# Modeling Phase Transitions in Dynamical Environments

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## Motivation and Cosmological Framework

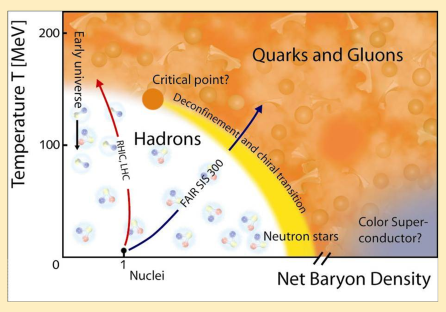

Figure: 1 The QCD phase diagram. In early universe the baryochemical potential $\mu$ is assumed to be close to zero.

An interesting way of testing the standard model is to look for possible consequences of predicted phase transitions in the early universe. Particularly, electro-weak and deconfinement/chiral phase transitions are often discussed in this context. Even stronger dynamical effects are expected in the fireballs created in relativistic heavy-ion collisions. Because of the fast expansion one should expect non-equilibrium effects, such as nucleation, spinodal decomposition, supercooling and reheating, to be important.

Due to theoretical uncertainties, we consider different possibilities regarding the type of a phase transition, the mechanism of the phase transformation and dynamics of the expansion. Since the early universe is almost baryon-antibaryon symmetric, its baryochemical potential is close to zero. As well established by lattice calculations, in this case the deconfinement phase transition occurs at a temperature below 200 MeV at time of $10^{-5}$ seconds after the big bang. We use an effective field-theoretical model to describe the QCD phase transition for different expansion rates. Assuming that the formation of a new phase proceeds via thermal fluctuations, we formulate an iterative scheme which detemines the Hubble parameter self-consistently for the case of cosmological expansion and finally the possibility of "small inflation" scenario is discussed.

- Around $10^{-34}$ after the big bang the universe is inflated and enters a homogeneous and isotropic rapidly expanding phase.
- Originally developed to explain flatness and other phenomena like density fluctuations.
- Expansion during the inflationary phase is provided by an unidentified scalar field (inflaton).
- Initial system is trapped in a flat potential while the expansion is driven by Friedman equations (1) and (2) from general relativity.

$$H^2 = \frac{8\pi G}{3}\rho \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{2}$$

$$H = \frac{\dot{a}}{a} \quad \rightarrow a(t) = a_0 \exp\left(\int_{t_0}^{t} H(t)dt\right) \tag{3}$$

If $H$ is constant (3) shows exponential expansion. We will now derive the dynamic equations for our model showing the interaction between field and radiation at the relevant phase

- Radiation
  - Equation of state in a radiation dominated universe
    $$P_R = \frac{\rho_R}{3} \tag{4}$$
  - Energy conservation
    $$\frac{d}{dt}\left(\rho a^3\right) + p\frac{d}{dt}\left(a^3\right) = 0 \rightarrow \dot{\rho}_R + 4H\rho_R = 0 \tag{5}$$
- Field
  - Energy momentum-tensor for scalar fields (without spatial terms)
    $$\rho = T^{00} = \frac{1}{2}\partial_t\phi\partial^t\phi + V_{\text{eff}} \tag{6}$$
    $$P = \frac{1}{3}\sum_i T^{ii} = \frac{1}{2}\partial_t\phi\partial^t\phi - V_{\text{eff}} \tag{7}$$
- Coupling of Field and Radiation
  $$\ddot{\phi} + \frac{dV}{d\phi} = -(3H + \Gamma)\dot{\phi} \tag{8}$$
  $$\dot{\rho}_R + 4H\rho_R = \Gamma\dot{\phi}^2 \tag{9}$$

The added $\Gamma$-term $\Gamma := \frac{g^2}{8\pi}m_\phi$, is responsible for the field decay into particles (e.g. $\phi \rightarrow q\bar{q}$).

