Debye screening mass to three-loop order in hot QCD

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I. Introduction

The **Debye mass** in hot QCD (for $N_\mathrm{f} = 0$) is determined to **three-loop** order as a matching coefficient of dimensionally reduced electrostatic QCD. The expansion in terms of both the strong coupling and the momentum requires the evaluation of **10^7 sum-integrals** which are reduced via IBP relations to a small number of master sum-integrals. These are then solved, using the method pioneered by Arnold and Zhai [1].

II. Dimensionally reduced effective QCD

Thermal QCD exhibits three different scales ($2\pi T$, gT and g^2T) of which the "ultra-soft" color-magnetic mode (g^2T) leads to a breakdown of the ordinary perturbative expansion [2].

The standard procedure for dealing with the scale hierarchy of thermal QCD is scale separation by isolating the soft and ultra-soft modes into 3d effective theories (EQCD and MQCD) [3]. In order to relate the effective theories to each other, a matching computation of the effective parameters is required. These theories can be studied non-perturbatively via lattice simulations.

The 3d **EQCD** Langrangian reads:

$$\begin{aligned}\mathcal{L}^{3d}_\mathrm{EQCD} &= -\frac{1}{4}F^a_{ij}F^a_{ij} + \mathrm{Tr}[D_i, A_0]^2 + \mathbf{m}^2_\mathrm{E}\mathrm{Tr}[A_0^2] \\ &\quad + \lambda_\mathrm{E}^{(1)}(\mathrm{Tr}[A_0^2])^2 + \ldots \\ D_i &= \partial_i - ig_\mathrm{E}A_i, \quad i,j = 1,2,3,\end{aligned}$$

where A_0 and A_i now correspond to the original electrostatic and magnetostatic gluon fields of QCD. Our task is to compute the effective mass parameter of EQCD m_E, to 3-loop order.

III. Matching computation

The general prescription is to require that various static quantities computed in both theories, match to the given order in a strict perturbative expansion with respect to the gauge coupling g. By using the background field gauge, we make sure that only the coupling constant g has to be renormalized in the end.

In practice, we define the screening mass m_el as the pole of the propagator of A_0 at $\mathbf{p}^2 = -m^2_\mathrm{el}$ and $p_0 = 0$. On the QCD side, we therefore have:

$$p^2 + \Pi_{00}(p^2) = 0$$

and on the EQCD side:

$$\mathbf{p}^2 + m^2_\mathrm{E,ren} + \delta m^2_\mathrm{E} + \Pi_\mathrm{EQCD}(\mathbf{p}^2) = 0,$$

where the mass parameter has to be renormalized here, due to known UV divergences in 3d EQCD.

The self-energy is written as an **expansion** in both the gauge coupling g and in the external momentum $\mathbf{p}$. The latter is justified due to the soft scale $p \propto gT$ at which the pole in the propagator arises.

$$\Pi^\mathrm{QCD}_{\mu\nu}(\mathbf{p}^2) = \sum_{n=1}^{\infty} \underset{\substack{\uparrow \\ n-\text{loop} \\ \text{vacuum sum-integrals}}}{\Pi_{\mu\nu,n}(0)}\,(g^2)^n + \mathbf{p}^2 \sum_{n=1}^{\infty} \underset{\substack{\uparrow \\ n-\text{loop} \\ \text{vacuum sum-integrals}}}{\Pi'_{\mu\nu,n}(0)}\,(g^2)^n + \ldots$$

This leads to the following expression for the screening mass:

$$\begin{aligned} m^2_\mathrm{el} &= g^2\Pi_1(0) + g^4\left[\Pi_2(0) - \Pi_1'(0)\Pi_1(0)\right] \\ &\quad + g^6\Big[\Pi_3(0) - \Pi_1'(0)\Pi_2(0) - \Pi_2'(0)\Pi_1(0) \\ &\quad + \Pi_1''(0)\Pi_1(0)^2 + \Pi_1(0)\Pi_1'(0)^2\Big] + \mathcal{O}(g^8). \end{aligned}$$

