Comparing Models of (Strongly Coupled) Anisotropic Plasma

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Motivation

The quark-gluon plasma (QGP) can nowadays be produced fleetingly in heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). Shortly after its production the plasma is expected to have sizable pressure anisotropies that potentially lead to interesting modifications compared to the isotropic system e.g.

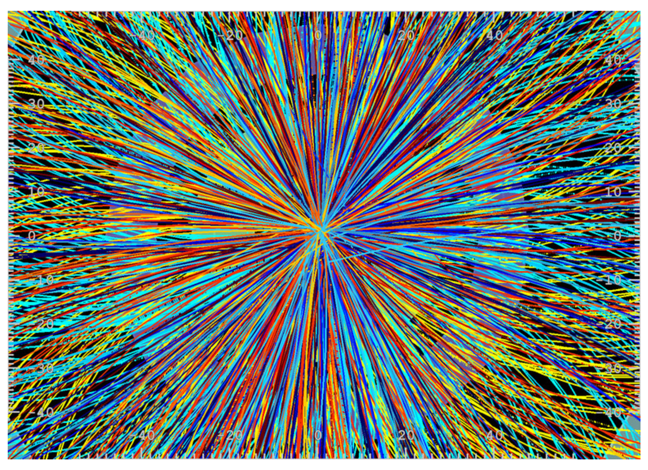
Figure: Collision of lead nuclei

- plasma instabilities
- more distinct transport coefficients
- directional dependence for jet quenching and heavy quark potentials
- ...

Holographic Models

Singular gravity dual (JW model) [1] The metric of the dual geometry in Fefferman-Graham (FG) coordinates

$$ds^2 = \frac{1}{u^2}\left(\gamma_{\mu\nu}(x^\sigma, u)dx^\mu d^x\nu + du^2\right)$$

can be related to the expectation value of the stress energy tensor. For a flat boundary metric at $u = 0$ and a gravity dual with a negative cosmological constant and no further matter fields the stress energy tensor

$$\langle T_{\mu\nu}(x^\sigma)\rangle = \frac{N_c^2}{2\pi^2}\gamma^{(4)}_{\mu\nu}(x^\sigma)$$

gives the boundary conditions for Einstein equations. Choosing $\langle T^\mu_\nu(x^\sigma)\rangle = \mathrm{diag}(\epsilon, P_\perp, P_\perp, P_z)$ the metric is of the form

$$ds^2 = \frac{1}{u^2}\left(-a(u)dt^2 + c(u)(dx^2 + dy^2) + b(u)dz^2 + du^2\right).$$

For $P_\perp \neq P_z$ a naked singularity appears, but it is still possible to compute correlators. However the limit $\omega \to 0$ is not meaningful which indicates the breakdown of our stationarity condition. For large enough ω we might be able to obtain the correct non-equilibrium behavior of an anisotropic conformal super Yang-Mills (SYM) plasma at infinite coupling.

Axion-dilaton gravity dual (MT model) [2] A regular and well behaved gravity dual can be obtained by adding a dilaton ϕ and an axion χ in the bulk

$$S_{bulk} = \frac{1}{\kappa^2}\int d^5x\sqrt{g}\left(\mathcal{R} + 12 - \frac{(\partial\phi)^2}{2} - e^{2\phi}\frac{(\partial\chi)^2}{2}\right)$$

and the metric is of the following form (here u is not the FG holographic coordinate)

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}(u)\mathcal{B}(u)dt^2 + dx^2 + dy^2 + \mathcal{H}(u)dz^2 + \frac{du^2}{\mathcal{F}(u)}\right).$$

- the model can be obtained from type IIB string theory
- the anisotropy is introduced by $\chi = az$
- $a = g_s dN_{D7}/dz$ can be viewed as a uniform density of D7 branes along the z-direction
- the boundary theory is not conformal $\langle T^\mu_\mu\rangle \propto a^4$ and therefore energy density and pressure depend separately on T/μ and a/μ.
- the boundary $\mathcal{N} = 4$ SYM gets deformed by

$$\delta S = \frac{1}{8\pi^2}\int \theta(z)\mathrm{Tr}\, F \wedge F \quad \text{with} \quad \theta(z) \propto az$$

