Excitations of 't Hooft-Polyakov monopoles [1]

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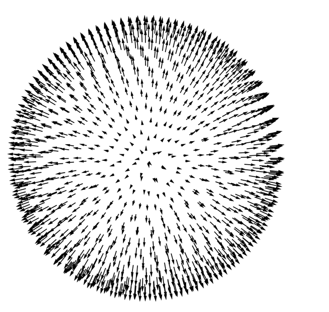
Introduction



- The 't Hooft-Polyakov monopole is a topological soliton with magnetic charge; 'hedgehog' scalar field stabilised by a (Wu-Yang) gauge field
- Most plausible grand unified theories predict 't Hooft-Polyakov monopoles – produced by Kibble-Zurek mechanism at phase transitions
- Quantum properties not particularly well understood: no complete one-loop mass correction calculation [2] – need to use lattice simulations
- MoEDAL experiment searching for monopoles produced at the LHC – how strongly might they interact with other particles?

Need techniques to probe properties of 't Hooft-Polyakov monopoles (pair production, interactions).

't Hooft-Polyakov monopoles on the lattice

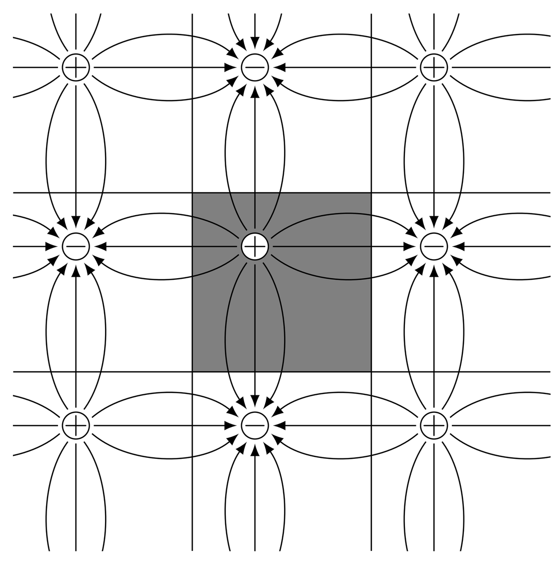
- Consider Georgi-Glashow: $\mathrm{SU}(2)$ YM with adjoint Higgs. Lattice action is

$$S = \sum_{\mathbf{x}} \left[2\sum_{\mu} \left(\mathrm{Tr}\Phi(\mathbf{x})^2 - \mathrm{Tr}\Phi(\mathbf{x})U_\mu(\mathbf{x})\Phi(\mathbf{x}+\hat{\mu})U_\mu^\dagger(\mathbf{x}) \right) + \frac{2}{g^2}\sum_{\mu<\nu}(2-\mathrm{Tr}U_{\mu\nu}(\mathbf{x})) + m^2\mathrm{Tr}\,\Phi^2 + \lambda(\mathrm{Tr}\,\Phi^2)^2 \right].$$

- Symmetry broken phase (classically $m^2 < 0$)
- Residual $\mathrm{U}(1)$: re-projected link angles

$$\alpha_\mu(\mathbf{x}) = \arg \frac{1+\hat{\Phi}(\mathbf{x})}{2} U_\mu(\mathbf{x}) \frac{1+\hat{\Phi}(\mathbf{x}+\hat{\mu})}{2}$$

from which we can get the lattice magnetic field and charge.



- Twisted boundary conditions [3]:

$$U_\mu(\mathbf{x}+L\hat{\jmath}) = \sigma_j U_\mu(\mathbf{x})\sigma_j$$
$$\Phi(\mathbf{x}+L\hat{\jmath}) = -\sigma_j \Phi(\mathbf{x})\sigma_j$$

– reverse the direction of magnetic flux (odd magnetic charge)

- Compare with C-periodic boundary conditions:

$$U_\mu(\mathbf{x}+L\hat{\jmath}) = = \sigma_2 U_\mu(\mathbf{x})\sigma_2$$
$$\Phi(\mathbf{x}+L\hat{\jmath}) = = -\sigma_2 \Phi(\mathbf{x})\sigma_2$$

– allow only even magnetic charge (including zero)

