The Bloch-Nordsieck Model at zero and finite temperature

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INTRODUCTION

PHOTON MASS = 0

- Long range interaction →NO free charged fermion
- From any EM (< ∞) energy ∞ number of photons

$$N=\lim_{\nu\to 0}\frac{I(\nu)}{h\nu}=\infty \text{ since } I(0)\neq 0$$

Fermion propagation in external field → soft gauge bosons are amitted and absorbed. (**Fig. 1.**)
What's the dressed fermion?→B-N Model: EXACT RESULT!

B-N Model ↔ Toy Model for QED

$$\mathcal{L}=\psi^\dagger u^0\left(iu^\mu\partial_\mu - m - eu^\mu A_\mu\right)\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$\gamma \leftrightarrow u$

Exact (B-N)	Perturbation Thm.	Resummations
• Solution by *path integral*	• 1-loop correction	• 2PI, T-DSE
• Hard to generalize	• Breaks down in *Infrared regime*	• *Works in IR*

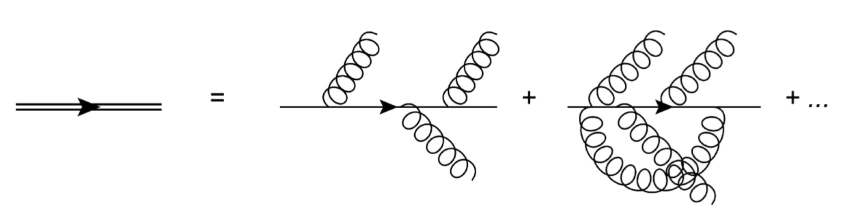


Fig. 1. *The "dressed" fermion*

We need to *sum over all possible* photon contributions!

SOLVING THE MODEL (T=0)

The Bloch-Nordsieck method

The free theory:

$$(iu^\mu\partial_\mu - m)G_0(x-y)=\delta(x-y)\ \rightarrow\ G_0(p)=\frac{1}{u_\mu p^\mu - m + i\epsilon}$$

The interacting case:

$$u^\mu\left(i\partial_\mu + eA_\mu(x)\right)-m]G(x,y|A)=\delta(x-y)$$

$$G(x,y|A)=i\frac{1}{2\pi^4}\int_0^\infty d\nu\int dp\exp\{-ip(x-y)-i\nu(up-m+i\epsilon)+iK(\nu|A)$$

$$G(x,y)=\frac{\int G(x,y|A)\langle T\exp\{ie\int\bar\psi(z)A(z)\psi(z)dz\}\rangle_{F_0}\mathcal{D}A}{\int\langle T\exp\{ie\int\bar\psi(z)A(z)\psi(z)dz\}\rangle_{F_0}\mathcal{D}A}$$

$$\boxed{G(p)=\frac{1}{(up)-m}\left(\frac{(up)}{m}-1\right)^{-\frac{e^2(3-\xi)}{8\pi^2}}}$$

(Bloch and Nordsieck [Phys. Rev. *52*, 54 (1937)])

- Retarded Greens function
- NO antifermions (**Fig. 2.**)

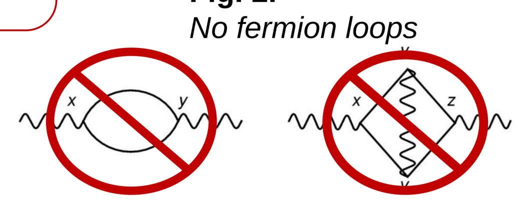


Fig. 2. *No fermion loops*

P.T. : 1-loop correction

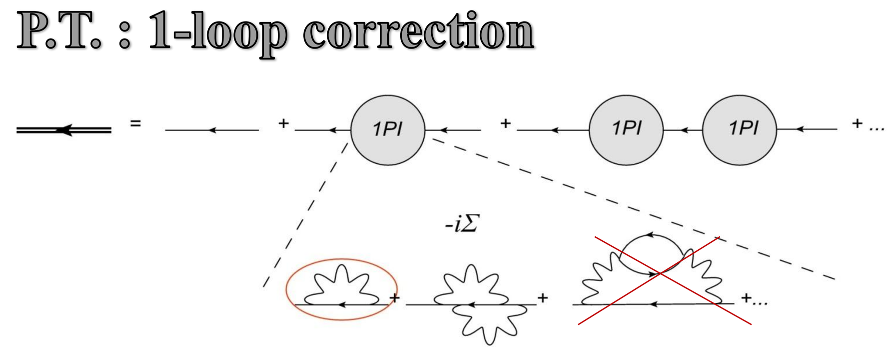


Fig. 3. *The expansion of the full propagator*

Self-energy $\sim\ I=-ie^2\frac{p^0-m+i\epsilon}{4\pi^2}\left[-\frac{2}{\delta}+\gamma_E-1-\frac{1}{2}\ln\pi+\ln\left(\frac{m-p^0-i\epsilon}{\lambda}\right)\right]$