Thermodynamics of θ-deformed gauge theory @ infinite and zero coupling

Since the gravity dual of the MT model has a regular horizon the thermodynamics of the system can be studied at **infinite coupling** [2]

$$S_{on-shell} = \frac{1}{T}\int d^3x\, f(T, N_{D7}/L_z) \quad (\text{where } f = F/V).$$

The pressures are

$$P_\perp = -f \quad \text{and} \quad P_z = -f + a\left(\frac{\partial f}{\partial a}\right)_T = -f + a\,\Phi$$

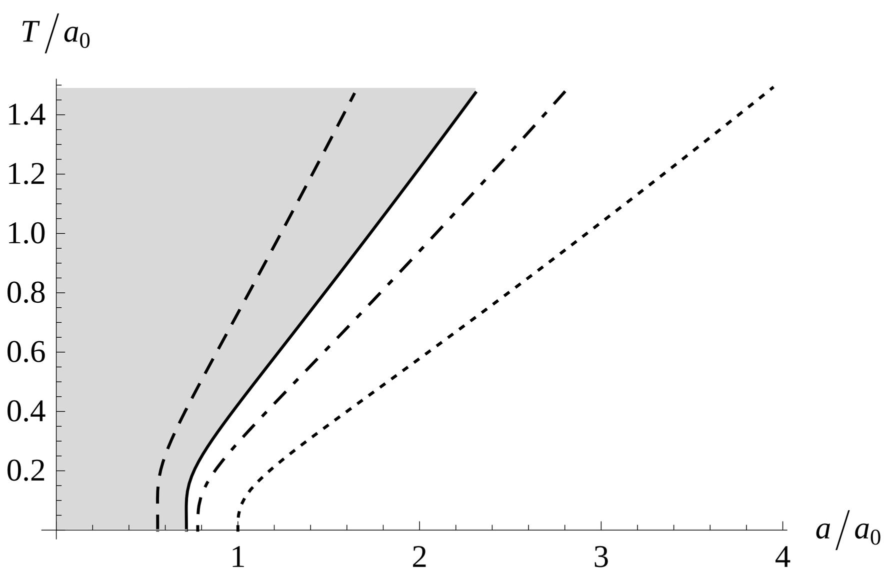

Figure: Phase diagram at infinite coupling. Left to the dashed line is the unstable region against filamentation in z-direction. The solid line is the boundary between metastable and stable region. The dot-dashed line indicates the boundary between prolate and oblate phases.

At **zero coupling** [3] we are led to consider a theory of free photons coupled to a source for 2+1 dimensional Chern-Simons operator

$$\omega_\pm^2 = \vec{k}^2 + \frac{a^2}{2}\left(1 \pm \sqrt{1 + \frac{4k_z^2}{a^2}}\right)$$

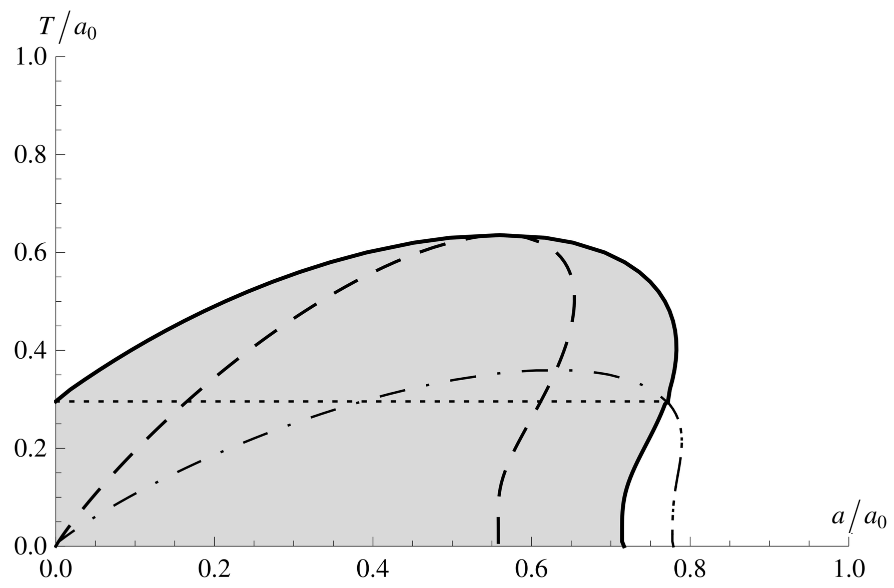

Figure: Phase diagram at zero coupling.