Form factors

- Form factor $\langle \mathbf{p}_2|\hat{\mathcal{O}}(0)|\mathbf{p}_1\rangle$ is most appropriate observable for studying interactions
- In the semiclassical limit, the form factor is given by the Fourier transform of the operator

$$f(\mathbf{p}_2,\mathbf{p}_1) = \langle \mathbf{p}_2|\hat{\mathcal{O}}(0)|\mathbf{p}_1\rangle \approx M\int \mathrm{d}^3x\; e^{i(\mathbf{p}_2-\mathbf{p}_1)\cdot\mathbf{x}}\mathcal{O}_{\mathrm{cl}}(\mathbf{x}),$$

- We will take the operator $\mathcal{O}$ to be $\mathrm{Tr}\,\Phi^2$ or $\mathbf{B}$
- Semiclassical results are then the Fourier transform of these operators in the monopole background
  - For $\mathbf{B}$, Coulomb result

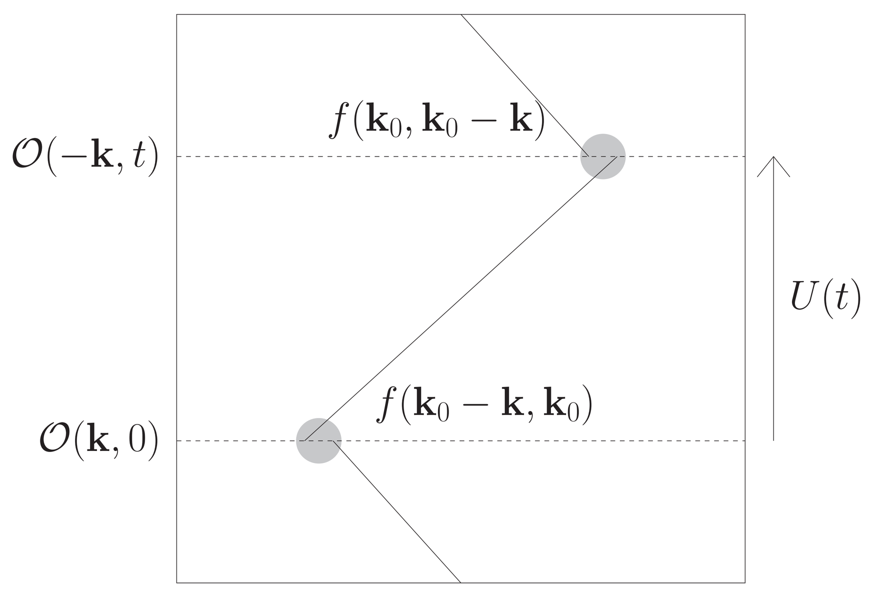
$$\langle \mathbf{k}|\hat{\mathbf{B}}(0)|\mathbf{0}\rangle = i\frac{4\pi M}{g}\frac{\mathbf{k}}{k^2}$$

  - For $\mathrm{Tr}\,\Phi^2$, obtain classical profile numerically and use to calculate $\langle \mathbf{k}|\mathrm{Tr}\hat{\Phi}^2(0)|\mathbf{0}\rangle$

Form factors on the lattice

- Consider the *worldline* of the monopole

$$\langle \mathcal{O}(0;\mathbf{k})\mathcal{O}(t;\mathbf{q})\rangle = \frac{\mathrm{Tr}\,U(T-t)\mathcal{O}(\mathbf{q})U(t)\mathcal{O}(\mathbf{k})}{\mathrm{Tr}\,U(T)}$$



- Defect recoils when interacting with particles of definite momentum k
- Assume large gap $\pi/L \gg k^2/2M$ to lowest two-particle state
- Work in limit of large T and t, do saddle point approximation for worldline
- With these approximations, we obtain

$$\langle \mathcal{O}(0,\mathbf{k})\mathcal{O}(t,\mathbf{q})\rangle = \frac{(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})}{L^3}\left(\frac{T}{M}\right)^{3/2}\frac{|f(\mathbf{k}_0,\mathbf{k}_0-\mathbf{k})|^2}{E_{\mathbf{k}_0-\mathbf{k}}E_{\mathbf{k}_0}W(\mathbf{k}_0)}e^{-S(\mathbf{k}_0)}.$$

(can derive equivalent expressions for other topological defects)

- Fully relativistic expression (rapidity change)
- Rearrange this to measure $|f(\mathbf{k}_0,\mathbf{k}_0-\mathbf{k})|^2$, and recover $f(\mathbf{k})$

