Taylor goes imaginary

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Introduction

Treating QCD at non-vanishing baryon density numerically suffers from the sign problem such that so far only approximate methods have been used to gain information at least at small values of the chemical potentials [1] which are, however, in the region phenomenologically relevant for RHIC and LHC physics.

In this paper we compare results obtained at imaginary values of the quark chemical potentials $\mu_i = i\mu_I$ - where lattice simulations are possible - with Taylor expansions. Moreover, we aim at an estimate of the curvature of the pseudocritical line in the $\mu - T$ plane.

Our study for staggered 2+1 flavors on lattices of size $16^3 \times 4$ clearly is exploratory. However, the quark mass values are close to the ones realized in nature. The Goldstone pion mass is tuned to about 220 MeV and the kaon acquires its physical mass. This corresponds to a (degenerate) light to strange quark mass ratio of $m_l/m_s = 1/10$. The action utilized is the p4fat3 action for which Taylor coefficients computed at $\mu_i = 0$ are available in our β range and at our quark masses [2].

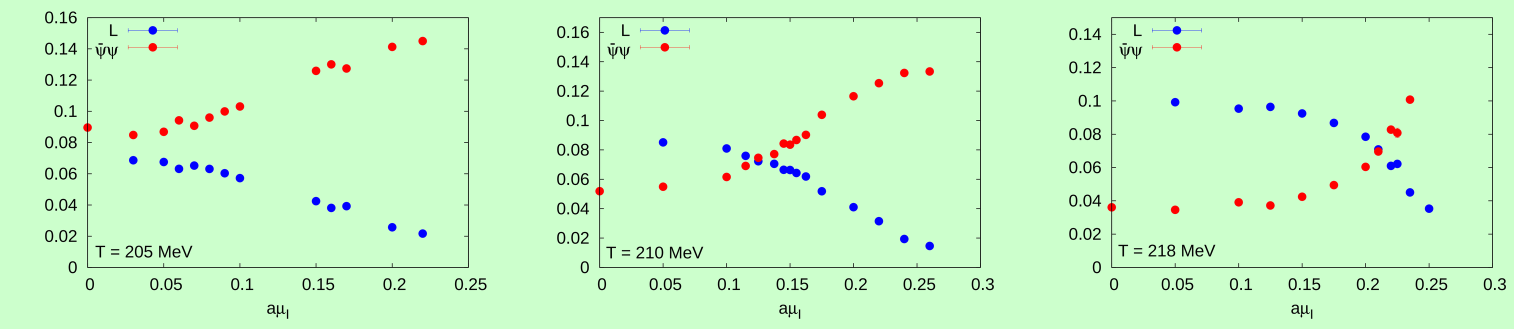
Locating the pseudocritical line

For small values of the quark chemical potentials $\mu_i, i = u, d, s$, the pseudo-critical line is expected to be described by

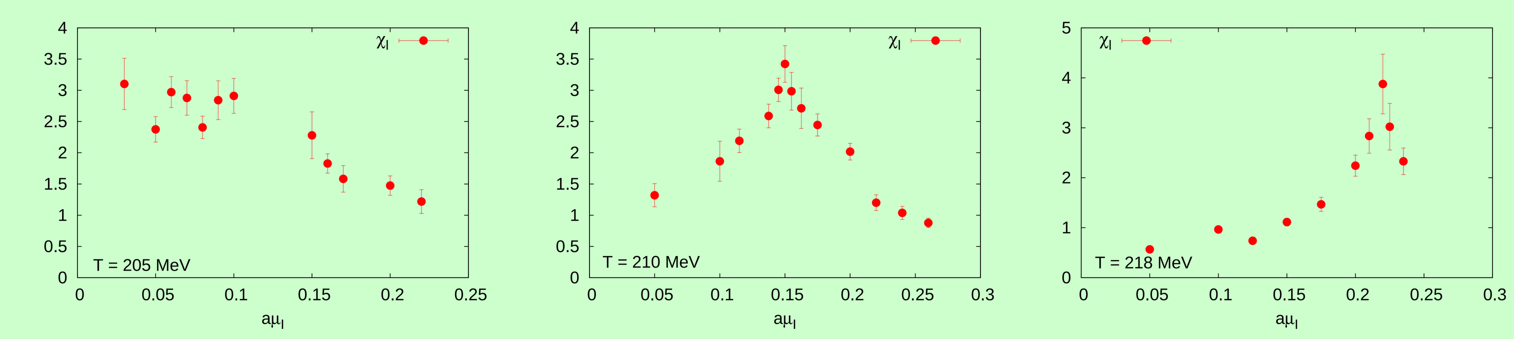
$$\frac{T}{T_c} = 1 - \sum_i \kappa_i \left(\frac{\mu_i}{T}\right)^2$$