- rich thermodynamics also at zero coupling
- filamentation instabilities present only up to a certain temperature
- high temperature limit is always prolate
- at zero coupling there also exists a prolate but unstable phase

Shear viscosity below the KSS bound

Generically a viscosity can be defined as

$$\eta_{\mu\nu\rho\sigma} = \lim_{\omega\to 0}\Im\frac{i}{\omega}\int dtd^3xe^{i\omega t}\theta(t)\langle[T_{\mu\nu}(t,0), T_{\rho\sigma}(0,0)]\rangle.$$

In an anisotropic fluid with axial symmetry the viscosity tensor has 5 independent components, two of which are shear viscosities

$$\eta_\perp = \eta_{x\ x}^{\ y\ y} \quad \text{and} \quad \eta_{||} = \eta_{x\ x}^{\ z\ z} = \eta_{y\ y}^{\ z\ z}.$$

In the MT model we consider metric fluctuations $\psi_\perp(u,q) = h^x_y(u,q)$ and $\psi_L(u,q) = h^x_z(u,q)$ and find

$$\eta_\perp = \frac{\Pi_\perp(u_h, q)}{i\omega\psi_\perp(u_h, q)} = \frac{s}{4\pi}$$

and

$$\eta_{||} = \frac{\Pi_L(u_h, q)}{i\omega\psi_L(u_h, q)} = \frac{s}{4\pi\mathcal{H}(u_h)},$$

with $\Pi_{\perp,L}(u,q)$ being the conjugate momentum with respect to u. The flow equations for $\Pi_\perp$ and Π_L are trivial in the hydrodynamic limit and therefore the membrane paradigm gives the correct viscosities of the boundary theory.

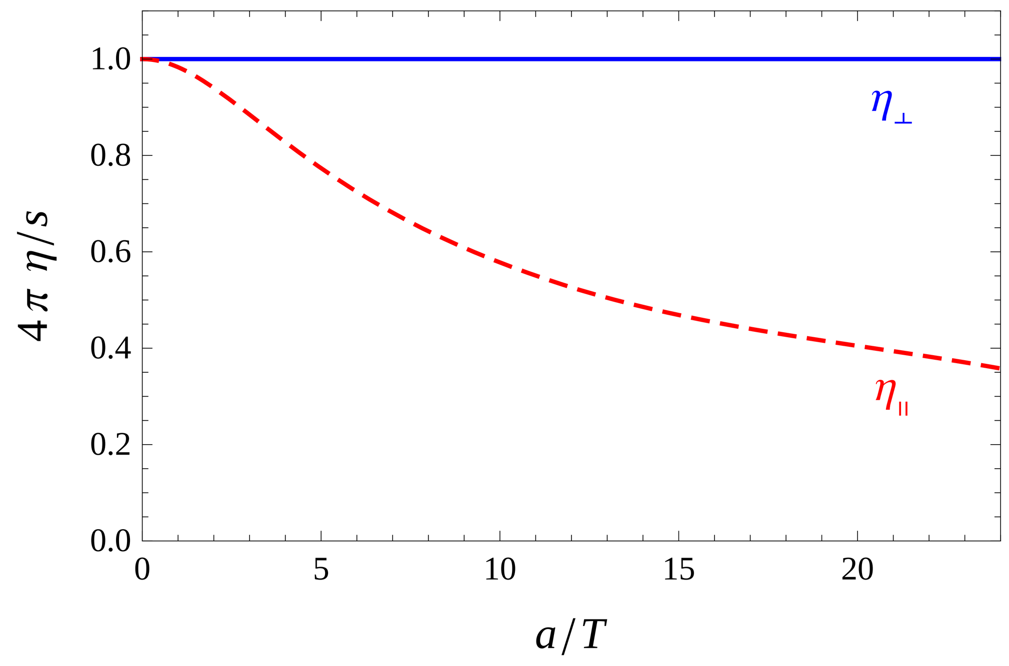

Figure: Transverse and longitudinal shear viscosity.