Mass results

- Need the monopole mass M to measure $f(\mathbf{k})$
- Compare two methods to measure the mass
  - Free energy [4] due to the twist $\Delta F/T$

$$\Delta F = -\ln\frac{Z_{\mathrm{tw}}}{Z_C} = \int dg \left[\left\langle \frac{\partial S}{\partial g}\right\rangle_{\mathrm{tw}} - \left\langle \frac{\partial S}{\partial g}\right\rangle_C \right]$$

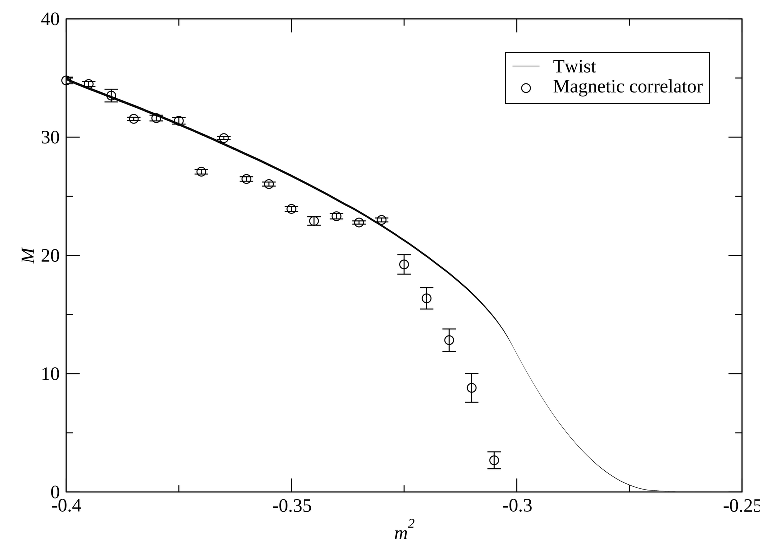
  – Then $\Delta F \to MT$ as $T\to\infty$

  - Correlator ('dispersion relation') when $k \ll M$,

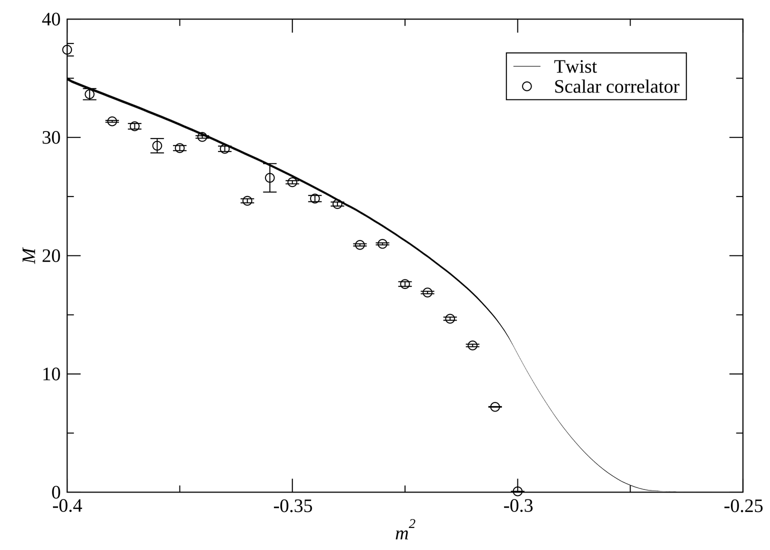
$$\langle \mathcal{O}(0,\mathbf{k})\mathcal{O}(t,\mathbf{q})\rangle = \frac{|f|^2}{M^2}e^{-\sqrt{M^2+\mathbf{k}_0^2}t-\sqrt{M^2+(\mathbf{k}-\mathbf{k}_0)^2}(T-t)+MT}.$$

  – solve for k_0 then obtain M by fitting to this expression

Measured with $B_i(x)$:



Measured with $\mathrm{Tr}\,\Phi^2$:



- Correlator measurements have relatively long autocorrelation time
  - Likely a consequence of large monopole mass
  - Semiclassical: need soliton worldline to be deformed
  - HMC doesn't seem to improve performance – used Metropolis and heatbath