In order to estimate the κ_i we simulated QCD for 2+1 flavors at a variety of imaginary values for the chemical potentials which were taken to be degenerate, $\mu_u = \mu_d = \mu_s = i\mu_I$, in the interval $0 \le \mu_I \le (\pi/3)T$, the Roberge-Weiss limit. The computations were carried out at two temperatures above and one below the pseudo-critical temperature at the light quark mass $m_u = m_d = m_l = (1/10)m_s$: $T = 205, 210, 218$MeV. The lowest temperature is above the critical temperature T_c in the chiral limit for the p4fat3 action at $N_\tau = 4$.

While the **Polyakov loop** L is not very sensitive to the transition, the **light quark chiral condensate** $\langle\bar\psi\psi\rangle_u$ is showing a fairly rapid rise when μ_I is increased.



The disconnected **chiral susceptibility** χ_u peaks at a pseudo-critical chemical potential.



Assuming degenerate curvature parameters κ_i, from a fit to the pseudo-critical values of μ_I we obtain

$$\kappa_i = 0.030(2)$$

Twice that value is to be compared with the value $\kappa_q = 0.059(2)(4)$ from [3]. Note however, that the latter value is the curvature in the chiral limit.

Since [4] has provided evidence that the light quark mass of $m_s/10$ is in the chiral $O(N)$ scaling window, we have been tempted to confront our data with $O(N)$ scaling behavior. The magnetization

$$M = m_s\langle\bar\psi\psi\rangle_u$$

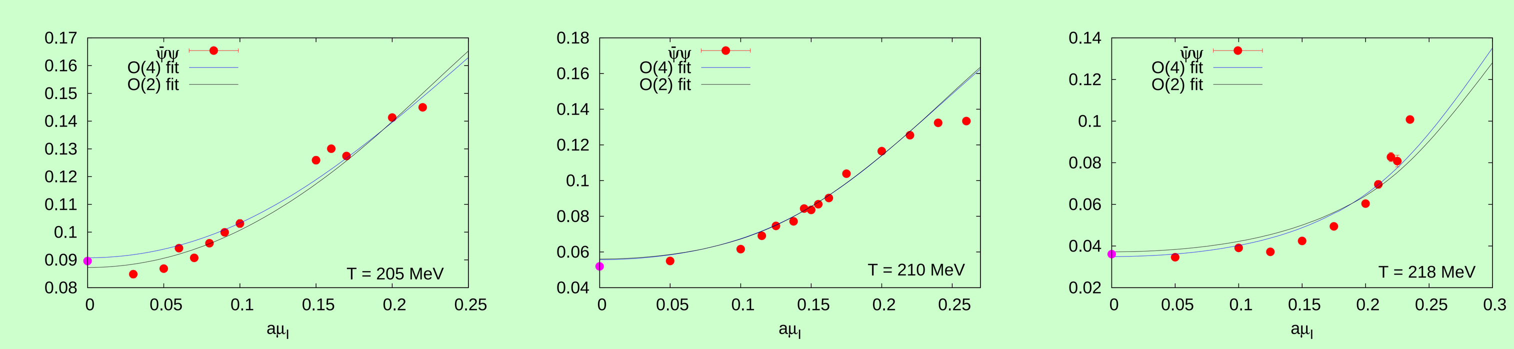
is thus fitted to the universal scaling function f_G as given in [5]

$$M = h^{1/\delta} f_G(z)$$

where

$$h = \frac{1}{h_0}\frac{m_l}{m_s} \qquad t = \frac{1}{t_0}\left[\frac{T}{T_c} - 1 - 3\kappa_i\left(\frac{\mu_I}{T}\right)^2\right] \qquad z = t\,h^{-1/\beta\delta} \qquad (1)$$

Note that the normalization constants h_0, t_0 as well as T_c, the critical temperature in the chiral limit for our action and at $N_\tau = 4$, are known from [4] such that we treat κ_i as the sole fit parameter. For larger μ_I this assumption could be modified due to several effects which are under investigation.



