Inverse Magnetic Catalysis in Dense Matter

A field theoretical and holographic perspective [1, 2]

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Motivation

Two QCD "laboratories" exhibit the strongest magnetic fields in the universe

- **non-central relativistic heavy ion collisions:** $B \sim 10^{18}$ G
- **compact stars:** up to $B \sim 10^{15}$ G at the surface, possibly $B \sim 10^{19}$ G in the core

At finite chemical potential μ and small temperature T we have to rely on models. We study the chiral phase transition influenced by a strong magnetic field in

- the **Nambu–Jona-Lasino** (NJL) **model**
- the **Sakai–Sugimoto model**

The NJL model

We use the Lagrangian

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\gamma^\mu D_\mu + \mu\gamma_0)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\psi)^2\right]$$

in a **background magnetic field** with $N_f = 1$ and apply the mean field approximation $(\bar{\psi}\psi)^2 \simeq -\langle\bar{\psi}\psi\rangle^2 + 2\langle\bar{\psi}\psi\rangle$.

This leads to

- **Landau level quantization** of the one-particle spectrum $\epsilon_{k_3,\ell} = \sqrt{k_3^2 + M^2 + 2\,|q|\,B\ell}$, where $M = -2G\langle\bar{\psi}\psi\rangle$
- with **degeneracy** $d_\ell = (2 - \delta_{0,\ell})|q|B/(4\pi^2)$

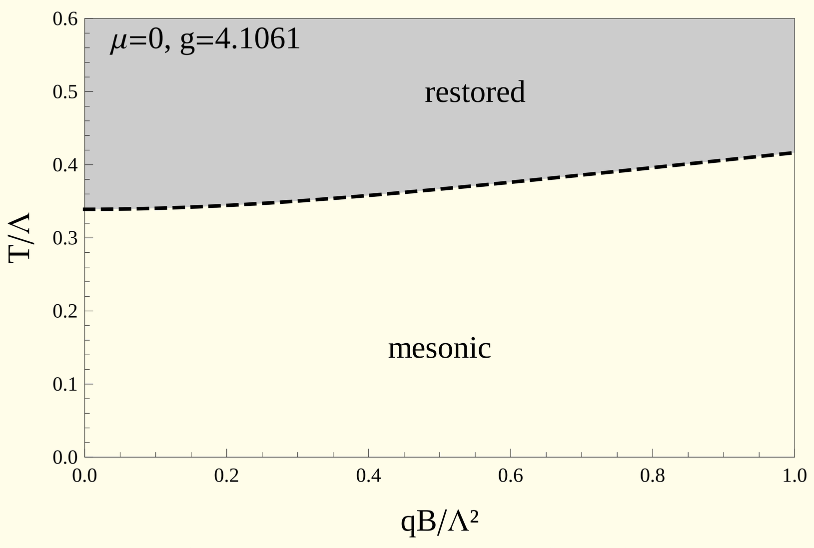
Solving the gap equation:

- B=0: only if $g := G\Lambda^2 N_c/(2\pi^2) > 1$ chiral symmetry is broken in vacuum
- **magnetic catalyis:** if $B > 0$ chiral symmetry is broken for any $g > 0$
- at $g \ll 1$ the gap looks similar to the BCS gap [3]

$$M = \sqrt{\frac{|q|B}{\pi}}\,\mathrm{e}^{-\frac{\pi^2}{|q|BN_cG}}$$

- the density of states (dos) at $k_3 = \ell = 0$ plays a similar role as the dos at k_F in BCS theory
- for $g > 1$ we find at $\mu = 0$

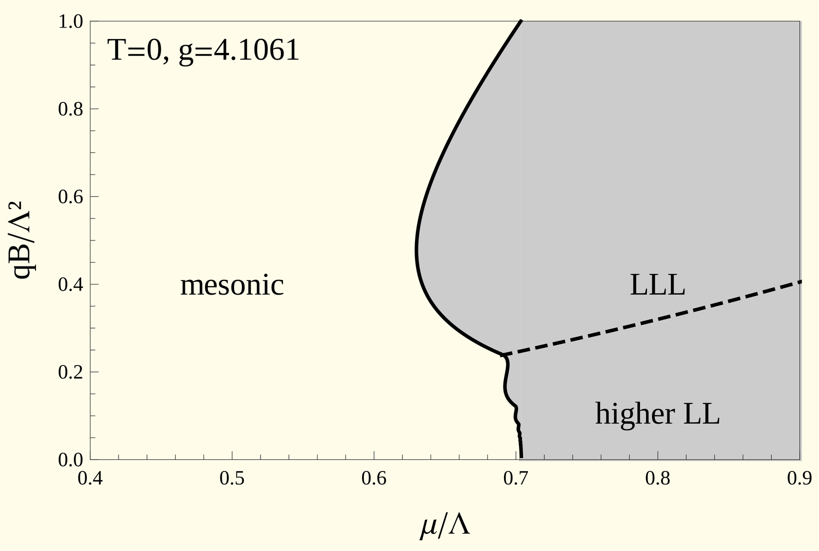
Phase diagram for chiral symmetry restoration at $\mu = 0$.



