Pulsars as probes of dense matter

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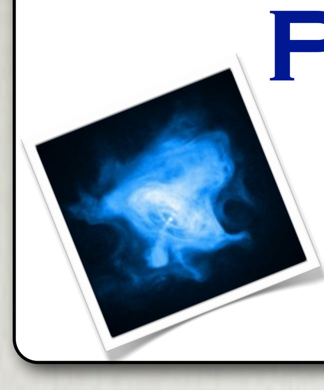
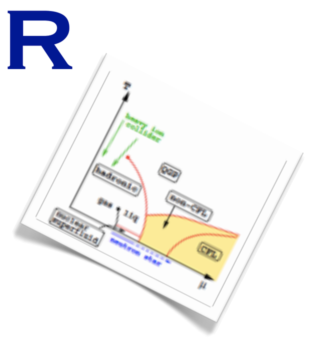




Abstract

The rotation frequencies of young pulsars are systematically below their theoretical limit. R-mode oscillations have been suggested as a possible explanation for this observation. With the help of semi-analytic expressions, that allow to assess the uncertainties due to the underlying microphysics, we perform a quantitative analysis of the r-mode scenario for the spin-down and the emitted gravitational waves of young pulsars. We find that the frequency to which r-modes spin down a young neutron star as well as the characteristic gravitational wave strain amplitude are extremely insensitive both to the microscopic details and the saturation amplitude. Comparing our result to astrophysical data, we show that for saturation amplitudes $0.01 \lesssim \alpha_{sat} \lesssim 0.1$ r-modes provide a coherent spindown scenario and that all observed young pulsars are very likely not spun down by r-modes any more. We find that the signal to noise ratio for gravitational waves would within the significant uncertainties still be large enough to detect gravitational wave emission due to r-modes with next generation detectors.

Dense matter in compact stars

Compact stars present the most dense form of matter in the universe ... and could consist of many novel phases of matter:

- To detect them requires to connect unique microscopic properties of a given phase to macroscopic observables
- Static star properties depend only on the equation of state and are very similar for different star compositions
- Dynamic properties depend on the low energy degrees of freedom and can vary strongly for different phases

  ... moreover they vary strongly with temperature and generally feature a power law behavior:
  - Shear viscosity $\eta = \tilde{\eta} T^{-\sigma}$ and bulk viscosity $\zeta = C^2 \tilde{\Gamma} T^{\delta} \omega^{-2}$ describe the damping of mechanical oscillations

    prefactors vary within a given phase of dense matter

    ① exponents are only different for distinct phases of dense matter

    all prefactors depend on the density and are very uncertain since we cannot solve dense QCD yet ...
  - Specific heat $c_V = \tilde{c}_V T^{\upsilon}$ and neutrino emissivity $\epsilon \approx \tilde{\epsilon} T^{\theta}$ describe the thermal properties of the star
- The macroscopic aspects depend only on the microscopic material properties integrated over the star:

② $\tilde{S} \equiv \frac{1}{R^5 \Lambda_{QCD}^{3+\sigma}} \int_{R_i}^{R_o} dr\, r^4 \tilde{\eta}$, $\tilde{V} \equiv \frac{\Lambda_{EW}^4}{R^3 \Lambda_{QCD}^{9-\delta}} \int_{R_i}^{R_o} dr\, r^2 A^2 C^2 \tilde{\Gamma} \, (\delta\Sigma)^2$, $\tilde{C}_V \equiv \frac{1}{R^3 \Lambda_{QCD}^{3-\upsilon}} \int_{R_i}^{R_o} dr\, r^2 \tilde{c}_V$ & $\tilde{L} \equiv \frac{1}{R^3 \Lambda_{EW}^4 \Lambda_{QCD}^{1-\theta}} \int_{R_i}^{R_o} dr\, r^2 \tilde{\epsilon}$

Hyperons?