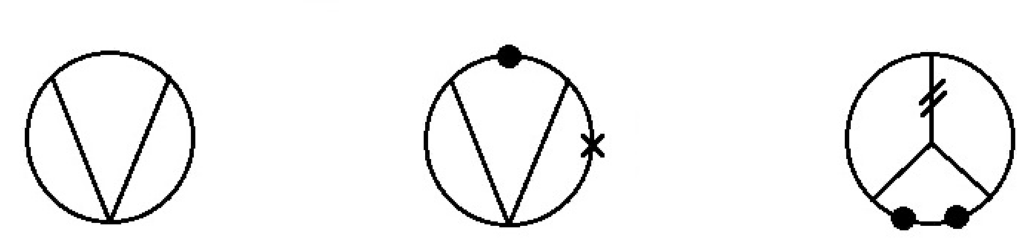
On the EQCD side, the m_E term is treated as a perturbation, leading to a massless tree-level A_0 propagator. By performing a Taylor expansion in $\mathbf{p}$, all the vacuum integrals vanish in dimensional regularization to all orders due to the absence of any other scale in the integrals: $\Pi_\mathrm{EQCD} = 0$.
By matching both computations, we obtain:

$$m_\mathrm{E,ren} = m_\mathrm{el} - \delta m_\mathrm{E}$$

IV. Evaluation of master sum-integrals

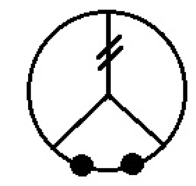
The evaluation of the QCD self-energy tensor $\Pi_{\mu\nu}(P)$ to three-loop order generates **$\approx$ 500 Feynman diagrams**. Therefore an automatized procedure is needed to handle such a tremendous task. Generation of the Feynman diagrams, the color algebra computation of $SU(N_c)$, the Lorentz contraction and the Taylor expansion into external momentum are performed using specific software (QGRAF, FORM). The remaining **$\approx 10^7$ sum-integrals** are reduced via IBP relations [4] to a set of $\approx$10 nontrivial 3-loop sum-integrals and other products of 1-loop sum-integrals [5].

Since the **IBP reduction** generates divergent pre-factors of $\mathcal{O}(\epsilon^{-2})$ for the master sum-integrals, a non-trivial basis transformation is performed to eliminate those. The remaining non-trivial 3-loop master sum-integrals are (the first has been computed in Ref. [6]):



For solving the remaining sum-integrals, we use the procedure pioneered by Arnold and Zhai [1], in which the 1-loop substructure of the sum-integrals is exploited.

- We improved the general prescription for extracting the divergent parts from the sum-integrals. This made it possible to develop a "semi-automatized" procedure for an analytic calculation of the divergent parts of a large class of V-type sum-integrals.
- Highly IR divergent parts are computed via further IBP reductions.
- The Mercedes type sum-integral with one inverse propagator is simplified using the dimensional method of Tarasov [7].



$$= -\frac{5}{36}\,\frac{T^2}{(4\pi)^4}\left(\frac{\mu^2}{4\pi T^2}\right)^{3\epsilon}\frac{1}{\epsilon^2}\times \\ \times\left[1 + \epsilon\left(\frac{71}{30} + \gamma_E + 2\frac{\zeta'(-1)}{\zeta(-1)}\right) + \mathcal{O}(\epsilon^2)\right]$$