The data points at vanishing μ_I have not been used in the fits. The fits seem to work reasonably well except at the highest μ_I values and return the following κ_i values:

T	205 MeV	210 MeV	218 MeV
$O(2)$	0.026(1)	0.031(1)	0.025(3)
$O(4)$	0.022(1)	0.028(1)	0.025(2)

Comparison with the Taylor expansion

In the Taylor expansion approach one writes the pressure p as a series in terms of the quark chemical potentials around $\vec\mu_0 = (\mu_{u0}, \mu_{d0}, \mu_{s0})$

$$\frac{p}{T^4}(\vec\mu) = \sum_{ijk} c^{uds}_{ijk}(\vec\mu_0)\left(\frac{\mu_u - \mu_{u0}}{T}\right)^i\left(\frac{\mu_d - \mu_{d0}}{T}\right)^j\left(\frac{\mu_s - \mu_{s0}}{T}\right)^k$$

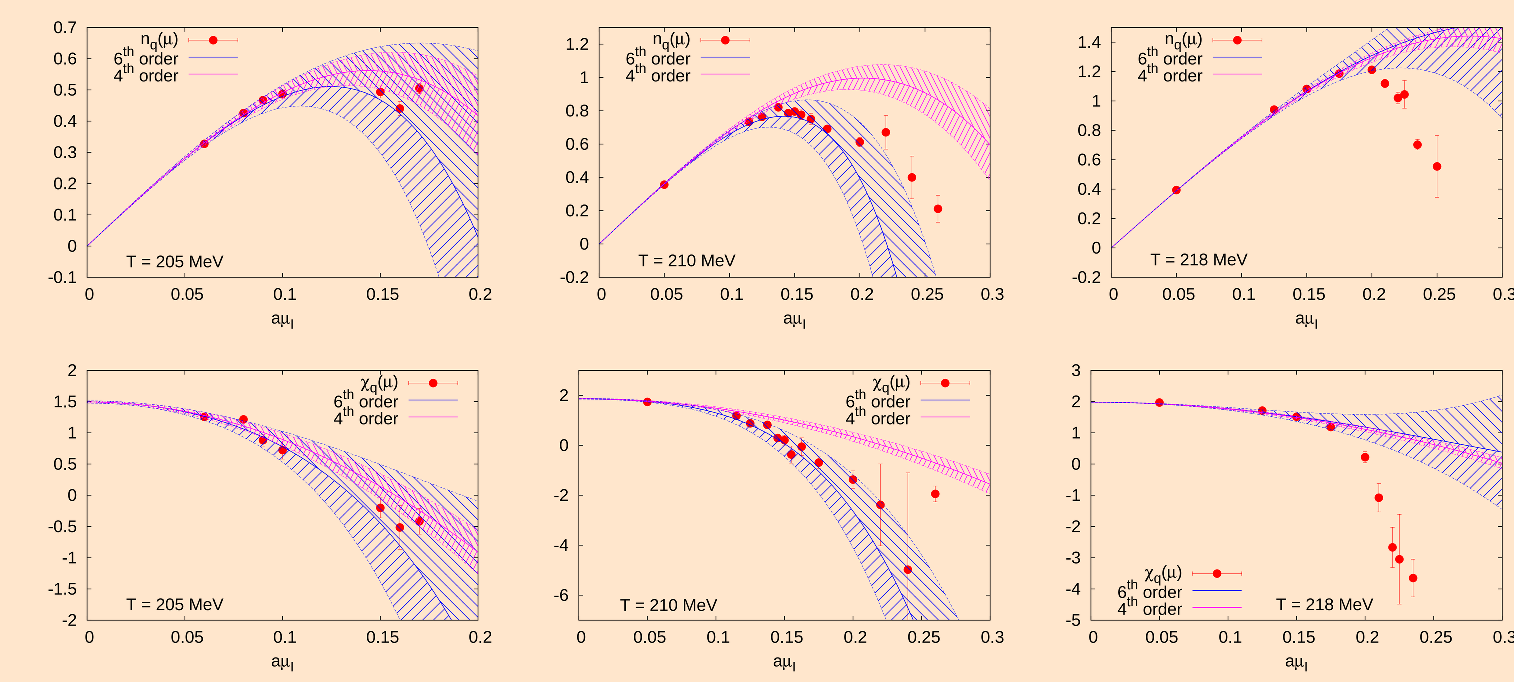
where usually $\vec\mu_0 = 0$ is chosen. Quark number densities n_i and susceptibilities χ_i, for instance

$$\frac{n_u}{T^3}(\vec\mu) = \frac{\partial(p/T^4)}{\partial(\mu_u/T)} = \sum_{ijk} i\,c^{uds}_{ijk}(\vec\mu_0)\left(\frac{\mu_u - \mu_{u0}}{T}\right)^{i-1}\left(\frac{\mu_d - \mu_{d0}}{T}\right)^j\left(\frac{\mu_s - \mu_{s0}}{T}\right)^k$$

$$\frac{\chi_u}{T^2}(\vec\mu) = \frac{\partial^2(p/T^4)}{\partial(\mu_u/T)^2} = \sum_{ijk} i(i-1)\,c^{uds}_{ijk}(\vec\mu_0)\left(\frac{\mu_u - \mu_{u0}}{T}\right)^{i-2}\left(\frac{\mu_d - \mu_{d0}}{T}\right)^j\left(\frac{\mu_s - \mu_{s0}}{T}\right)^k$$

are then easily obtained.

