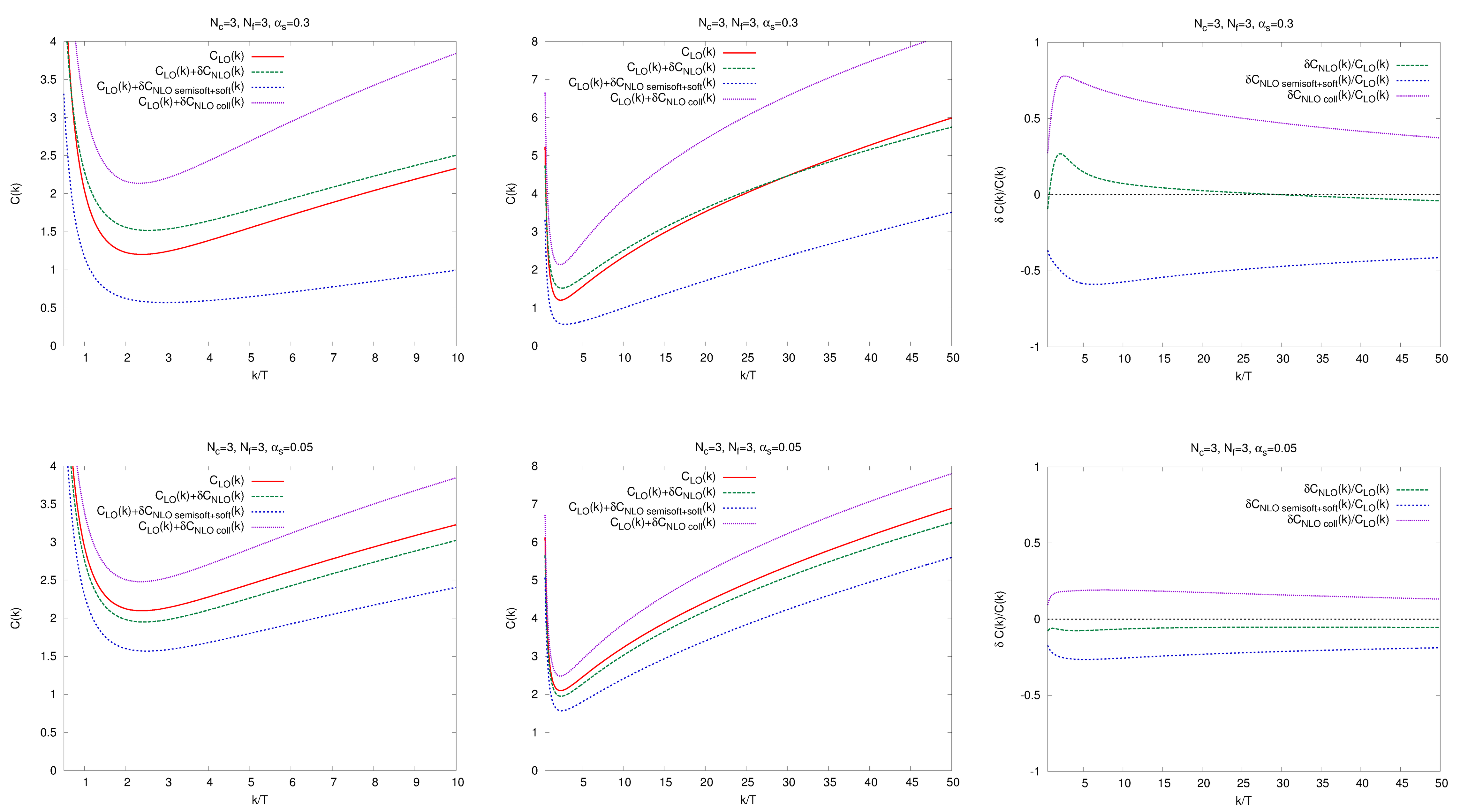
Results

- Leading order rate

$$(2\pi)^3\frac{d\Gamma}{d^3k}\bigg|_{\mathrm{LO}} = \mathcal{A}(k)\overbrace{\left[\log\frac{T}{m_\infty}+C_{2\to2}(k)+C_{\mathrm{coll}}(k)\right]}^{C_{\mathrm{LO}}(k)} \qquad \mathcal{A}(k)=\alpha_{\mathrm{EM}}\,g^2\,C_FT^2\frac{n_{\mathrm{F}}(k)}{2k}\sum_f Q_f^2d_f$$

- NLO correction to the rate

$$(2\pi)^3\frac{d\delta\Gamma}{d^3k}\bigg|_{\mathrm{NLO}} = \mathcal{A}(k)\overbrace{\Bigg[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty}+\frac{\delta m_\infty^2}{m_\infty^2}C_{\mathrm{semisoft}}(k)}_{\delta C_{\mathrm{NLO\,semisoft+soft}}(k)}+\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\mathrm{coll}}^{\delta m}(k)+\frac{g^2C_AT}{m_D}C_{\mathrm{coll}}^{\delta\mathcal{C}}(k)}_{\delta C_{\mathrm{NLO\,coll}}(k)}\Bigg]}^{\delta C_{\mathrm{NLO}}(k)}$$



Comments and conclusions

- The NLO contribution is made of four terms, with a semisoft/soft log
- These four terms combine in two large and opposite contributions that largely cancel giving a relatively small NLO correction
- Is the cancellation accidental? At $\alpha_s=0.3$ the NLO is initially positive, then turns negative and keeps growing at large k/T. At small α_s ($\alpha_s=0.05$) the correction is always negative
- Contrary to the heavy-quark diffusion case, here only $C_{\mathrm{coll}}^{\delta\mathcal{C}}(k)$ is purely non-abelian ⇒ this might explain observed relative smallness of NLO
- In the phenomenologically interesting window up to $k/T\sim10$ the NLO correction is 10%-20% for $\alpha_s=0.3$
- A lot of new and interesting technology has been developed: sum rules ⇒ help in tackling new NLO calculations

References

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