$NRQC_2D$ in Non-zero Baryon Density Environment

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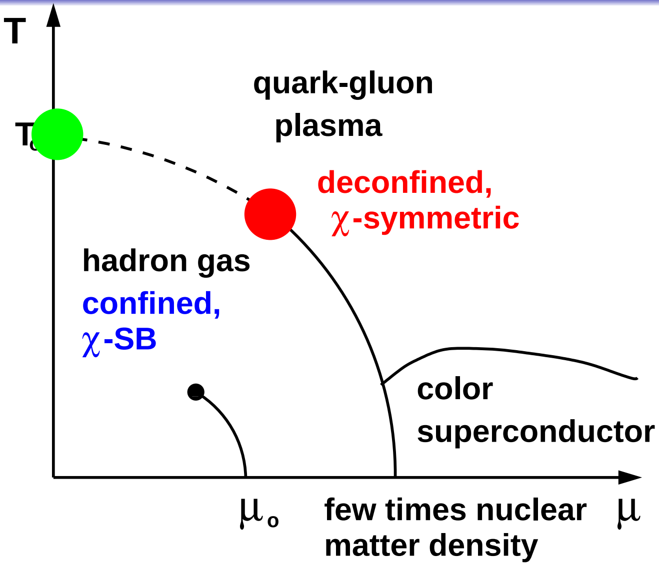
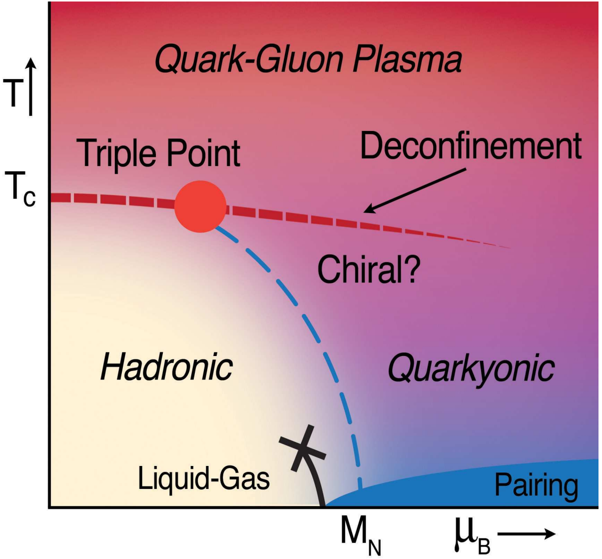
based on PLB 711(2012) 199, arXiv:1202.4353

Outline

QCD Phase Diagram



QCD Phase Diagram



from arXiv:0911.4806

QCD Phase Diagram

- lattice study of QCD in baryon rich environment is difficult due to "complex action" problem:

$$\mathcal{L}_q = \overline{\psi}(\not{D} + m + \mu\gamma_4)\psi \tag{1}$$

With grassmann integral,

$$\rightarrow \det(\not{D} + m + \mu\gamma_4) \tag{2}$$

Define $M(\mu) = \not{D} + m + \mu\gamma_4$. Then

$$\det(M(\mu)) = \det(M(\mu)\gamma_5^2) = \det(\gamma_5 M(\mu)\gamma_5) = \det(-\not{D} + m - \mu\gamma_4) \tag{3}$$

QCD Phase Diagram

$$\det(M(\mu = 0)) = \det(M(\mu = 0)^{\dagger}) = \det(M(\mu = 0)^{*}) \qquad (1)$$

In general,

$$\det(M(\mu)) = \det(M(-\mu)^{\dagger}) = \det(M(-\mu)^{*}) \qquad (2)$$

• “complex action” means that the usual “importance sampling” of Monte Carlo method has exponential difficulty in converging to a correct answer

2-Color QCD

- QCD-like theory: two-color QCD or QCD with adjoint quarks
  (Kogut et al, Nucl. Phys. B582 (2000) 477)

- Consider $N_f = 2$ Wilson fermion, two-color QCD

$$\mathcal{S} = \overline{\psi}_1 M(\mu)\psi_1 + \overline{\psi}_2 M(\mu)\psi_2 - J\bar{\psi}_1(C\gamma_5)\tau_2\bar{\psi}_2^{tr} + \bar{J}\psi_2^{tr}(C\gamma_5)\tau_2\psi_1, \quad (3)$$

where

$$M_{xy}(\mu) = \delta_{xy} - \kappa\sum_{\nu}\Big[(1-\gamma_\nu)e^{\mu\delta_{\nu 0}}U_\nu(x)\delta_{y,x+\hat{\nu}} + (1+\gamma_\nu)e^{-\mu\delta_{\nu 0}}U_\nu^\dagger(y)\delta_{y,x-\hat{\nu}}\Big]. \quad (4)$$

