we don’t need to add any additional terms to the entropy current
all the chiral properties are

- III term: Chiral Dipole Wave (dipole moment induced by B-field)
- I term: Chiral Magnetic Effect (electric current along B-field)

Effective Lagrangian
two-component fluid
low-dimensional defects
Effective model for sQGP
- total effective Euclidean Lagrangian
- Free quarks and strong B-field:

\[ \theta = 0 \]

So the entropy production is always nonnegative. This fact tells us that
and obey the Landau conditions

- Free quarks (see 0808.3382): \( \Lambda = \sqrt{2}\sqrt{T^2 + \frac{\mu^2}{T^2}} \propto \rho_{\text{sphaleron}} \)
- Free quarks and strong B-field: \( \Lambda = 2\sqrt{\|B\|} \)
- Dynamical lattice fermions (1105.0385): \( \Lambda \approx 3 \text{ GeV} > \Lambda_{\text{QCD}} \)

Change in entropy and higher order gradient corrections

The terms \( \tau^{\mu\nu} \), \( \nu^{\mu} \) and \( \nu^{I} \) denote higher-order gradient corrections and obey the Landau conditions

\[ u_{\tau^{\mu\nu}} = 0, \quad u_{\nu^{\mu}} = 0, \quad u_{\nu^{I}} = 0 \]

Using both hydrodynamic equations and constitutive relations one can derive

\[ \frac{\partial}{\partial t}(\rho u^\mu + \frac{\mu}{T} u^{I}) = -\frac{1}{T}(\partial_{\mu}u_{\nu}+\nu^{\nu}(\partial_{\mu}\rho_{\tau}-\frac{1}{T}E_{\mu})-\nu^{I}\partial_{\mu}u^{I} - T) \]

so the entropy production is always nonnegative. This fact tells us that
- \( \tau^{\mu\nu} \) and \( \nu^{\mu} \) and \( \nu^{I} \) have no new additional orders to the currents.
- In absence of dissipative corrections we obtain \( \partial_{\mu}(u_{\tau^{\mu\nu}} + \frac{\mu}{T} u^{I}) = 0 \), i.e.
only the “normal” component contributes to the entropy current, while the “superfluid” component has zero entropy.

Fermionic spectrum at finite temperature

\[ \nu(\lambda) \]

there are two separated parts of the spectrum at intermediate temperatures!
- a strong external magnetic field does not destroy the picture
- all the chiral properties are described by the near zero modes

Hydrodynamic equations

Equations of motion for the quadratic effective Minkowski Lagrangian

\[ \partial^{\nu}\partial^{\mu}\theta = \frac{C}{4f_{B}^{2}} F_{\mu\nu} F_{\lambda}^{\mu} + \frac{g^{2}}{32\pi^{2}} f_{5}^{2} g_{\mu\nu} g_{\lambda}^{2}, \]

\[ \partial_{\mu}F_{\mu\nu} = -f^{\nu\theta} + C(\partial_{\lambda}\theta) F_{\mu\nu}, \]

\[ \partial_{\mu}F_{\mu\nu} = 0. \]

Varying the quadratic Lagrangian with respect to axial-vector \( A_{\mu} = \partial_{\mu}\theta \) we obtain the axial current \( j_{x} = -f^{\nu\theta} \) (curl-free).

Conservation law \( \partial_{\mu}(T^{\mu\nu} + \theta^{\nu}) = 0 \) makes it possible to express divergency of the fluid energy-momentum tensor \( T^{\mu\nu} \) via the one of the electromagnetic stress-energy tensor \( \theta^{\mu\nu} = F_{\mu\lambda} F_{\nu}^{\lambda} - \frac{1}{2}g_{\mu\nu} F^{2} \).

In summary, the hydrodynamic equations are

\[ \partial_{\mu}T^{\mu\nu} = F^{\tau\lambda}(j_{\lambda} + C F_{\lambda\nu}^{\sigma}u^{\nu}) \equiv F^{\nu\lambda}(j_{\lambda} + j_{\beta}^{S}), \]

\[ \partial_{\nu}j_{\nu}^{S} = \frac{C}{4f_{B}^{2}} F_{\mu\nu} F_{\lambda}^{\mu} - \frac{g^{2}}{32\pi^{2}} f_{5}^{2} g_{\mu\nu} g_{\lambda}^{2}, \]

\[ \partial_{\mu}j_{\mu}^{S} = 0. \]

plus the Josephson equation \( u_{\nu} \partial_{\nu}\theta + G_{\mu\nu} = 0 \). Corresponding constitutive relations in gradient expansion are

\[ T^{\mu\nu} = (x + P) u^{\mu} u^{\nu} + P u^{\mu} u^{\nu} + \tau^{\mu\nu} \theta^{\nu}, \]

\[ j_{\mu}^{S} = \frac{C}{4f_{B}^{2}} F_{\mu\nu} F_{\lambda}^{\mu} - \frac{g^{2}}{32\pi^{2}} f_{5}^{2} g_{\mu\nu} g_{\lambda}^{2}, \]

\[ j_{\mu}^{S} = -\tau^{\mu\nu} \theta^{\nu} + \nu^{\mu}_{5}. \]

The stress-energy tensor \( T^{\mu\nu} \) consists of two parts, an ordinary fluid component and a pseudoscalar “superfluid” component. This modifies the equation of state by adding to the r.h.s. a new \( \theta \)-dependent term

\[ dP = s dT + \rho d\mu - \frac{1}{2} d\left[ \frac{1}{2} (\partial_{\mu}u_{\nu} + \frac{\mu}{T} u^{I}) \right], \]

where \( s \) is the entropy density.

Phenomenological output

Electric and magnetic fields in the fluid rest frame are defined as

\[ E^{\mu} = F^{\mu\nu} u_{\nu}, \quad B^{\mu} = F^{\mu\nu} u_{\nu} + \frac{1}{2} \mu^{\nu\alpha\beta} u_{\alpha} F_{\beta} \]

An additional electric current, induced by \( \theta \)-field

\[ j_{5}^{S} = C F_{\tau\lambda} \theta^{\lambda} = -C_{\mu\lambda} B_{\lambda} + C_{\tau\mu\nu} u^{\nu} \partial_{\tau} E_{\lambda} - \rho_{\lambda} (\partial_{\mu} B). \]

- I term: Chiral Magnetic Effect (electric current along B-field)
- II term: Chiral Electric Effect (electric current transverse to E-field and to both normal and superfluid velocities)
- III term: Chiral Dipole Wave (dipole moment induced by B-field)
- The field \( \theta(x, t) \) itself: Chiral Magnetic Wave (propagating imbalance between the number of left- and right-handed quarks)