

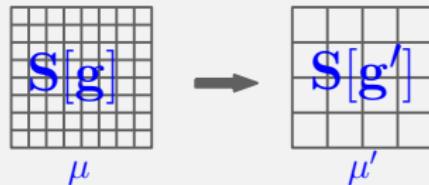
# Thermodynamic Lattice Study for Walking Dynamics in Strongly Flavored Gauge Theory

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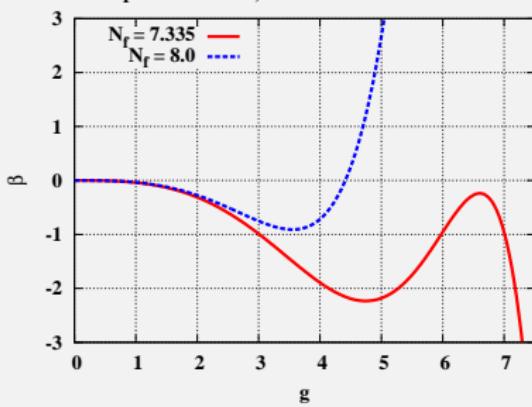
Poster Presentation at SEWM, Swansea, July 11, 2012

# (Pre-) Conformal Gauge Theory

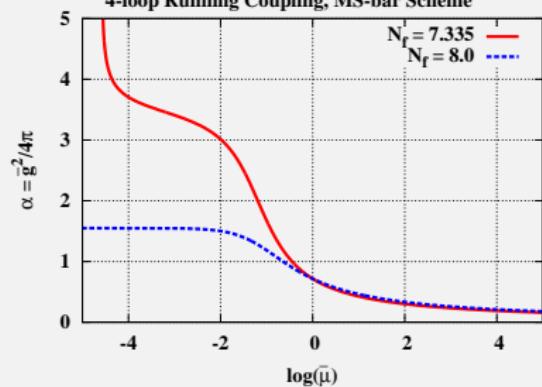


$$\beta(g, N_c, N_f) = \mu \frac{dg}{d\mu} = -g^3 \sum_{n=0}^{\infty} b_n(N_c, N_f) g^{2n}. \quad (1)$$

4-loop beta-function, MS-bar Scheme



4-loop Running Coupling, MS-bar Scheme



# Motivation and Goal

$$\frac{\langle \bar{\Psi} \Psi \rangle|_{\text{ETC}}}{\langle \bar{\Psi} \Psi \rangle|_{\text{TC}}} = \exp \left[ \int_{\Lambda_{\text{TC}}}^{\Lambda_{\text{ETC}}} d(\log \mu) \gamma[g^2(\mu)] \right] \xrightarrow{\text{Conformal}} \left( \frac{\Lambda_{\text{ETC}}}{\Lambda_{\text{TC}}} \right)^{\gamma[g_*^2]} . \quad (2)$$

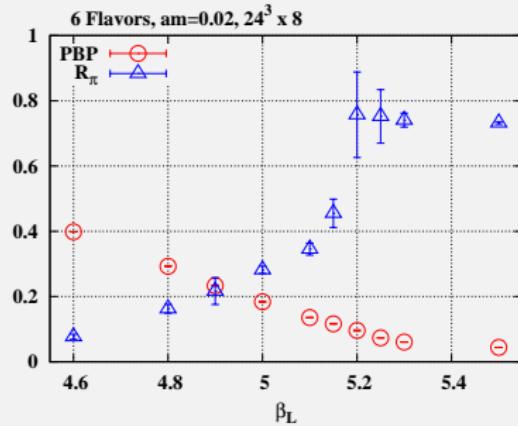
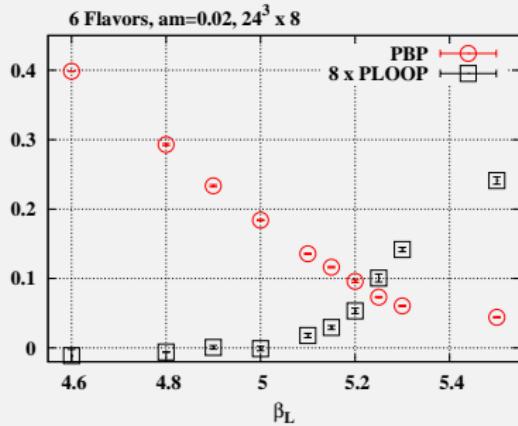
## Motivations

- Application to the Walking Technicolor Model.
- Fundamental Question: Is there Conformal and/or Walking?
- Connection to the AdS/CFT Conjecture.