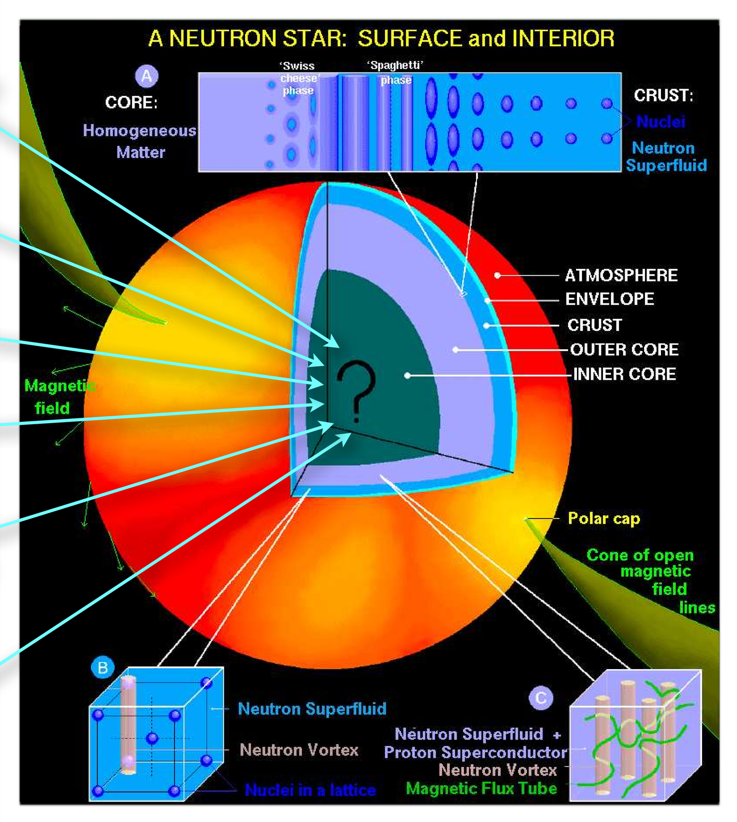
Meson condensates?

Mixed phases?

Quark matter?

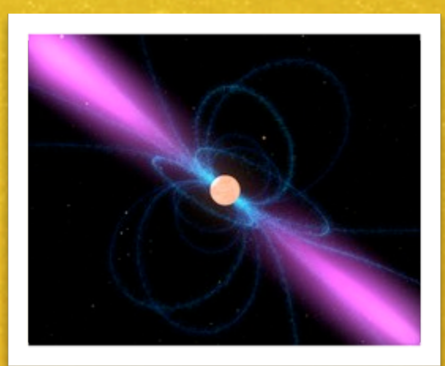
Anisotropy/ Inhomogeneity?

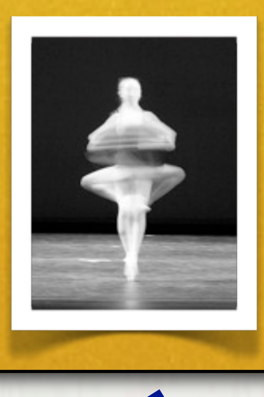
Color super-conductivity?



Observational pulsar data

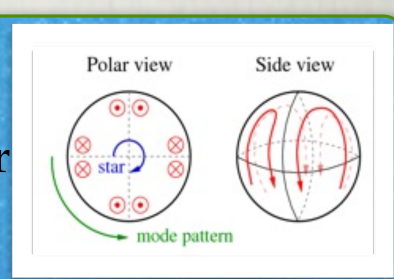
- Pulsars are rotating compact stars with very collimated beams that are observed as extremely regular pulses
- Pulsar timing data (frequencies & time derivatives) present by far the most precise observational property of compact stars ... frequencies known up to 13 digits
- Frequencies change over time by magnetic dipole radiation or gravitational wave emission due to oscillations of the star
- Whereas old pulsars can spin very fast $\lesssim kHz$ the spin rates of young pulsars are more than an order of magnitude lower, although they should be created with large frequencies due to the spin-up during the core-bounce ...



