# Chiral Superfluidity of the Quark-Gluon Plasma

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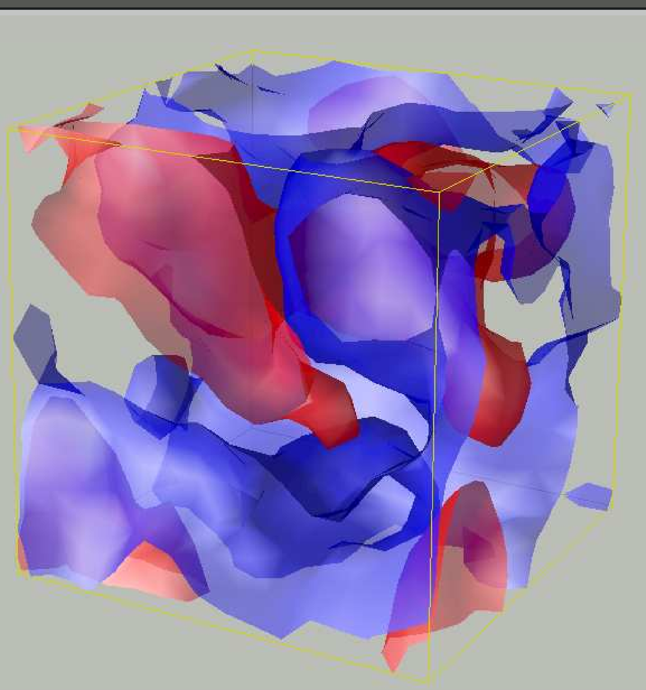
## Motivation

- **Irregular structure of the QCD vacuum**
  - fractal distribution of the topological charge density
  - low-dimensional defects
- **Effective model for sQGP**
  - effective Lagrangian
  - two-component fluid
- **Axion-like field within QCD**



## Bosonization with a finite cut-off

Euclidean functional integral for $\mathrm{SU(N_c)} \times \mathrm{U_{em}(1)}$ is given by

$$\int D\bar{\psi} D\psi \, \exp\left\{ - \int_V d^4x \, \bar{\psi}(\not{D} - im)\psi + \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5)$$

- integrate out quarks below a cut-off Dirac eigenvalue $\Lambda$
- add gauge-invariant terms to the Lagrangian to match the chiral anomaly
- consider a pure gauge $A_{5\mu} = \partial_\mu \theta$ for the auxiliary axial field
- and the chiral limit $m \longrightarrow 0$

The **total effective Euclidean Lagrangian** reads as

$$\begin{aligned} \mathcal{L}_E^{(4)} = & \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu \\ & + \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu\theta \partial_\mu\theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \widetilde{G}^a_{\mu\nu} + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \widetilde{F}_{\mu\nu} \\ & + \frac{N_c}{24\pi^2} \theta \Box^2 \theta - \frac{N_c}{12\pi^2} (\partial^\mu\theta \partial_\mu\theta)^2 \end{aligned}$$

So we get an axion-like field with decay constant $f = \frac{2\Lambda}{\pi}\sqrt{N_c}$ and a negligible mass $m_\theta^2 = \lim_{V\to\infty} \frac{\langle Q^2 \rangle}{f^2 V} \equiv \chi(T)/f^2$. We tend to interpret it as a quasiparticle moving along the low-dimensional defects!

## Interpretation of the scale Λ

From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = \frac{1}{2}\left(\frac{\Lambda}{\pi}\right)^2 \mu_5 + \frac{1}{3\pi^2}\mu_5^3$$

- Free quarks (see 0808.3382): $\Lambda = \pi\sqrt{\frac{2}{3}}\sqrt{T^2 + \frac{\mu_5^2}{\pi^2}} \propto \rho_{sphaleron}^{-1}$
- Free quarks and strong B-field: $\Lambda = 2\sqrt{|eB|}$
- Dynamical lattice fermions (1105.0385): $\Lambda \simeq 3\,\mathrm{GeV} \gg \Lambda_{QCD}$