- $\det M(\mu)$ is real (but doesn't mean that it is positive)

$$\gamma_5 M(\mu)\gamma_5 = M^\dagger(-\mu) \quad (5)$$

$$(C\gamma_5)\tau_2 M(\mu)(C\gamma_5)^{-1}\tau_2 = M^*(\mu) \quad (6)$$

2-Color QCD

• Grassman variable integration turns the fermion part of the action into a determinant

$$\int d\overline{\psi} d\psi e^{-S_F} \rightarrow \mathrm{det} M(\mu) \tag{3}$$

since $\mathrm{det}M(\mu) = \mathrm{det}M^*(\mu)$, $\mathrm{det}M(\mu)$ is real

• redefine $\overline{\phi} = -\psi^{\mathrm{tr}}{}_2 C \tau_2, \phi = C^{-1} \tau_2 \psi^{\mathrm{tr}}{}_2$

$$\mathcal{S} = (\bar{\psi} \quad \bar{\phi}) \begin{pmatrix} M(\mu) & J\gamma_5 \\ -\bar{J}\gamma_5 & M(-\mu) \end{pmatrix} \begin{pmatrix} \psi \\ \phi \end{pmatrix} \equiv \bar{\Psi} \mathcal{M} \Psi. \tag{4}$$

2-Color QCD

- With the redefinition

$$\mathcal{M}^\dagger\mathcal{M} = \begin{pmatrix} M^\dagger(\mu)M(\mu)+|\bar{J}|^2 & \\ & M^\dagger(-\mu)M(-\mu)+|J|^2 \end{pmatrix} \tag{3}$$

- Monte Carlo feasible despite the quark chemical potential

2-Color QCD

- $\mathrm{QC_2D}$ is asymptotically free

- There is spontaneous chiral symmetry breaking
  $\rightarrow$ pion is light

- But qq is a color singlet $\rightarrow$ diquark condensate does not break color symmetry

- If gluon dynamics is important for QCD phase diagram, $\mathrm{QC_2D}$ is close to QCD in $1/N_c$ sense.

Lattice NRQC$_2$D

• calculate quarkonium correlator in the background color gauge field which has dynamical light quark effect and non-zero temperature/non-zero baryon density effect

• UKQCD, DiRAC, DEISA lattices at $\beta = 1.9, \kappa = 0.168$ ($a = 0.186(8)$fm = 1/1.060(45) GeV^{-1}, $m_\pi a = 0.68(1), m_\pi/m_\rho = 0.80(1)$

N_s	N_t	j	μ	No. of Conf.
12	24	0.02-0.04	0.0 - 1.10	~ 50
12	16	0.04	0.0 – 0.90	$100 \sim 500$
16	12	0.04	0.0 – 0.60	$200 \sim 1000$

Table: summary for the lattice data set

• Two flavor $SU(2)$ unimproved Wilson fermion, unimproved Wilson gauge action

Lattice NRQC$_2$D

- Non-relativistic QCD in FT

$$G(\vec{x}, t=0) = S(x) \tag{3}$$

$$G(\vec{x}, t=1) = \left[1 + \frac{1}{2n}\,\frac{\vec{D}^2}{2m_b^0}\right]^n U_4^\dagger(\vec{x}, t)\left[1 + \frac{1}{2n}\,\frac{\vec{D}^2}{2m_b^0}\right]^n G(\vec{x}, 0) \tag{4}$$

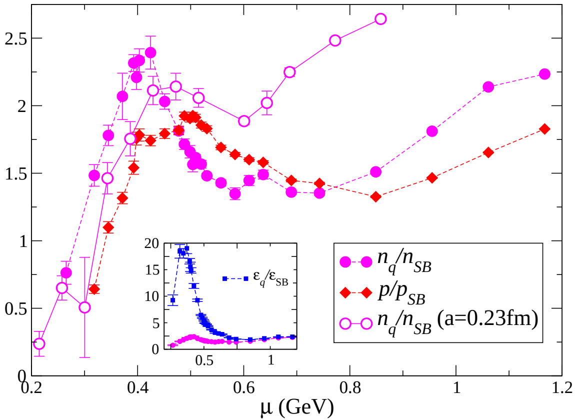
$$G(\vec{x}, t+1) = \left[1 + \frac{1}{2n}\,\frac{\vec{D}^2}{2m_b^0}\right]^n U_4^\dagger(\vec{x}, t)\left[1 + \frac{1}{2n}\,\frac{\vec{D}^2}{2m_b^0}\right]^n [1 - \delta H]\, G(\vec{x}, t) \tag{5}$$