At finite chemical potential one finds

- a finite axial current [4]: $\mathcal{J}_5^3 = N_c|q|B\mu/(2\pi^2)$ if $M = 0$ at any T coming solely from the LLL
- **inverse magnetic catalysis** at finite μ if $g > 1$

Phase diagram for chiral symmetry restoration at $T = 0$.



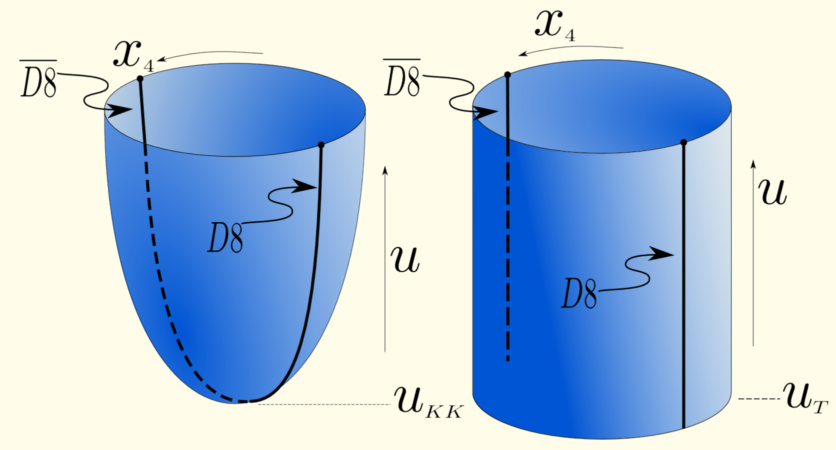
- **energy costs for condensation increase** with μ but **also with B** because of the degeneracy factor. The LLL contribution is $N_c|q|B\mu^2/(4\pi^2)$
- **analogue to Clogston limit** for superconductors: $\Delta\Omega \propto B(\mu^2 - M^2/2)$ for $g \ll 1$, with $\overline{\mu} \leftrightarrow B$ and $\mu \leftrightarrow \delta\mu$

The Sakai–Sugimoto model

The Sakai–Sugimoto model [5, 6] is a top–down aproach to a gravity theory dual to large N_c QCD. It exhibits

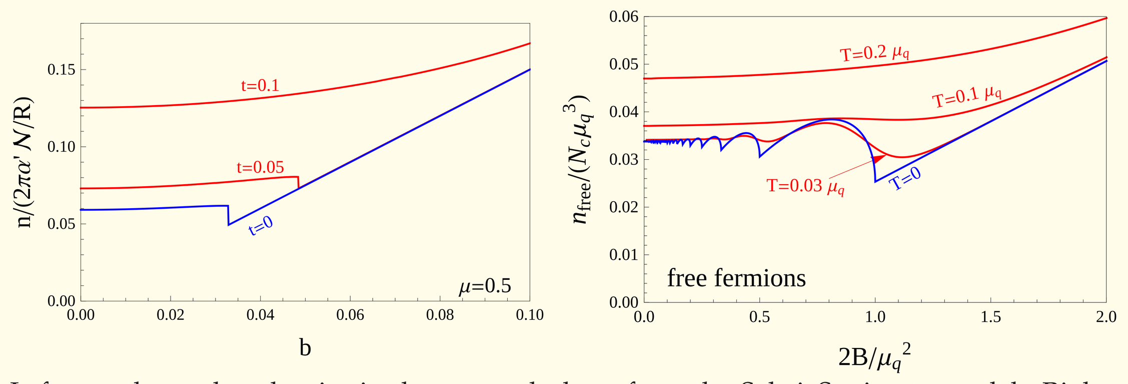
- **broken supersymmetry** by introducing an extra dimensional Kaluza–Klein circle
- confinement–deconfinement transition via a Hawking–Page transition between different geometries
- fundamental matter by D8- and anti–D8-branes separated on the Kaluza–Klein circle
- **spontaneous chiral symmetry breaking** by joining the D8- and anti–D8-branes in the bulk
- an **interpolation** between (non-local) **NJL** and **QCD** [7] by tuning the D8-brane separation parameter
- a splitting of chiral symmetry restoration and deconfinement by increasing the magnetic field for sufficiently small separation
- a constituent quark mass given by the location of the tip of the joined D8-branes