## Change in entropy and higher order gradient corrections

The terms $\tau^{\mu\nu}$, $\nu^\mu$ and $\nu_5^\mu$ denote higher-order gradient corrections and obey the Landau conditions

$$u_\mu \tau^{\mu\nu} = 0, \quad u_\mu \nu^\mu = 0, \quad u_\mu \nu_5^\mu = 0.$$

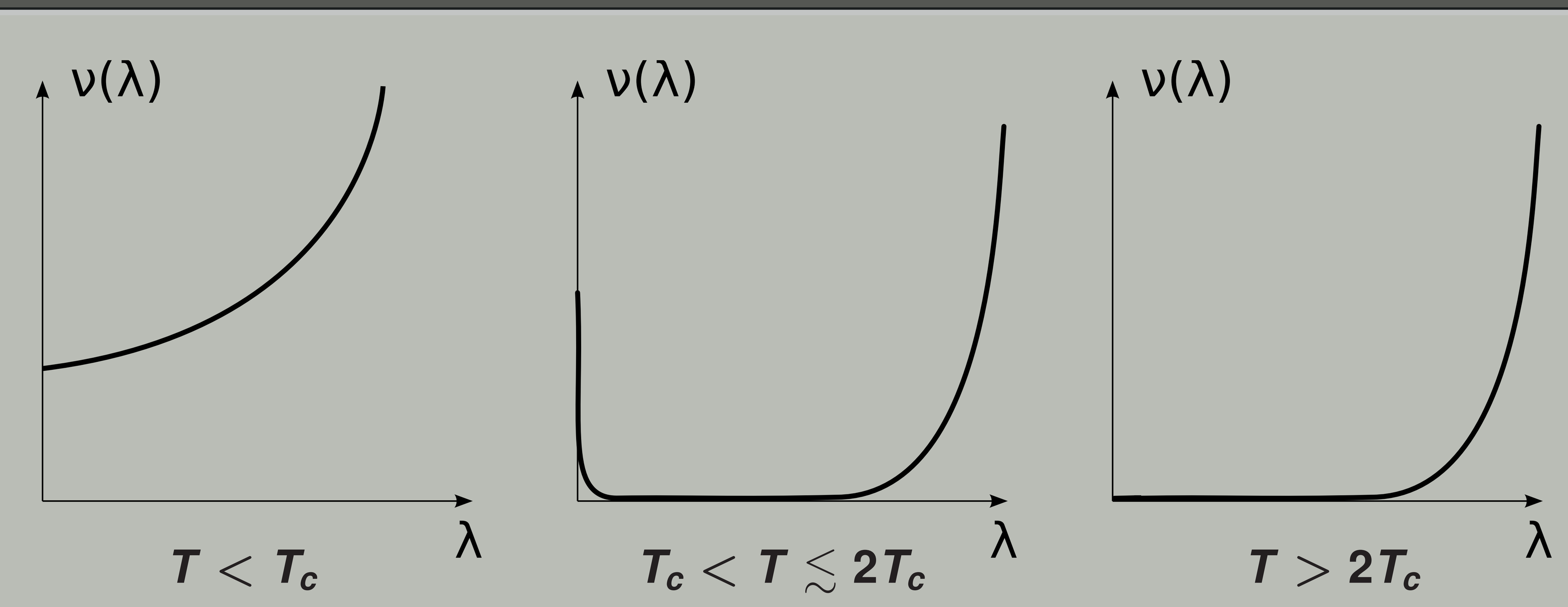
Using both hydrodynamic equations and constitutive relations one can derive

$$\partial_\mu\left(s u^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu_5}{T}\nu_5^\mu\right) = -\frac{1}{T}(\partial_\mu u_\nu)\tau^{\mu\nu} - \nu^\mu\left(\partial_\mu \frac{\mu}{T} - \frac{1}{T}E_\mu\right) - \nu_5^\mu \partial_\mu \frac{\mu_5}{T}.$$

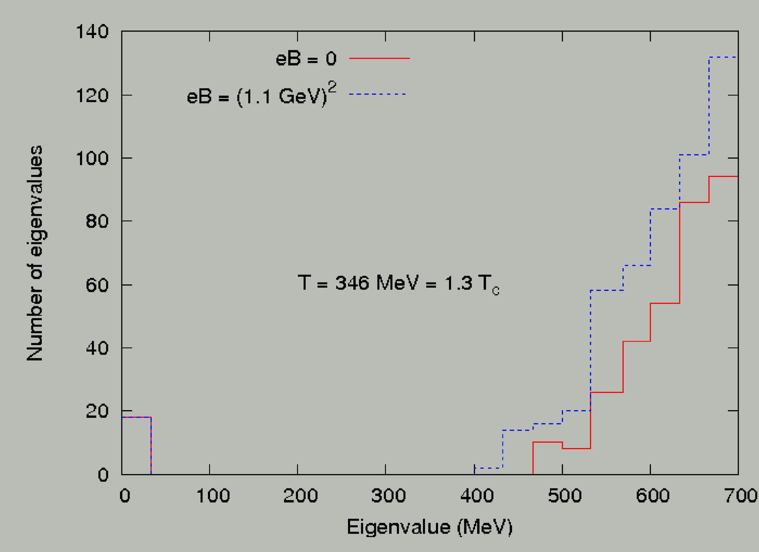
so the entropy production is always nonnegative. This fact tells us that