Renormalized self-energy: $\Sigma_r=-e^2\frac{p^0-m}{4\pi^2}\ln\left(\frac{m-p^0}{\lambda}\right)$

Dyson-series ⟶ $iG(p^0)=\frac{i}{G_0^{-1}-\Sigma}$

$$\boxed{G_{1-loop}(p^0)=\frac{1}{p^0-m-\Sigma_r}=\frac{1}{p^0-m}\left[1-\frac{e^2}{4\pi^2}\ln\left(\frac{p^0-m}{\lambda}\right)\right]}$$

(in u=(1,0,0,0) frame)

RESULTS

T = 0

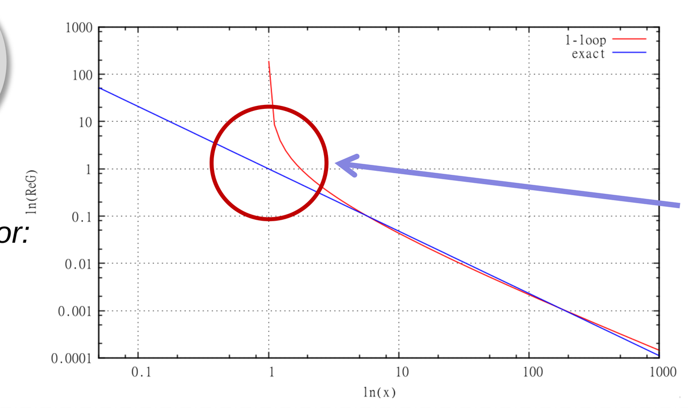


Fig. 7. *Propagator: 1-loop*

P.T.

Breaks down in the IR regime (x=up-m)

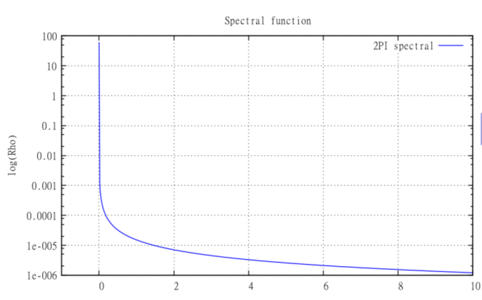


Fig. 8. *Spectral function: 2PI* (x=up-m)

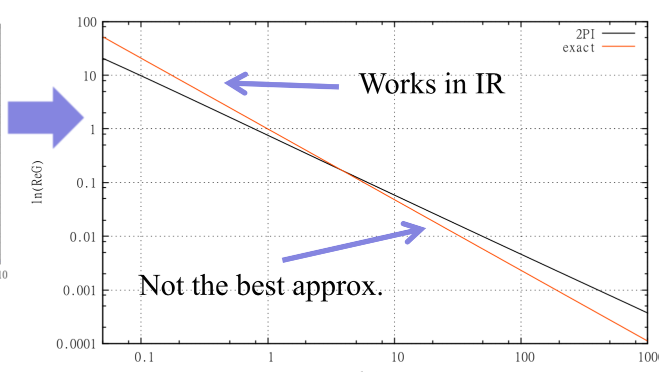
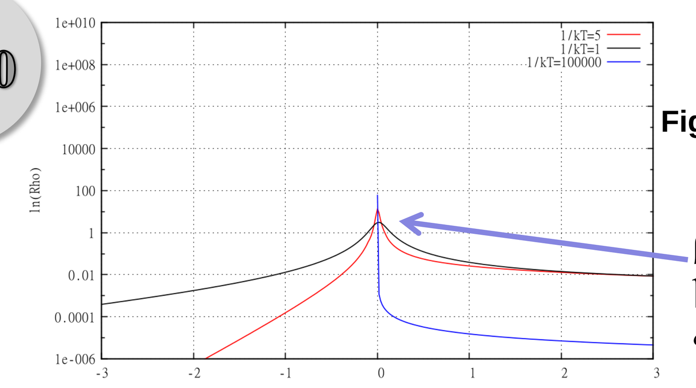


Fig. 9. *Propagator: 2PI* (x=up-m)

2PI

MOD. 2PI ⟶ gives back the exact result!