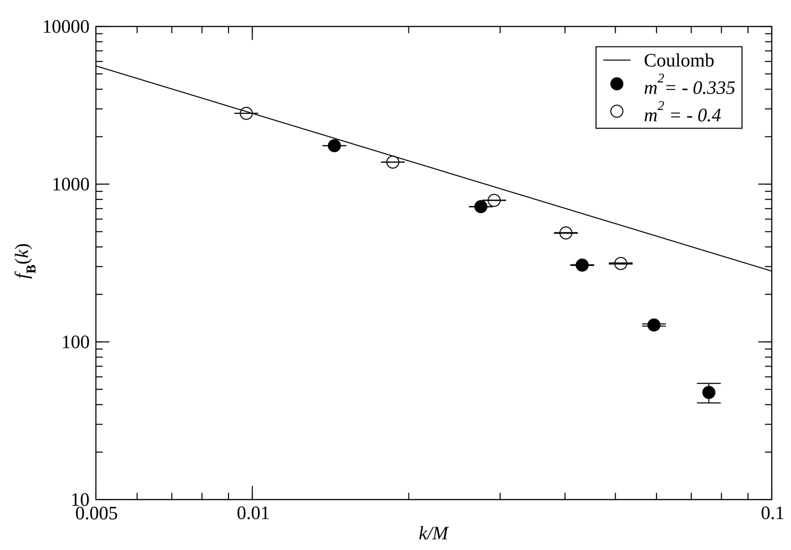
Form factor results

- Measure the form factor using

$$f(\mathbf{k}) = \pm i\sqrt{\langle \mathcal{O}(0,\mathbf{k})\mathcal{O}(t,-\mathbf{k})\rangle}\left(\frac{M}{T}\right)^{3/4}\sqrt{E_{k_0-k}E_{k_0}W(k_0)}e^{S(k_0)/2}$$

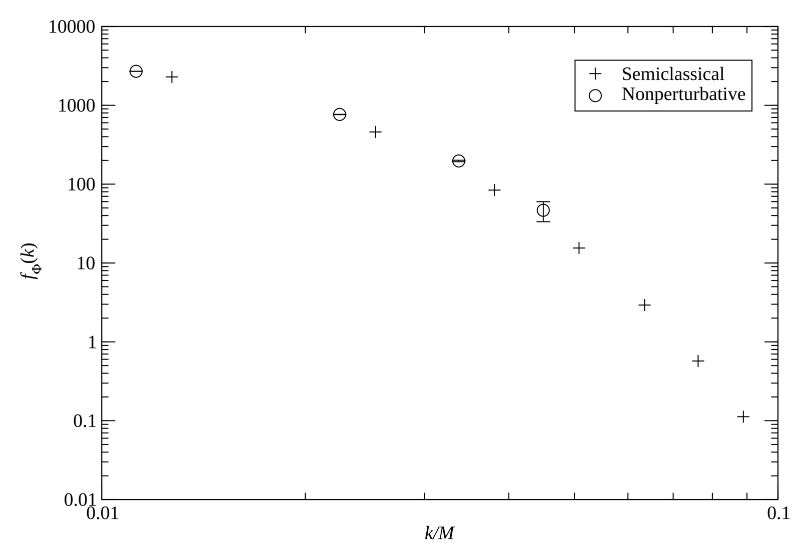
- Rotation invariance means that the magnetic field form factor $f_B(\mathbf{k})$ is always parallel to $\mathbf{k}$ – treat it as a (pure imaginary) scalar quantity
- $\mathrm{Tr}\,\Phi^2(\mathbf{k})$ is (by contrast) periodic – $f_\Phi(\mathbf{k})$ should be pure real

Magnetic field (photon)



Two values of m^2 shown.

Scalar field



Comparison with numerical semiclassical result.

- Scalar field behaviour is close to semiclassical expectations
- Magnetic field behaviour is dramatically different – deviation from Coulomb result
  - Possibly due to charge fluctuations in the core of the monopole over area $1/m_{\mathrm{H}}$
  - Expect that charge fluctuations have a finite continuum limit
  - Deeper in the broken phase, charge fluctuations smaller, closer to Coulomb result
- Signal for both is very clean, compared to correlator measurements of mass

Conclusions

- Mass and form factors of monopole measured using correlation functions
- Long autocorrelation times for mass, but good signal for form factor
- Charge fluctuations mean the quantum 't Hooft-Polyakov monopole does not appear pointlike either for the scalar or magnetic field
- Future work
  - Investigate lattice artefacts due to pinning when $m_{\mathrm{H}} \gtrsim 1$
  - Pair creation – need to analytically continue result
  - Work with actual photon operator rather than B_i
  - Smaller lattice spacing, lighter monopoles – charge distribution in the continuum limit
- Other applications
  - Kinks [5], domain walls (in preparation), cosmic strings
  - Intrinsic width of confining strings

Key references

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