Lattice NRQC$_2$D

where $S(x)$ is the source and

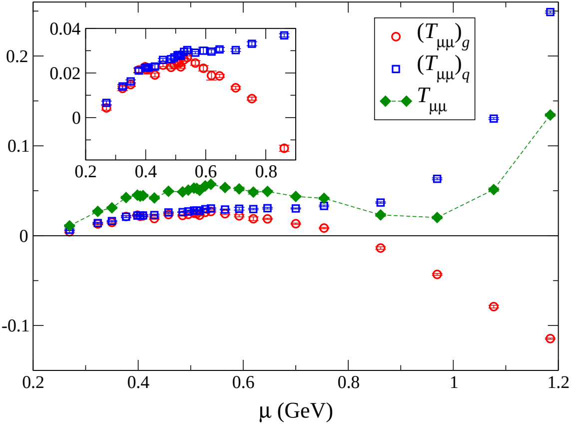
$$
\begin{aligned}
\delta H &= -\frac{(\vec{D}^{(2)})^2}{8(m_b^0)^3} + \frac{ig}{8(m_b^0)^2}(\vec{D}\cdot\vec{E} - \vec{E}\cdot\vec{D}) \\
&- \frac{g}{8(m_b^0)^2}\vec{\sigma}\cdot(\vec{D}\times\vec{E} - \vec{E}\times\vec{D}) - \frac{g}{2m_b^0}\vec{\sigma}\cdot\vec{B} \\
&+ \frac{a^2\vec{D}^{(4)}}{24m_b^0} - \frac{a(\vec{D}^{(2)})^2}{16n(m_b^0)^2}
\end{aligned}
\tag{3}
$$

remind you



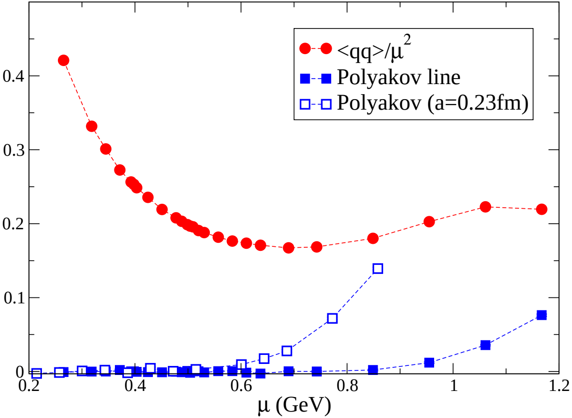
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
μ (GeV)
20
15
10
5
0
0.5
1
ε_q/ε_SB
n_q/n_SB
p/p_SB
n_q/n_SB (a=0.23fm)

remind you

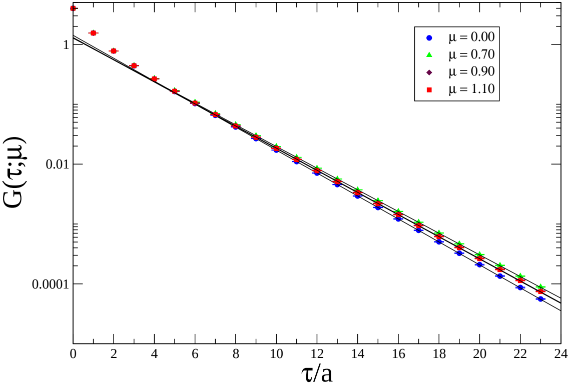


0.04
0.02
0
0.2
0.4
0.6
0.8
$(T_{\mu\mu})_g$
$(T_{\mu\mu})_q$
$T_{\mu\mu}$
0.2
0.1
0
-0.1
0.2
0.4
0.6
0.8
1
1.2
μ (GeV)

remind you

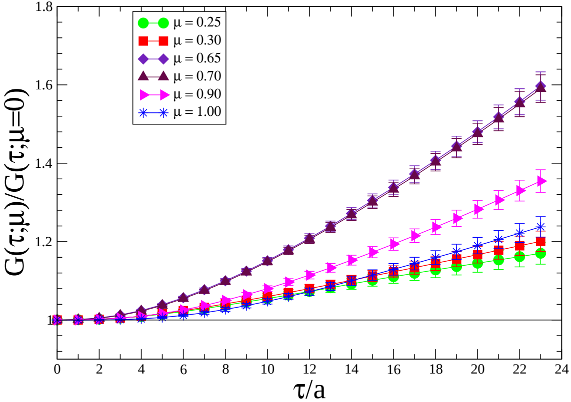

<qq>/μ^2
Polyakov line
Polyakov (a=0.23fm)
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
1
1.2
μ (GeV)