In the following we compare the light **quark number density** n_q and its **susceptibility** χ_q computed at non-vanishing μ_I with the predictions of a Taylor expansion around $\vec\mu_0 = 0$:



The error bands have been obtained by adding the errors on the Taylor coefficients in absolute value, and such are overestimated.

In principle one can do the expansion around various imaginary $\vec\mu_0$ and continue to real $\vec\mu$ thereby checking for reliability. Moreover, the error of the expansion could be estimated. This was so far beyond reach within the available computing resources.

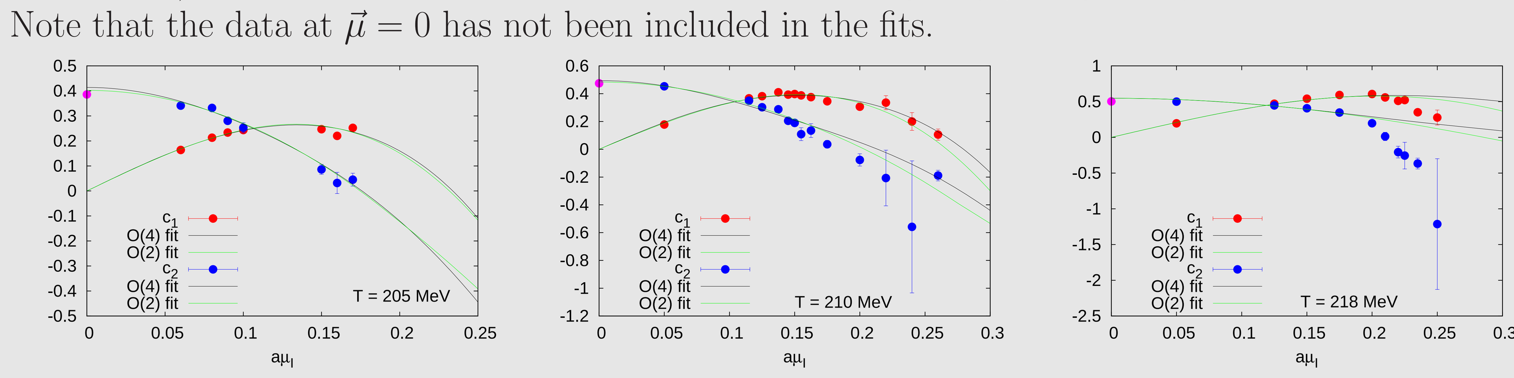
Quark number density and susceptibility vs. O(N)

When the singular part of the free energy density f_s is known the scaling behavior of n_q and χ_q, or equivalently of the appropriate Taylor coefficients, e.g. $c_1 = c^{uds}_{100}, c_2 = c^{uds}_{200}$, can be predicted as long as the μ_i dependence of t can safely be approximated by (1). For $O(4)$ symmetric model $f_s = h_0 h^{1+1/\delta} f_f(z)$ has been reconstructed in [5] via the relation

$$f_G(z) = -\left(1 + \frac{1}{\delta}\right) f_f(z) + \frac{z}{\beta\delta} f_f'(z) \quad .$$

In similar fashion, an interpolation of $f_G(z)$ in $O(2)$ has been undertaken in [6].

These interpolations for $f_s(z)$, together with a regular contribution $\sim t$ (h is fixed throughout all our simulations), have been used to fit the data for c_1, c_2 at imaginary μ. Again, κ_i is the sole fit parameter. Note that the data at $\vec\mu = 0$ has not been included in the fits.



The fit results for κ_i are summarized in the following table:

	T	205 MeV	210 MeV	218 MeV
$O(2)$	c_1	0.021(1)	0.021(1)	0.016(1)
	c_2	0.023(1)	0.021(1)	0.016(2)
$O(4)$	c_1	0.039(1)	0.036(1)	0.023(2)
	c_2	0.042(2)	0.036(1)	0.023(3)

While the fit results for κ_i, at least for $O(2)$, do not seem to be inconsistent, it is clear that at values for μ_I close to the Roberge-Weiss limit, other scaling studies need to be performed in the future.

References

1. see e.g. a recent review by L. Levkova, PoS LATTICE 2011 (2011) 011.
2. M. Cheng et al., Phys. Rev. D79 (2009) 074505.
3. O. Kaczmarek et al., Phys. Rev. D83 (2011) 014504.
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