## Effective Field Theory

An effective model should simulate the symmetry features of QCD to investigate such a phase transition by its fields' dynamics. One important symmetry is chiral symmetry. In a chiral model like the linear sigma model we consider a scalar field $\phi$ within the following potential (10).

$$V(\phi) = \frac{\lambda}{4}\left[\sigma^2 + \vec{\pi}^2 - \sigma_0^2\right]^2 - \epsilon\sigma + \text{T-dependent terms} \tag{10}$$

At $T < T_C$ the symmetry is spontaneously broken which is associated with the chiral condensate. Its excitations are Goldstone Bosons (Pions). At $T > T_C$ the symmetry is restored and the melting condensate forms a quark gluon plasma. For scale transformations ($x_\mu \rightarrow \lambda x_\mu$, $\phi \rightarrow \lambda^{-d}\phi$), $\mathcal{L}_{QCD}$ is invariant at classical level. However this is broken by the QCD trace anomaly. Therefore an effective model with an additional field $\chi$ (Dilaton), breaking scale invariance is needed. Now we introduce the scaled $\sigma$-model which obeys both, chiral symmetry and scale invariance.

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$$T^\mu_\mu = <\frac{\beta_{QCD}}{2g}G^a_{\mu\nu}G^{\mu\nu}_a> = 4U(\chi_0) - \frac{\partial U}{\partial\chi}\chi_0|_{\chi=\chi_0} = -b\chi_0^4 \tag{11}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\phi,\chi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + U(\chi^2) \tag{12}$$

$$V(\phi,\chi) = \frac{\lambda}{4}\left[\phi^2 - \sigma_0^2\frac{\chi^2}{\chi_0^2}\right]^2 - \epsilon\sigma \tag{13}$$

$$U(\chi^2) = 2b\chi^4\ln\left(\frac{\chi^2}{\Lambda^2}\right) \tag{14}$$

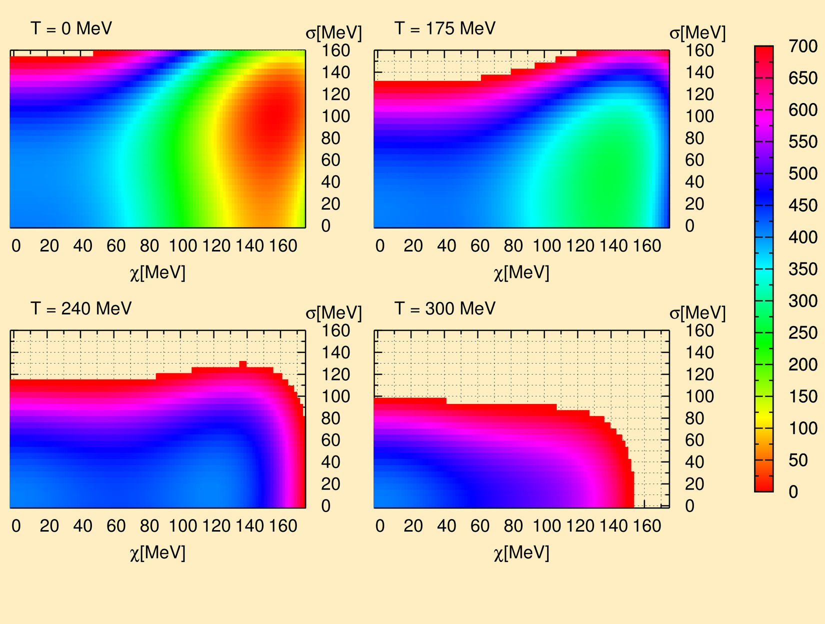

Figure: 2 $V(\sigma,\chi)$-potential for various temperature. We see that at $T_C$ = 240 MeV both states are on the same level. Energy levels are in MeV/fm$^3$

This $\chi$-field simulates scale properties of QCD and thus its vacuum excitations represent glueballs (11). The mixed term in the potential (13), scaled by the dilaton field also drives chiral symmetry.

## Reheating Processes

We introduce a thermal bath $<\phi^2>= T^2$ and a modified potential

$$U(\chi^2) = 2b\chi^4\ln\left(\frac{\chi^2}{\Lambda^2}\right) + A\chi^2T^2 \tag{15}$$

as shown in Figure (3)