## Goal of Our Project

- To study the (pre-) conformal dynamics in lattice gauge theory at finite  $T$ .
- To elucidate the Braun-Gies  $(T_c - N_f)$  phase diagram.
- To reveal the Miransky-Yamawaki  $(N_f - g_c)$  phase diagram.

# Finite $T$ Chiral Phase Transition



- Larger  $\beta_L \rightarrow$  Larger  $T = [a(\beta_L)N_t]^{-1}$ , with  $N_s \gg N_t$ .
- In the chiral limit ( $ma \rightarrow 0$ ),

$$R_\pi \equiv \frac{\chi_\sigma}{\chi_\pi} = \begin{cases} 0 & (\text{Chiral Broken}) \\ 1 & (\text{Chiral Restored}) \end{cases} . \quad (3)$$

# From $\beta_L^c$ to $T_c(N_f)$

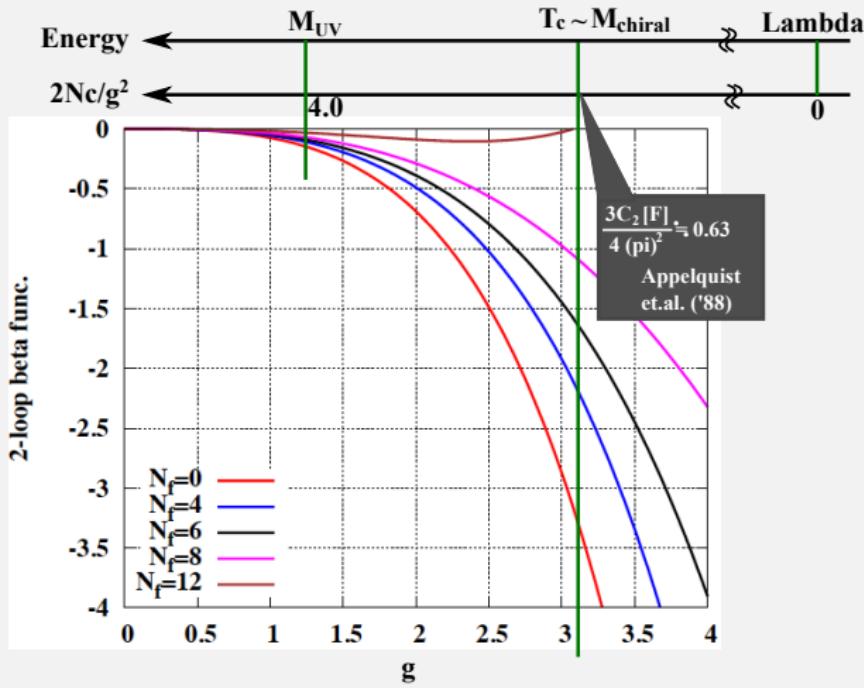
**Table:** The summary table of  $\beta_L^c$ , which has been obtained by using the same action except a choice of  $N_f$ . Blue: Deuzeman-Lombardo-Pallante ('08).

$N_f \setminus N_t$	4	6	8	12
0	-	$7.88 \pm 0.05$	$8.20 \pm 0.10$	-
4	-	$5.89 \pm 0.03$	-	
6	$4.675 \pm 0.025$	$5.025 \pm 0.025$	$5.225 \pm 0.025$	$5.45 \pm 0.05$
8	-	$4.1125 \pm 0.0125$	-	$4.34 \pm 0.04$