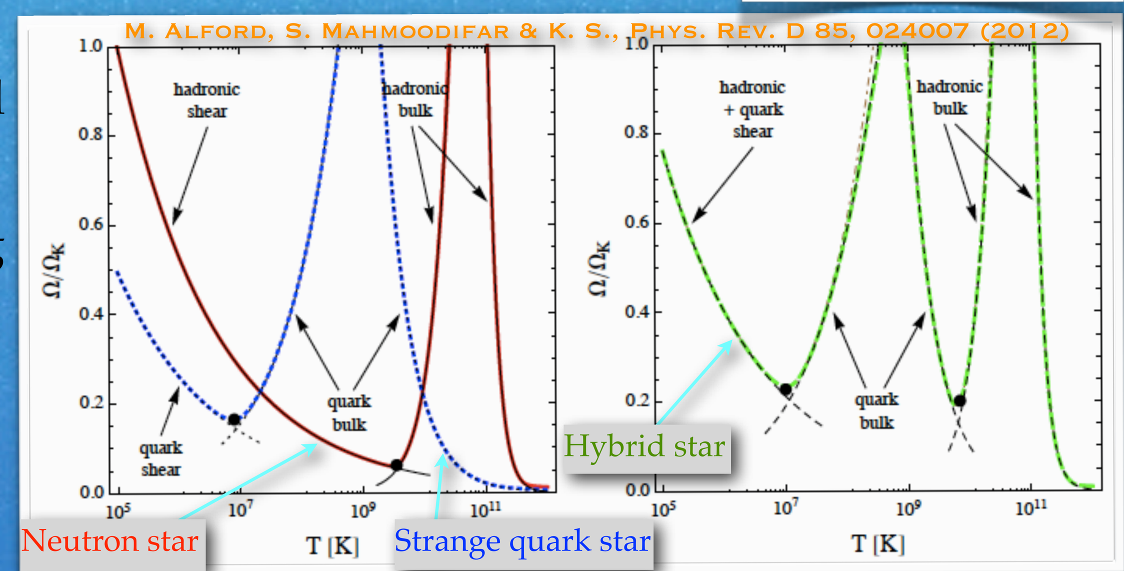


R-mode oscillations

- Oscillation modes allow to directly probe the star's interior just like "seismology" probes the composition of the earth
- R-mode oscillations are unstable to gravitational wave emission and require viscous damping
- R-modes are unstable only in characteristic instability regions
- Semi-analytic results for the boundary are very insensitive to quantitative microphysical details ②
- but depend strongly on the exponents ① for qualitatively different phases



M. Alford, S. Mahmoodifar & K. S., Phys. Rev. D 85, 024007 (2012)



Pulsar spindown

- The pulsar evolution of r-mode amplitude α, angular velocity Ω and temperature T is determined by conservation equations:

$$\frac{d\alpha}{dt} \approx \alpha \left(\frac{1}{|\tau_G|} - \frac{1}{\tau_V} \right), \quad \frac{d\Omega}{dt} \approx -\frac{2\Omega Q \alpha^2}{\tau_V} \quad \text{and} \quad \frac{dT}{dt} \approx -\frac{1}{C_V} \left(L_\nu - P_V \right)$$

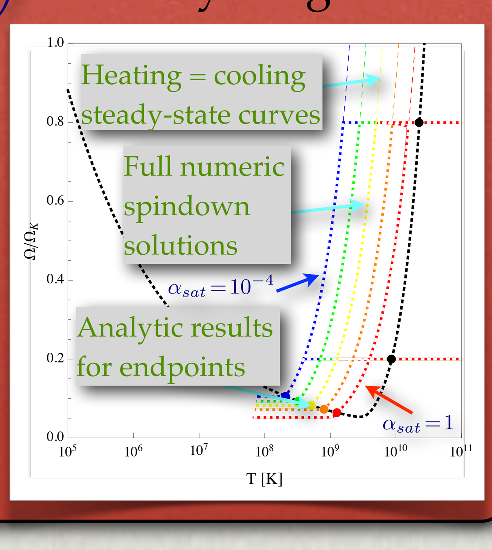
gravitational r-mode instability time scale; viscous damping time scale; specific heat capacity; neutrino luminosity; dissipated power