$2d$ cut through the background geometry in x_4-u-space denoting the coordinate of the Kaluza-Klein circle and the AdS-radius respectively. Left: confined-chirally broken. Right: deconfined-chirally symmetric



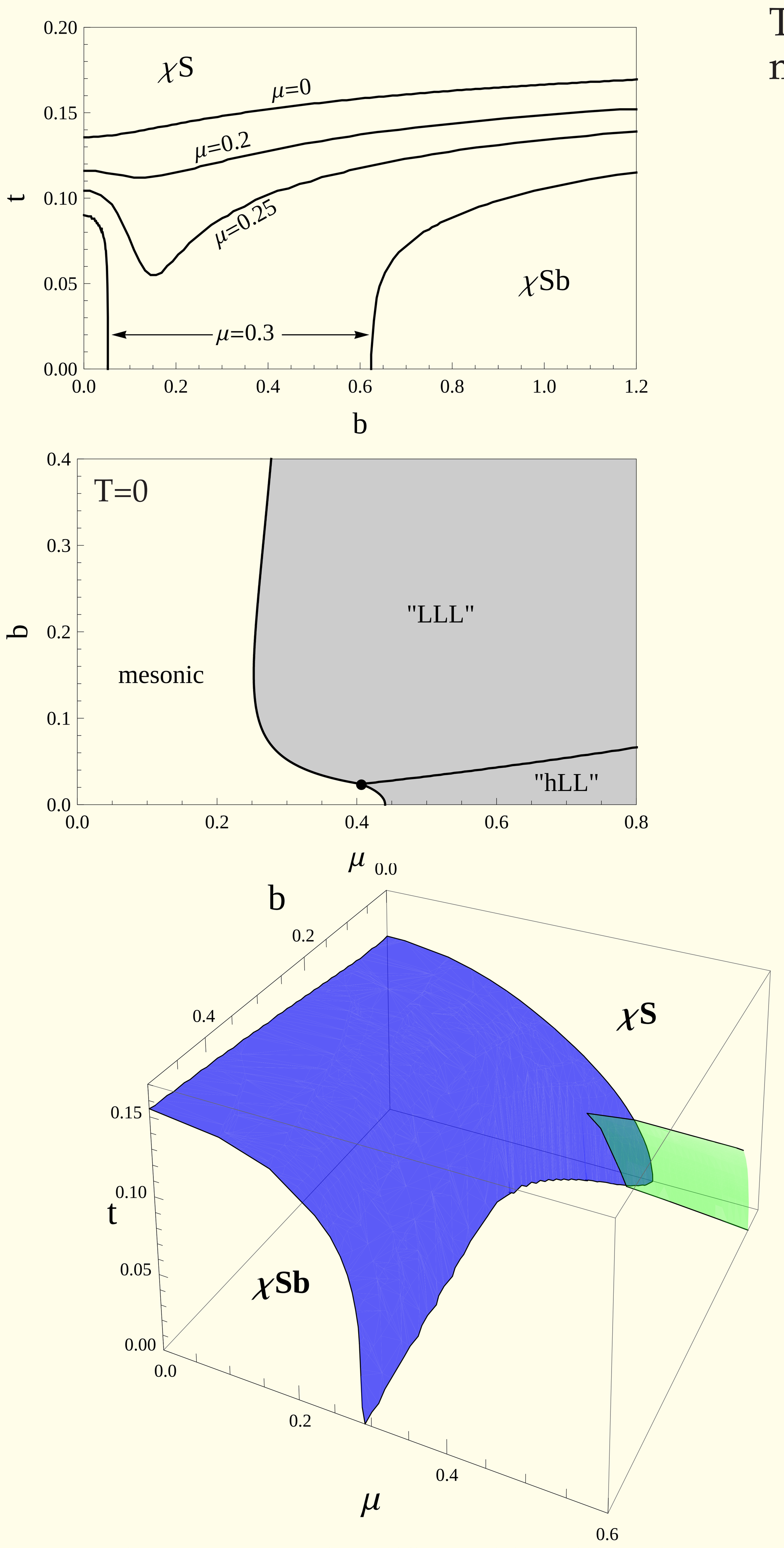
There exists a **phase transition within the chirally symmetric phase** first discussed in [8].