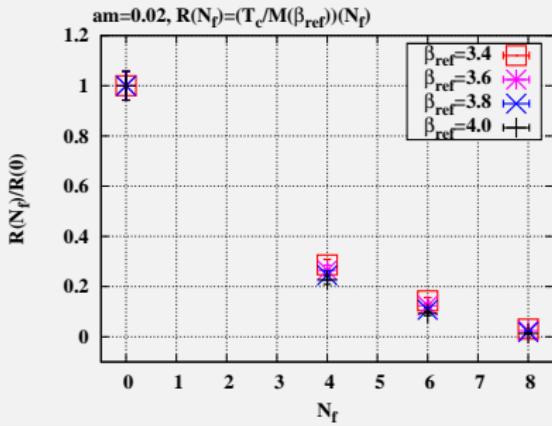
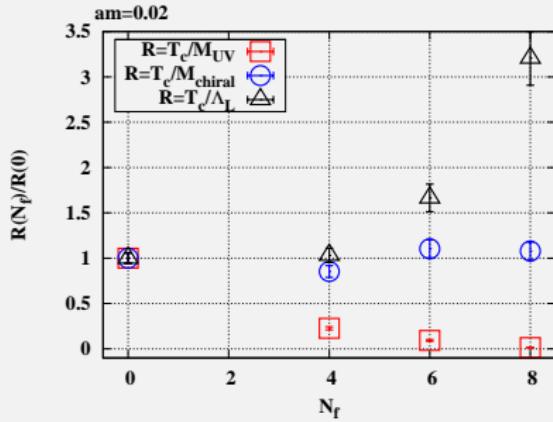
$$M(\beta_L^{\text{ref}}) \times a(\beta_L^c) = \left( \frac{b_0^2}{b_1} \frac{\beta_c + 2N_c b_1/b_0}{\beta_L^{\text{ref}} + 2N_c b_1/b_0} \right)^{b_1/(2b_0^2)} \exp \left[ -\frac{\beta_L^c - \beta_L^{\text{ref}}}{4N_c b_0} \right]. \quad (4)$$

$$\frac{1}{N_\tau} = \frac{T_c}{M(\beta_L^{\text{ref}})} (N_f) \times M(\beta_L^{\text{ref}}) a(\beta_L^c(N_f, N_t)). \quad (5)$$

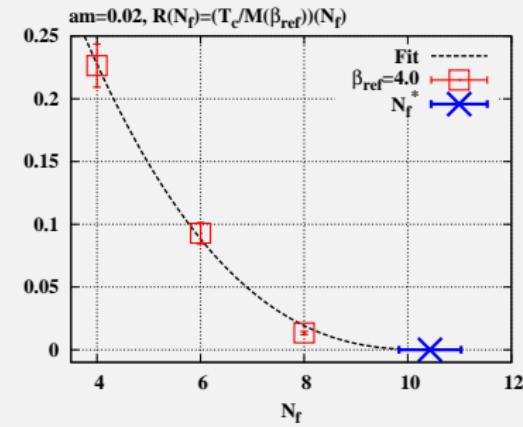
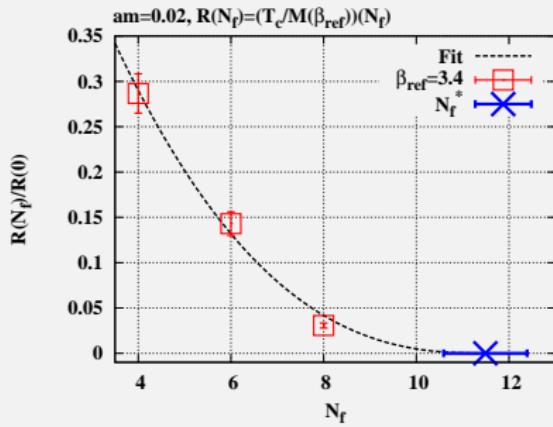
# Reference-Scale $M$