T ≠ 0



2PI

Fig. 11. *Spectral function: 2PI (T<∞)*

ρ(p) spreads out because of the "Heat bath"

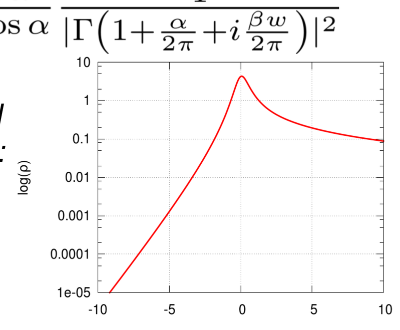
MOD. 2PI

$$\rho(w)=\frac{\exp^{\beta w/2}\sin\alpha}{\cosh(\beta w)-\cos\alpha}\frac{1}{|\Gamma(1+\frac{\alpha}{2\pi}+i\frac{\beta w}{2\pi})|^2}$$

u_0 = 1 / not 1

Numerical approach needed

Fig. 11. *Spectral function: M 2PI (T<∞)*



2PI resummation

Dealing with IR divergencies⟶ *resummations*

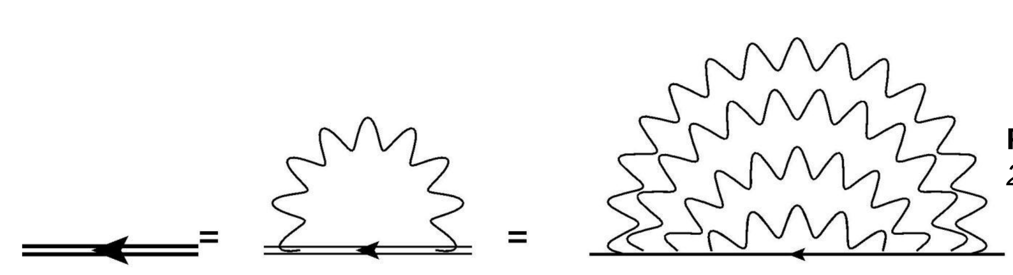


Fig. 4. *2PI*

$$G[\Sigma]=\frac{1}{G_0^{-1}-\Sigma}\ \leftrightarrow\ \Sigma[G]=\frac{-ie^2}{(2\pi)^4}\int dk^4\,\frac{G(p^0-k^0)}{k^2+i\epsilon}$$

Self-consistent equations

Numerical solution

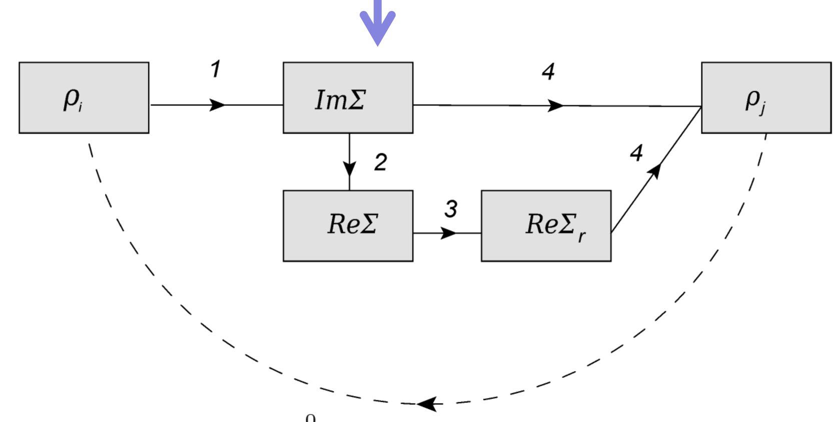


Fig. 5. *Algorithm*

1. $\mathrm{Im}\Sigma(p^0)=\frac{1}{4\pi^2}\int_0^{p^0}dk^0\ k^0\rho^{FR}(p^0-k^0)$
2. $\mathrm{Re}\Sigma(p^0)=\mathcal{P}\frac{1}{\pi}\int_{-\infty}^{\infty}dq^0\ \frac{\mathrm{Im}\Sigma(q^0)}{q^0-p^0}$
3. $\mathrm{Re}\Sigma_r(p^0)=\mathrm{Re}\Sigma(p^0)-\left(\mathrm{Re}\Sigma(p_t^0)-\frac{\partial\mathrm{Re}\Sigma(p_t^0)}{\partial p^0}\Big|_{p_t^0}(p^0-p_t^0)\right)$ — Renormalization
4. $\rho^{FR}(p^0)=\frac{2\mathrm{Im}\Sigma_r}{\mathrm{Re}[G_0^{-1}-\Sigma_r]^2+[\mathrm{Im}\Sigma_r]^2},\ (\rho(x)=\langle\{\psi(x),\psi^\dagger(0)\}\rangle_0)$ — Spectral function

Modified 2PI: Truncated Dyson-Schwinger eqs.