The longitudinal shear viscosity **violates the holographic viscosity bound** without recourse to higher-derivative gravity and with fully known gauge-gravity correspondence [4].
First simulations with `MUSIC`, a fully 3+1 dimensional hydrodynamic simulation code for heavy ion collisions [5], done with reduced longitudinal shear viscosity [6]: changes in rapidity dependence of v_2 that are however too small to be constrained by experiment.

Heavy quark potentials and jet quenching

Comparing the real part of anisotropic **heavy quark potential** (HQP) obtained from weakly coupled hard anisotropic loop effective theory (HAL) with holographic models we see that whether two quarks separated along or transverse to the anisotropy are direction are more strongly bound depends on the whether the plasma is oblate or prolate. This feature is also observed for the JW model [7].

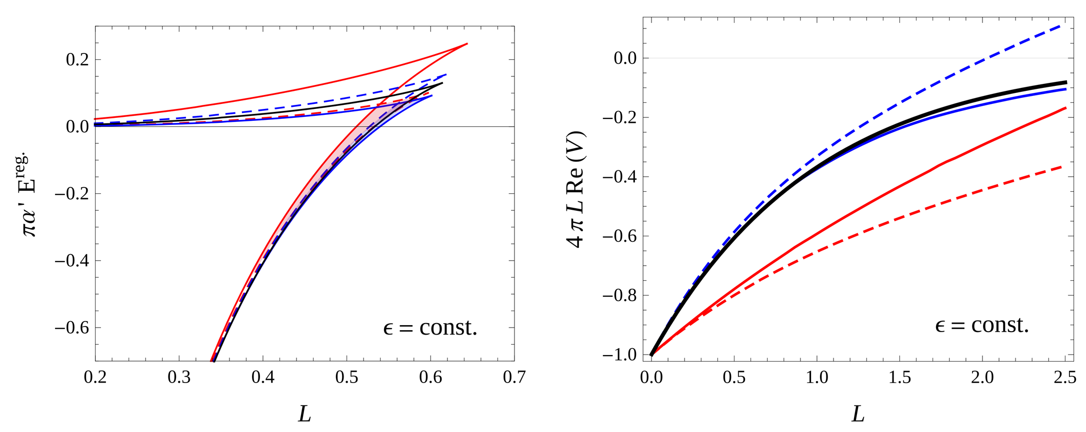

Figure: Anisotropic HQP for JW model (left) and with HAL formalism (right). Separation of quarks along z- (full lines) and transverse (dashed lines) direction for oblate (blue) and prolate (red) plasma

In the MT model quarks separated in the transverse direction are always more strongly bound [7, 8]. However this is also seen in the zero coupling boundary theory.

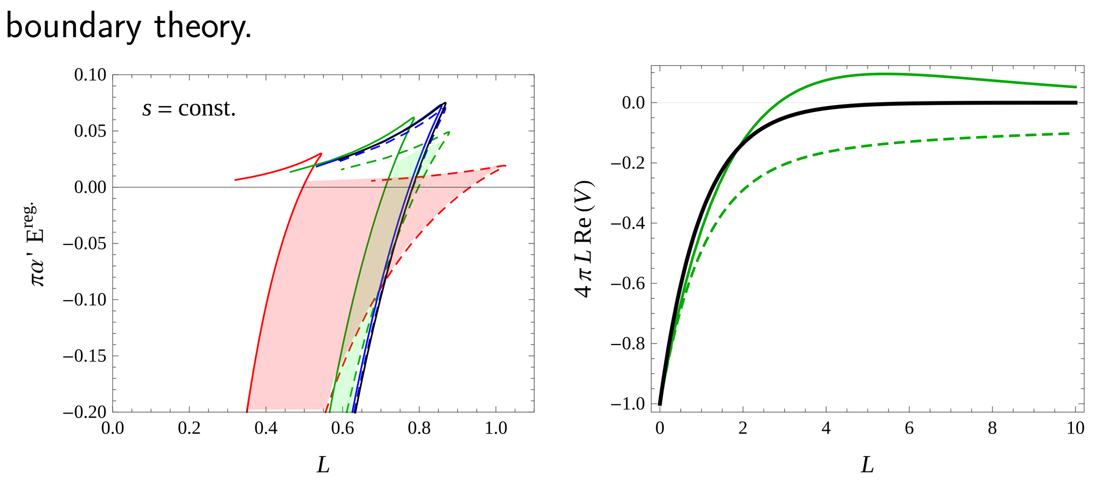

Figure: Anisotropic HQP for MT model (left) and theta deformed gauge theory at zero coupling (right). In the left plot $a/s^{1/3}$ increases from red (oblate phase) to green and blue (both prolate phase). In the right plot whether the plasma is oblate or prolate depends on the scale chosen.