- Requires a non-linear saturation mechanism that saturates the r-mode at a finite amplitude α_{sat} where $\tau_V = |\tau_G|$
- At saturation the thermal evolution is always faster than the spin-down so the evolution follows a thermal steady state curve
- This allows to derive the analytic result for the final frequency

$$\Omega_f = \left(\left(\frac{3^8 5^3}{2^{17} \pi} \right)^{\theta+\sigma} \left(\frac{4\pi}{5} \right)^{\sigma} \frac{\tilde{S}^{\theta} \tilde{L}^{\sigma} \Lambda_{QCD}^{3\theta+9\sigma}}{\Lambda_{EW}^{4\sigma} \left(\tilde{J}^2 G M^2 R^3 \right)^{\theta+\sigma} \alpha_{sat}^{2\sigma}} \right)^{\frac{1}{6\theta+8\sigma}}$$

of a young star

- For a standard neutron star $\theta = 8$ (modified Urca) and $\sigma = 5/3$ (leptons) so that $\Omega_f \sim \tilde{S}^{\frac{3}{23}} \tilde{L}^{\frac{5}{184}}$ is extremely insensitive to the microphysics!
- This allows to give an error estimate $f_f^{(NS)} \approx (61.4 \pm 9.4)\,\mathrm{Hz}\ \alpha_{sat}^{-\frac{5}{92}}$
- Spindown times depend strongly on α_{sat}

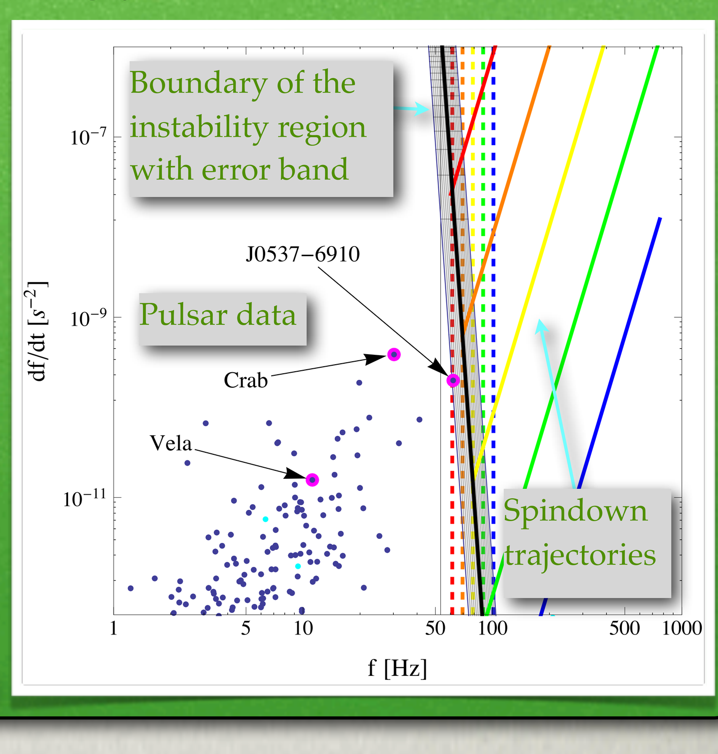


Direct connection between microphysics and macrophysical observables

Control over the uncertainties allows robust predictions

Comparison to pulsar timing data

- The result for the frequency to which r-modes can spin down a star allows a direct connection to pulsar timing data
- The fastest spinning observed young pulsar J0537-6910 could be inside of the r-mode instability region (T not known)
- In addition to the frequency also the spindown rate is know for many pulsars R. Manchester, et. al., Astron. J. 129, 1993 (2005)
- The spindown rate depends strongly on the saturation amplitude α_{sat}
- J0537-6910 is just at the lower boundary of the error band
  - Very likely no young pulsar currently spins down due to r-modes!
- R-modes can provide a quantitative explanation for the observed absence of fast young pulsars for: $0.01 \lesssim \alpha_{sat} \lesssim 0.1$



Gravitational wave detection

- The gravitational wave emission of pulsating compact stars is a source for next generation detectors, like advanced LIGO
- The optimal gravitational wave amplitude of an oscillating compact star observed on earth is analytically given by

$$h_c(\nu) = \sqrt{\frac{9 G I \nu}{20 D^2}}$$

in terms of the gravitational wave frequency $\nu = 2/(3\pi)\,\Omega$, the moment of inertia I and the distance D

B. Owen, et. al., Phys. Rev. D 58 (1998) 084020

- The above expression for the final frequency yields an analytic result for the amplitude close to the boundary of the instability region, where the evolution spend the most time, and allows to estimate the uncertainty to be within a factor 4
- Advanced LIGO should thereby be able to see such sources ... G.M. Harry, et. al., Class. Quant. Grav. 27 (2010) 084006