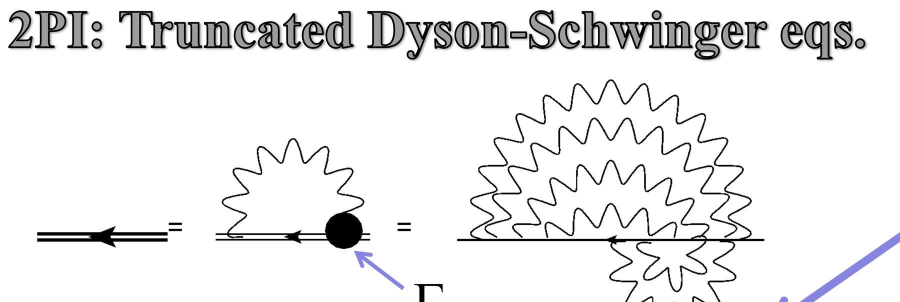


Fig. 6. *T D-S eqs.*

"Modified 2PI" = 2PI + vertex corrections

$$k_0\Gamma^0(p,p-k,k)=G^{-1}(p)-G^{-1}(p-k)\quad(\textit{Exact Ward-id.})$$

$$\Sigma(p)=\frac{-ie^2}{(2\pi)^4}\int dk^4\frac{1}{k^2+i\epsilon}G(p-k)u_\mu\Gamma^\mu(k;p-k,p)\ \xrightarrow{\text{analytic solution!!}}\ \boxed{G(p^0)=\frac{const.}{(p^0-m)^{(1+\frac{\alpha}{\pi})}}\equiv \text{B-N}}$$

SOLVING THE MODEL (T≠0)

What's new at T<∞ ?

- *Can't use only u*=(1,0,0,0): The plasma points out a reference frame
- *Fermi-Dirac, Bose-Einstein* distributions
- New loop integrals (*Retarded* self-energy→R/A)

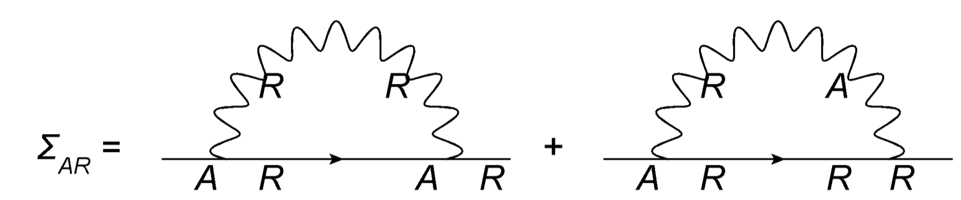


Fig. 10. *Retarded 1-loop self-energy in R/A formalism at* **T<∞**

The propagators:

γ:
- $\mathcal{G}_{RA}(k)=\frac{1}{(k_0+i\epsilon)^2-\underline{k}^2}\rightarrow \mathcal{D}isc\mathcal{G}_{RA}(k)=\rho_\gamma(k)$
- $\mathcal{G}_{RR}(k)=(1/2+n_b(k_0))\rho_\gamma(k)$

e⁻:
- $G_{RA}(p-k)=\frac{1}{u(p-k)-m+i\epsilon}\rightarrow \mathcal{D}isc G_{RA}(p-k)=\rho_\gamma(p-k)$
- $G_{RR}=(1/2+n_f(p_0-k_0))\rho(p-k)$ [$n_f(p_0-k_0))=0$ in this model]

P.T.: 1-loop

$$\mathcal{D}isc\Sigma_{AR}(p)=\frac{e^2}{2\pi}\Theta(up-m)(up-m)+\frac{Te^2}{4\pi u}\ln\left(\frac{1-e^{-\beta|up-m|/(u_0-u)}}{1-e^{-\beta|up-m|/(u_0+u)}}\right)$$

2PI resummation

$$\mathcal{D}isc\Sigma_{AR}(p)=\frac{e^2}{8\pi^2u}\int_{-\infty}^{\infty}dz\rho(z)\frac{1}{u}\int_{\frac{z-up}{u_0-u}}^{\frac{z-up}{u_0+u}}dkn(k)$$

Modified 2PI ⟶ integral equation for ρ(p)

$$\rho(w)+\frac{\alpha}{\pi}\int dq\frac{1}{2u}\int_{u_0-u}^{u_0+u}\frac{ds}{s^2}(1+n(\tfrac{q}{s}))\rho(w-q)=0$$

CONCLUSION

1. Exact B-N
   - Power law behaviour
   - Hard to generalize
2. 1-loop PT
   - Breaks down in IR
3. Resummations
   - 2PI: works in IR but poor approximation
   - T D-S eqs.: a new way to solve the model
4. Finite T
   - Resummations are working well
5. Outlook
   - Adapting the method to QED
   - Investigate the IR physics
   - Examine bound states
   - Adapting the method to non-Abelian gauge fields (?)

LITERATURE

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