Studying **jet quenching** at weak coupling by one-loop calculation using HAL we find that $\hat{q}_L > \hat{q}_\perp$ for oblate pressure anisotropy and $\hat{q}_L < \hat{q}_\perp$ for the prolate case [10]. In the JW model the ordering is the opposite and in the MT model $\hat{q}_L > \hat{q}_\perp$ always. Therefore here the MT model agrees with an oblate plasma at weak coupling while for HQP it looked prolate compared with weak coupling calculations [7, 9].

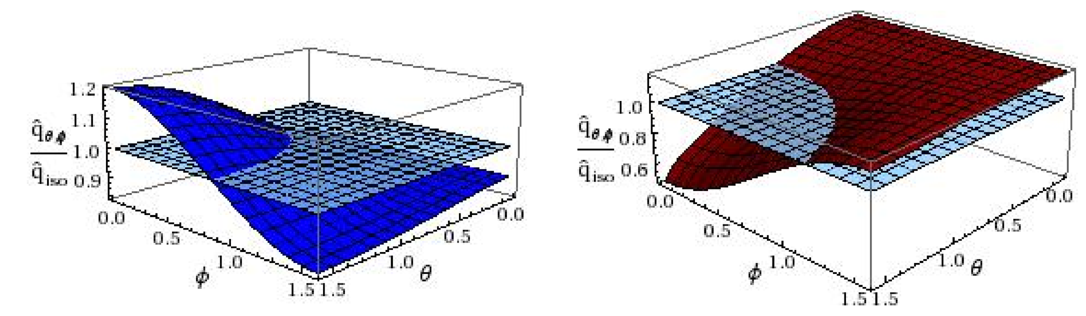

Figure: Jet quenching in JW model for oblate (left) and prolate (right) plasma at constant energy density. θ defines direction of moving quark away from the z-axis in the xz-plane and ϕ the direction of the momentum loss away from the y-axis in the plane transverse to the moving quark.

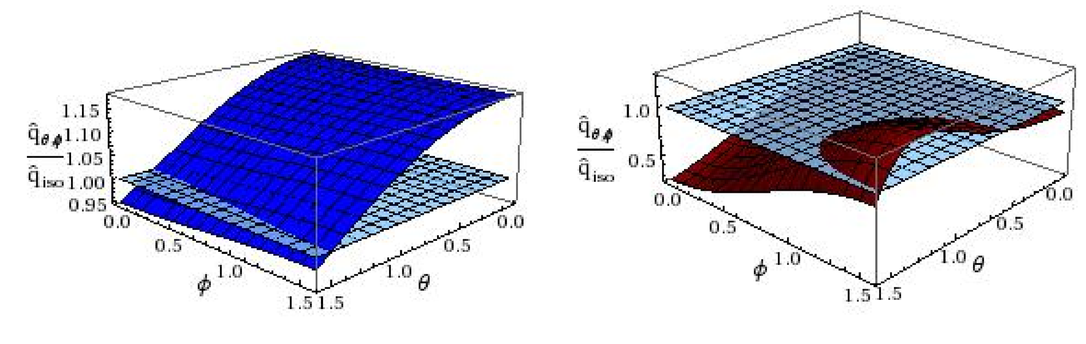

Figure: Jet quenching in MT model for oblate (left) and prolate (right) plasma at constant entropy density.

Anisotropic jet quenching could also be dominated by large chromomagnetic fields generated by plasma instabilities that give rise to $|B_\perp| > |E_\perp|$ and $|E_L| > |B_L|$ and it was argued that this gives $\hat{q}_L > \hat{q}_\perp$ in an oblate plasma [11]. In fact (different) instabilities for prolate anisotropies turn out to give exactly the same answer, meaning that $\hat{q}_L > \hat{q}_\perp$ also for prolate plasma [7].

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