# $T_c/M$ as a function of $N_f$



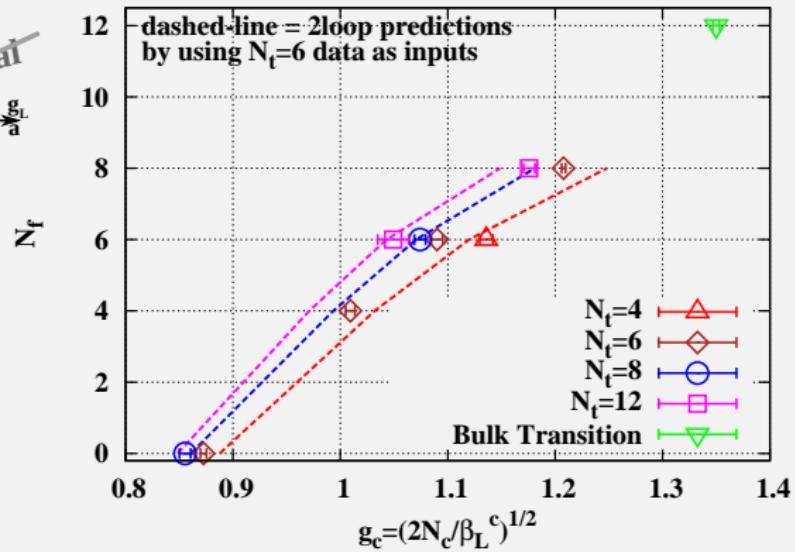
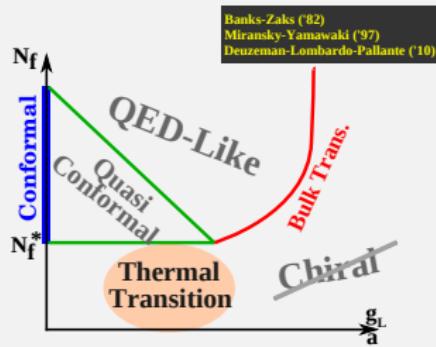
# Critical Flavor Number $N_f^*$



$$T_c(N_f) = K |N_f - N_f^*|^{-(2b_0^2/b_1)(N_f^*)}, \quad (\text{c.f. Braun-Geiss ('11)}) \quad (6)$$

$$N_f^* = \begin{cases} 11.49 \pm 0.9 & (\beta_{ref} = 3.4), \\ 10.43 \pm 0.6 & (\beta_{ref} = 4.0). \end{cases} \quad (7)$$

# Miransky-Yamawaki Phase Diagram



# AdS/CFT: As a Guide (Kiryitsis et.al. ('08))

- 5D Einstein-Dilaton Action:

$$\mathcal{S} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R - \frac{4}{3}(\partial_\mu \phi)^2 + V(\phi) \right]. \quad (8)$$

- Black-Hole Metric Ansatz:

$$ds^2 = b^2(z) \left[ -f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)} \right]. \quad (9)$$

Outputs from Einstein Eqs. are  $f(z)$ ,  $b(z)$ , and  $\lambda(z) = e^{\phi(z)} \sim N_c g^2(z)$ .

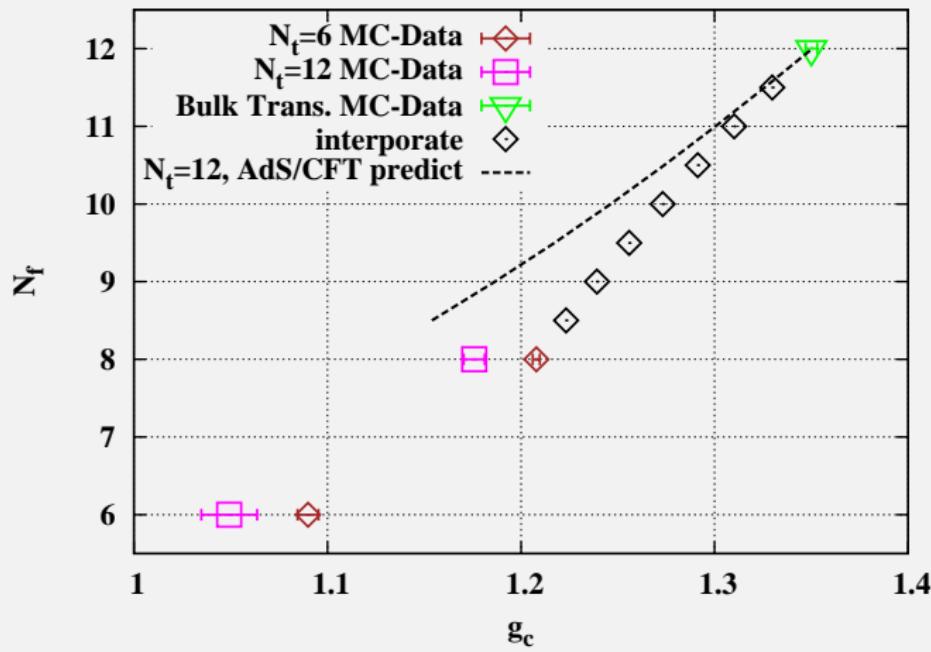
- beta-function Ansatz, (Alanen et.al. ('10)):

$$\beta(\lambda) = b(z) \frac{d\lambda(z)}{db(z)} = -c\lambda^2 \frac{(1-\lambda)^2 + e}{1 + 2c\lambda^3/3}. \quad (10)$$