S- and P-wave Correlators



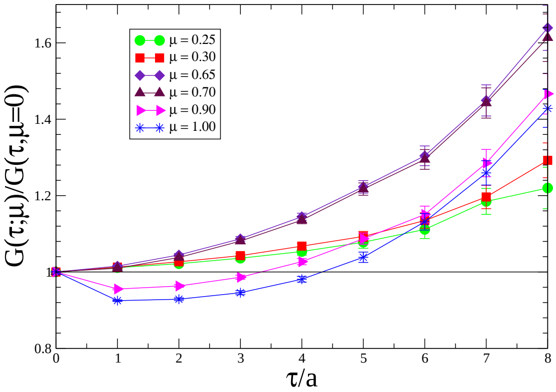
- correlators for the ${}^1S_0$ state with heavy quark mass $Ma = 5.0$ and $j = 0.02$ on $12^3 \times 24$ lattice together with single exponential fit

S- and P-wave Correlators



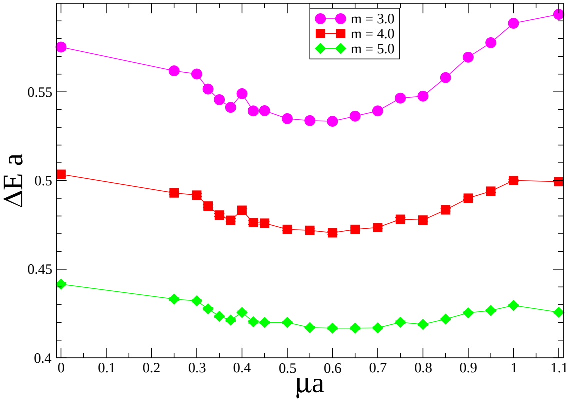
- the ratio $\sum_{\vec{x}} G(\vec{x},\tau;\mu) / \sum_{\vec{x}} G(\vec{x},\tau;0)$ for ${}^1S_0$ correlators $12^3 \times 24$ with $Ma = 5.0$

S- and P-wave Correlators



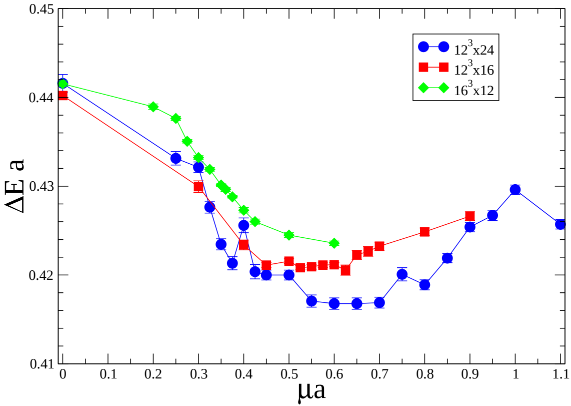
- the ratio $\sum_{\vec{x}} G(\vec{x},\tau;\mu)/\sum_{\vec{x}} G(\vec{x},\tau;0)$ for ${}^1P_0$ correlators on $12^3 \times 24$ with $Ma = 5.0$. Due to the noisiness of the P-wave data, only a limited τ range is shown

fitted mass of S-wave



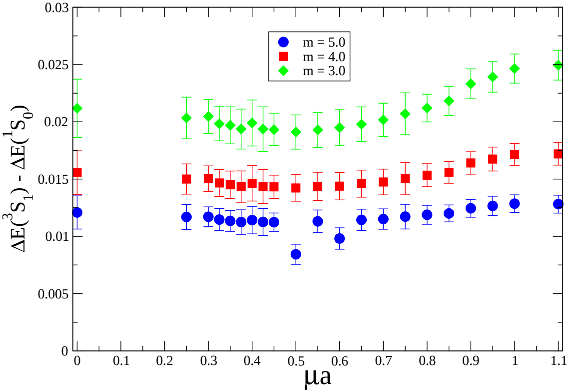
- energy of the $^1S_0$ state vs. quark chemical potential μ for heavy quark mass $Ma = 3.0, 4.0$ and 5.0 with $j = 0.02$ on $12^3 \times 24$ lattice

fitted mass of S-wave



• temperature dependence of the $^1S_0$ state energy vs. μ for $Ma = 5.0$ with $j = 0.04$

fitted mass of S-wave



- the splitting between the $^3S_1$ state energy and $^1S_0$ state energy for three different M on $12^3 \times 24$

Conclusion

- for two-color QCD, lattice $\mathrm{NRQC_2D}$ study of heavy quarkonium allows an interesting probe for gluon effect in non-zero temperature/non-zero baryon density.

- S-wave correlators shows a bound state behavior upto $\mu a = 1.10$ at $Ta = 1/24$

- P-wave correlators shows an interesting μ-dependce but are too noisy to extract stable fit results.

- masses from S-wave correlators show three distinct physical regions of $\mathrm{QC_2D}$ at low temperature high baryon density.

- physical reason(s) behind for such behaviors need further theoretical investigation.