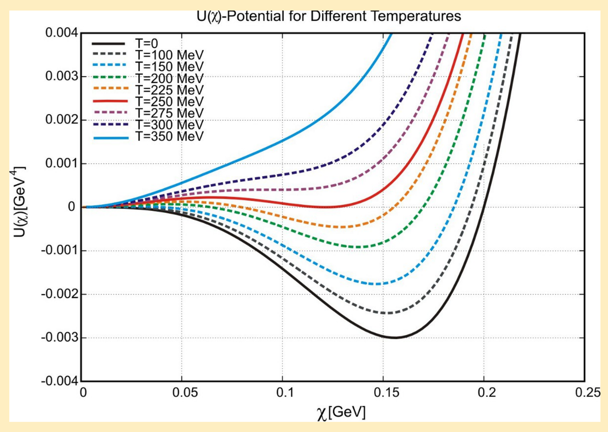

Figure: 3 Modified $U(\chi^2)$-potential . Mind the closed lines such as $T = 0$ (black line), $T_C$ = 250MeV (red line) and $T > T_C$ (blue line, 350MeV) relevant to display the transitional behaviour.

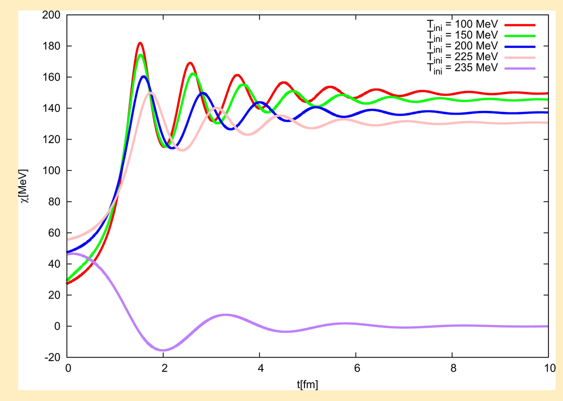

Figure: 4 Solution of equation of motion starting at potential barrier for different temperatures.

We assume that thermal fluctuations given by

$$P(\chi,\dot{\chi}) \simeq \exp\left[-\frac{\dot{\chi}^2 V_{max}}{2T}\right] \tag{16}$$

($V_{\max}$ being the maximum volume) create necessary kinetic energy to start a phase transition as a rolling down process. It is accompaigned by the energy dissipation due to the $\Gamma$-term.

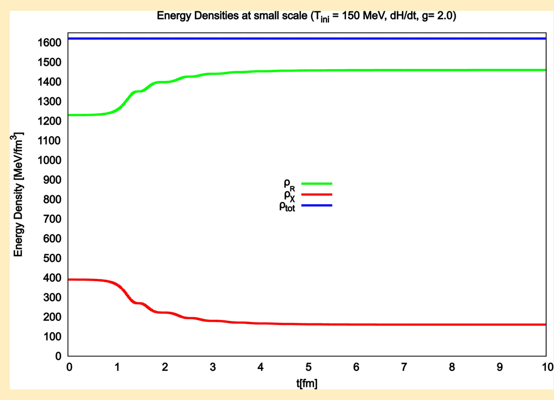

Figure: 5 Energy transfer from field to radiation during the phase transition at small scale (fermi) , displaying field- (red), radiation- (green), and total energy density (blue). The change in H and its influence on $\rho_{\text{total}}$ is not visible at this scale. The latent heat is given by (??)

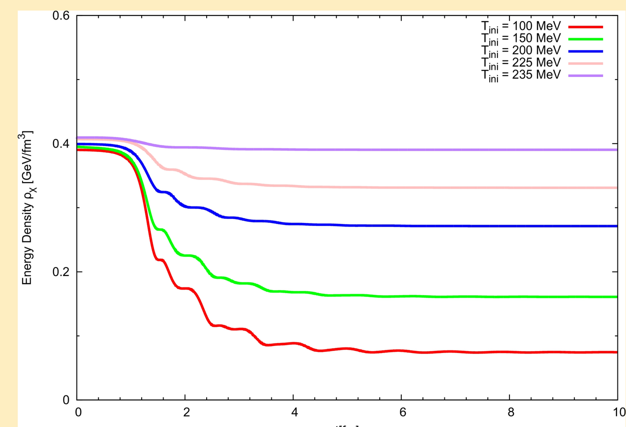

Figure: 6 Energy transfer as in previous plot, for different initial temperatures normalized in a way that the time delay ($\Delta t \sim \mathcal{O}(fm)$) until the transition occurs is not displayed.