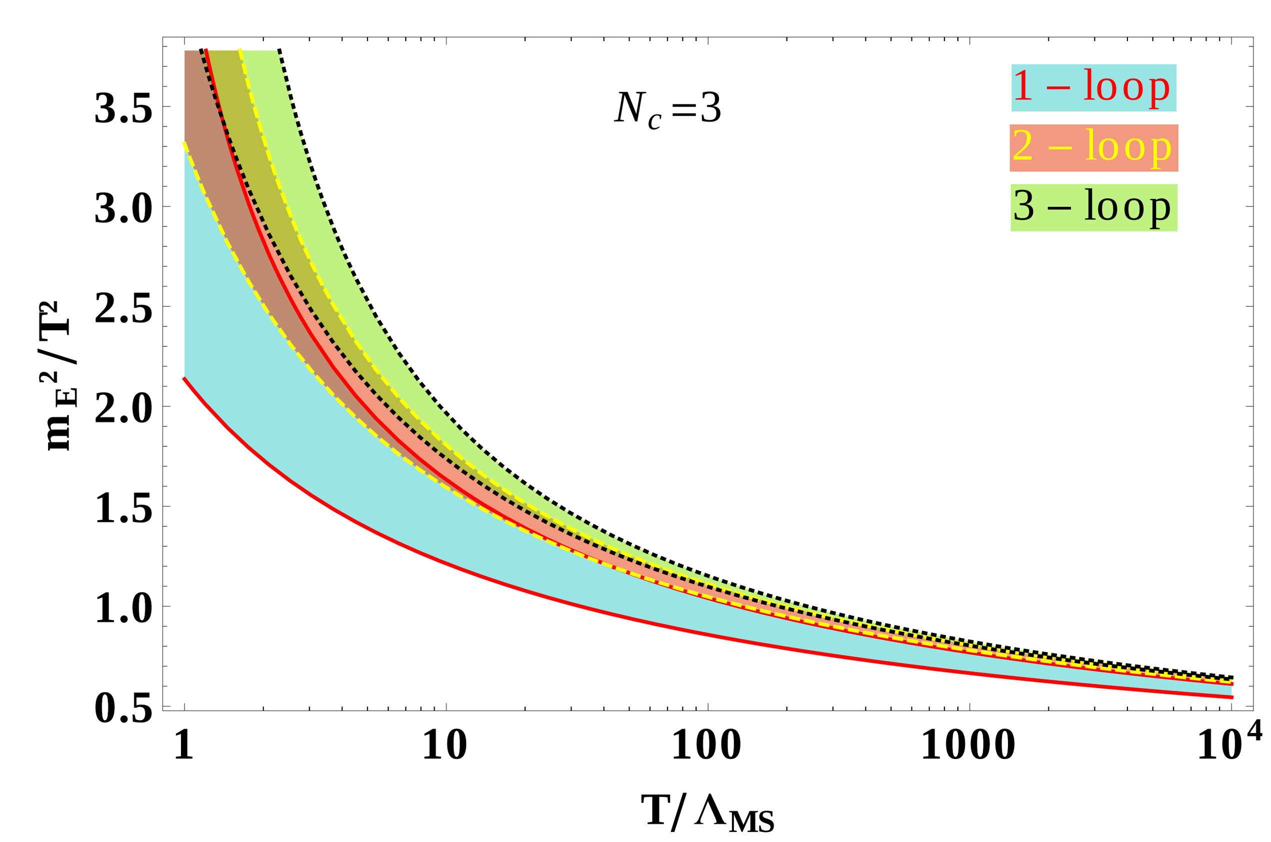
V. Results

After renormalizing the QCD gauge coupling (here $g \equiv g_\mathrm{R}$), the **renormalized Debye mass** up to 3-loop order is given by:

$$\begin{aligned}\mathbf{m}^2_\mathrm{E,ren} = T^2g^2(\bar\mu)\frac{C_A}{3}\Bigg\{&1 + \frac{g^2(\bar\mu)}{(4\pi)^2}\frac{C_A}{3}\left(22\ln\frac{\bar\mu e^{\gamma_\mathrm{E}}}{4\pi T} + 5\right) \\ &+ \frac{g^4(\bar\mu)}{(4\pi)^4}\left(\frac{C_A}{3}\right)^2\Bigg(484\ln^2\frac{\bar\mu e^{\gamma_\mathrm{E}}}{T} - 116\ln\frac{\bar\mu e^{\gamma_\mathrm{E}}}{T} + \frac{1091}{2} \\ &-144\gamma_\mathrm{E} + 324\ln 4\pi - 180\frac{\zeta'(-1)}{\zeta(-1)} - \frac{56}{5}\zeta(3)\Bigg) + \mathcal{O}(g^6)\Bigg\}\end{aligned}$$

where the 1-loop and 2-loop contributions have been calculated in Ref. [8].

For analysing the running of the Debye mass with respect to the temperature (T), we have used the solution of the RGE equation of the 4d coupling g, in which the QCD β-function was truncated after $\mathcal{O}(g^8)$. The parameter $\Lambda_{\overline{\mathrm{MS}}}$ corresponds to the QCD scale defined in Ref. [8] ($\Lambda_{\overline{\mathrm{MS}}} \approx 200$ MeV). The arbitrary scale $\bar\mu$, was chosen at the point where the effective coupling g_E has a minimal sensitivity to it: $\bar\mu_\mathrm{opt}/T \approx 2\pi$. The 3 bands in the plot arise by varying $\bar\mu = (0.5\ldots2.0)\times\bar\mu_\mathrm{opt}$.



The plot shows a slight **increase** of the Debye mass with respect to the 2-loop result. In addition, the sensitivity with respect to the arbitrary scale $\bar\mu$ decreases, which indicates that the perturbative expansion up to 3-loop order shows **good convergence** properties.

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