- we don't need to add any additional terms to the entropy current like in Son and Surowka (0906.5044)
- no additional first-order corrections to the currents.
- in absence of dissipative corrections we obtain $\partial_\mu(s u^\mu) = 0$, i.e. only the "normal" component cotributes to the entropy current, while the "superfluid" component has zero entropy.

## Fermionic spectrum at finite temperature




- there are two separated parts of the spectrum at intermediate temperatures!
- a strong external magnetic field does not destroy the picture
- all the chiral properties are described by the near zero modes




## Hydrodynamic equations

Equations of motion for the quadratic effective Minkowski Lagrangian

$$\begin{aligned} \partial^\mu \partial_\mu \theta &= \frac{C}{4f^2} F^{\mu\nu} \widetilde{F}_{\mu\nu} + \frac{g^2}{32\pi^2 f^2} G_a^{\mu\nu} \widetilde{G}^a_{\mu\nu}, \\ \partial_\mu F^{\mu\nu} &= -j^\nu + C(\partial_\sigma\theta)\widetilde{F}^{\sigma\nu}, \\ \partial_\mu \widetilde{F}^{\mu\nu} &= 0. \end{aligned}$$

Varying the quadratic Lagrangian with respect to axial-vector $A_{5\mu} = \partial_\mu\theta$ we obtain the axial current $j_5^\mu = -f^2 \partial^\mu\theta$ (curl-free!). Conservation law $\partial_\mu(T^{\mu\nu} + \Theta^{\mu\nu}) = 0$ makes it possible to express divergency of the fluid energy-momentum tensor $T^{\mu\nu}$ via the one of the electromagnetic stress-energy tensor $\Theta^{\mu\nu} = F^{\mu\lambda}F^\nu{}_\lambda - \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$.

In summary, the hydrodynamic equations are

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu\lambda}(j_\lambda + C\widetilde{F}_{\lambda\sigma}\partial^\sigma\theta) \equiv F^{\nu\lambda}(j_\lambda + j_{S\lambda}), \\ \partial_\mu j_5^\mu &= -\frac{C}{4} F^{\mu\nu}\widetilde{F}_{\mu\nu} - \frac{g^2}{32\pi^2} G_a^{\mu\nu}\widetilde{G}^a_{\mu\nu}, \\ \partial_\mu j^\mu &= 0, \end{aligned}$$

plus the Josephson equation $u^\mu \partial_\mu \theta + \mu_5 = 0$.
Corresponding constitutive relations in gradient expansion are

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu\theta \partial^\nu\theta + \tau^{\mu\nu}, \\ j^\mu &= \rho u^\mu + \nu^\mu, \\ j_5^\mu &= -f^2 \partial^\mu\theta + \nu_5^\mu \end{aligned}$$

The stress-energy tensor $T^{\mu\nu}$ consists of two parts, an ordinary fluid component and a pseudoscalar "superfluid" component. This modifies the equation of state by adding to the r.h.s. a new $\theta$-dependent term

$$dP = s dT + \rho d\mu - f^2 d\left[\frac{1}{2}\partial^\mu\theta \, \partial_\mu\theta\right],$$

where $s$ is the entropy density.

## Phenomenological output

Electric and magnetic fields in the fluid rest frame are defined as

$$E^\mu = F^{\mu\nu}u_\nu, \qquad B^\mu = \tilde{F}^{\mu\nu}u_\nu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$$

An additional electric current, induced by $\theta$-field

$$j_\lambda^S = C\widetilde{F}_{\lambda\sigma}\partial^\sigma\theta = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\sigma\beta}u^\alpha \partial_\sigma\theta E_\beta - u_\lambda(\partial\theta \cdot B)$$

- I term: Chiral Magnetic Effect (electric current along B-field)
- II term: Chiral Electric Effect (electric current transverse to E-field and to both normal and superfluid velocities)
- III term: Chiral Dipole Wave (dipole moment induced by B-field)
- The field $\theta(\vec{x}, t)$ itself: Chiral Magnetic Wave (propagating imbalance between the number of left- and right-handed quarks)

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