- large B phase with $n = N_c/(2\pi^2)B\mu$ **looks like LLL**
- no oscillations from "higher Landau levels" and the LLL transition is first order



Left: quark number density in the restored phase from the Sakai–Sugimoto model. Right: quark number density in the restored phase from the NJL model

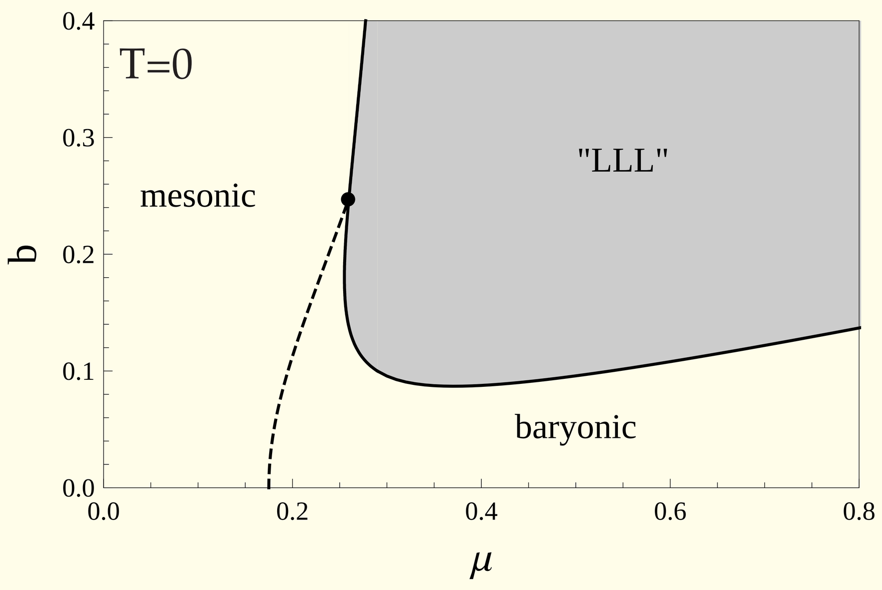
Inverse magnetic catalysis



Top: critical temperature at several μ. Middle: critical chemical potential at $T = 0$. Bottom: full 3d phase diagram; $b = 2\pi\ell_s^2 B$, etc. (dimensionless)

The chiral phase transition in the Sakai-Sugimoto model:

- **magnetic catalysis** at small μ [9]
- increasing B can restore chiral symmetry at finite μ, i.e. we observe **inverse magnetic catalysis** (IMC)
- this is due to the energy cost for condensation $\propto B$ in the LLL, where IMC is most pronounced, and because the gap is not catalysed strongly enough ($M \sim \alpha + \beta B^2$ in both models at small B)
- IMC up to $(\mu, B) \sim (230\ \mathrm{MeV}, 10^{19}\ \mathrm{G})$
- at large B, where MC is present, we find a **holographic analogue to the Clogston limit** $\Delta\Omega \propto B[\mu^2 - M^2\sqrt{\pi}4\Gamma(3/5)/(9\Gamma(1/10))]$
- including **large N_c baryons** [2]:
  - chiral symmetry is broken at any μ for small temperatures and small B.
  - the **IMC is more pronounced**
  - baryon onset (second order) increases with B and ends in the chiral phase transition line



Chiral phase transition including baryons in the Sakai–Sugimoto model

- **magnetars**: quark matter favored by a strong magnetic field?

References and funding

[1] F. Preis, A. Rebhan and A. Schmitt, JHEP **1103**, 033 (2011) [arXiv:1012.4785 [hep-th]].
[2] F. Preis, A. Rebhan and A. Schmitt, J. Phys. G G **39**, 054006 (2012) [arXiv:1109.6904 [hep-th]].
[3] V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, Phys. Rev. Lett. **73**, 3499 (1994) [Erratum-ibid. **76**, 1005 (1996)] [hep-ph/9405262].
[4] M. A. Metlitski and A. R. Zhitnitsky, Phys. Rev. D **72**, 045011 (2005) [hep-ph/0505072].
[5] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005) [hep-th/0412141].
[6] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **114**, 1083 (2005) [hep-th/0507073].
[7] E. Antonyan, J. A. Harvey, S. Jensen and D. Kutasov, hep-th/0604017.
[8] G. Lifschytz and M. Lippert, Phys. Rev. D **80**, 066007 (2009) [arXiv:0906.3892 [hep-th]].
[9] C. V. Johnson and A. Kundu, JHEP **0907**, 103 (2009) [arXiv:0904.4320 [hep-th]].

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