Reheating begins when energy is transferred from field to particles ($\sigma \rightarrow 2\pi$; $\chi \rightarrow q\bar{q}$) as shown in Figures (4), (5) and (6), given an amount of latent heat by $\Delta\rho = AT_C^2(\chi^2_{max} - \chi_0^2)$, transferred into radiation.

## Inflation-like Cosmic QCD Phase Transition

In a self-consistent model we calculate analytically how interaction between energy densities and Hubble expansion takes place in a radiation dominated universe

For the exact analytic solution calculated from

$$H^2 = \zeta^2\rho_{\text{tot}}, \tag{17}$$

whereas $\zeta^2 = \frac{8\pi}{3M_{\text{Planck}}}$, one has to take into account that the total energy density $\rho_{\text{tot}} = \rho_R + B$ in fact contains also a constant vacuum energy $B = b\chi_0^4$ before phase transtion. Knowing that $\dot{\rho}_{\text{tot}}$ equals $\dot{\rho}_R$

$$\dot{\rho}_R = -4\zeta(\rho_R + B)^{1/2}\rho_R \tag{18}$$

leading to this solution:

$$\rho_{\text{tot}} = B\frac{1}{\tanh^2\left(-\frac{t}{2\tau}\right)} \tag{19}$$

and $\tau = \frac{1}{4\zeta\sqrt{B}}$ being the relaxation time of our system. Considering limiting cases for our total energy density we obtain

$$\rho_{\text{tot}} = \begin{cases} \frac{4B\tau^2}{t^2} = \frac{1}{4\zeta^2t^2} & t << \tau \\ B & t >> \tau \end{cases} \tag{20}$$

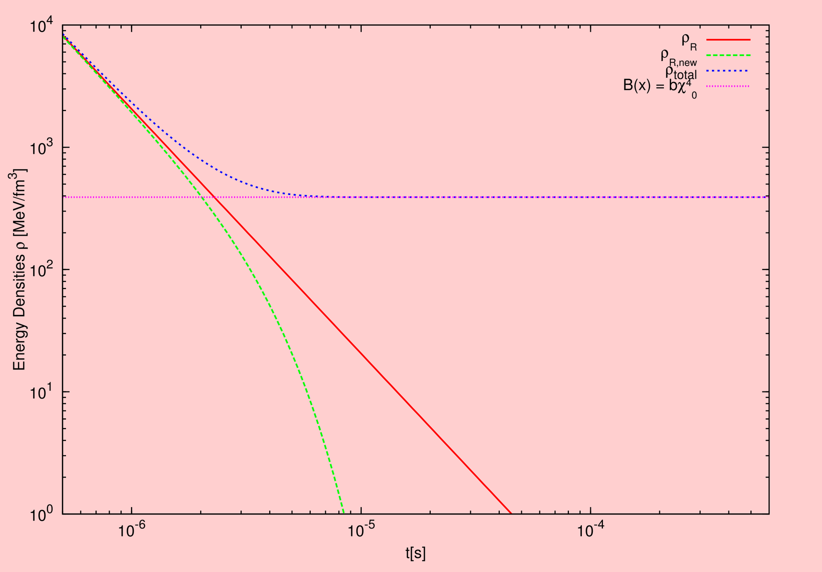

Figure: 7 Logscale description of the behaviour of energy densities in the early universe over time. The red line shows the evolution of $\rho_R$ in a standard radiation dominated universe, the green line display the same for a radiation dominated universe with an additional vacuum energy $B = b\chi_0^4$ (violet line) before transition. The blue line is the total energy density. For early times the total energy density has quite the same amount as the standard radiation energy density. For late times the radiation part appears to vanish and it obtains the value for the added vacuum energy $B$.

According to the dynamics in Figure (7) we observe a corresponding behaviour for the Hubble constant and the scale factor, showing the simple and exact analytic solutions. We clearly see that $H$ does not simply decay but arrives at a constant value, corresponding to the elaborated exponential increase in the scale factor.

## Conclusions

- A first order cosmic QCD phase transition can generate "mini-inflation" if supercooling is very strong ($\Delta t \sim \mu s$).
- In heavy ion collisions expansion is much faster and realistic calculations within a fast dynamical background are needed.
- Droplet formation with a finite radius should be considered.
- Full thermodynamical potential has to be impemented for a more realistic study.
- Future projects focus on studying possible first order transition in electroweak theories.