- Thermodynamics:

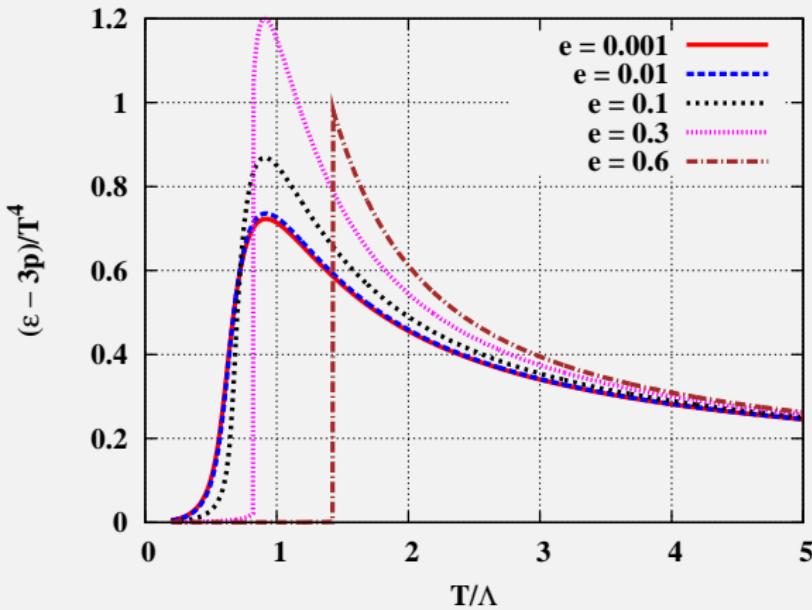
$$4\pi T_h = -df(z_h)/dz, \quad f(z_h) = 0, \quad s(T_h) = \frac{b^3(\lambda(z_h))}{4G_5}. \quad (11)$$

## Walking Signal in MY Phase Diagram, From AdS/CFT



# Trace Anomaly, From AdS/CFT

Based on the model proposed Alanen et.al. ('08), we investigate the trace anomaly with variations of conformal breaking effects  $e$ :



# Summary

## Summary

- We have investigated the chiral phase transition (crossover) at finite  $T$  in the strongly flavored QCD-Like theory by using the lattice gauge theory.
- We have discussed the lower edge of conformal window  $N_f^*$  by using our lattice data with the generalized 2-loop asymptotic scaling. We have obtained  $N_f^* \sim 9.8 - 12.4$
- We have discussed the signal of the walking dynamics in the Miransky Yamawaki Diagram and the Trace Anomaly with a help of AdS/CFT.

## Future Works

- To set a scale  $a^{-1}$  and complete  $T - N_f$  Phase Diagram:  
The potential measurement is on progress.
- Critical behavior near the IR-Fixed Pt.
- The color  $SU(N_c = 2)$  with 8 flavors at finite  $T$ .

# References

## References

- K. Miura, M. P. Lombardo and E. Pallante, "Chiral phase transition at finite temperature and conformal dynamics in large  $N_f$  QCD," Phys. Lett. B **710** (2012) 676.
- K. Miura, M. P. Lombardo and E. Pallante, "Thermodynamic Study for Conformal Phase in Large  $N_f$  Gauge Theory," PoS Lattice 2011, arXiv:1111.1098 [hep-lat].

## Computers and Code

- **MILC Code:** [http://www.physics.utah.edu/~detar/milc/milc\\_qcd.html](http://www.physics.utah.edu/~detar/milc/milc_qcd.html)
- **Argolithm:** Rational Hybrid Molecular-Dynamics with Omelyan-Integrator
- **Computers:** SP6 and BG/P in CINECA, SP16000 in YITP, and Italian-Grid-Infrastructures