# High amplitude oscillations in dense matter

Mark Gower Alford
Washington University in St. Louis

M. Alford, S. Reddy, K. Schwenzer, arXiv:1110.6213
(Phys. Rev. Lett. 108, 111102 (2012)

M. Alford, S. Mahmoodifar, K. Schwenzer,
Phys. Rev. D85 024007 and 044051 (2012)

# Overview

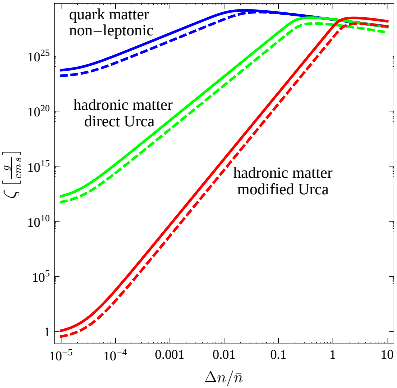
- Some oscillations of neutron stars can reach high amplitude. E.g. violent local accretion impacts, or unstable "r-modes".
- Response of different phases of dense matter to compression may provide signatures of their presence in neutron stars.
- At high enough amplitude and low enough temperature, processes that depend on flavor equilibration have amplitude-dependent ("suprathermal") enhancements.
- Suprathermal enhancement may be strong enough to overcome suppression due to
  - slowness of flavor equilibration
  - Cooper pairing of relevant fermions

# Overview

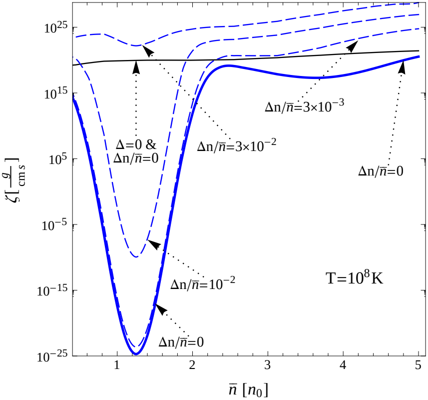
- Some oscillations of neutron stars can reach high amplitude. E.g. violent local accretion impacts, or unstable "r-modes".
- Response of different phases of dense matter to compression may provide signatures of their presence in neutron stars.
- At high enough amplitude and low enough temperature, processes that depend on flavor equilibration have amplitude-dependent ("suprathermal") enhancements.
- Suprathermal enhancement may be strong enough to overcome suppression due to
  - slowness of flavor equilibration
  - Cooper pairing of relevant fermions
- Consequences for high amplitude oscillations:
  - Enhancement of heating and neutrino emission
  - Superfluid phases can have unsuppressed bulk viscosity and neutrino emission
  - Enhanced bulk viscosity is capable of stopping r-mode growth (but only at *very* high amplitude).

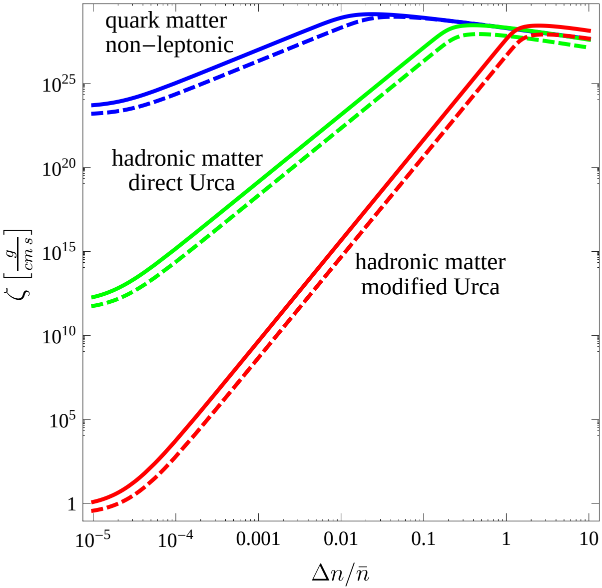
# Suprathermal enhancement of bulk viscosity

Unpaired matter, $T = 10^6\,\mathrm{K}$

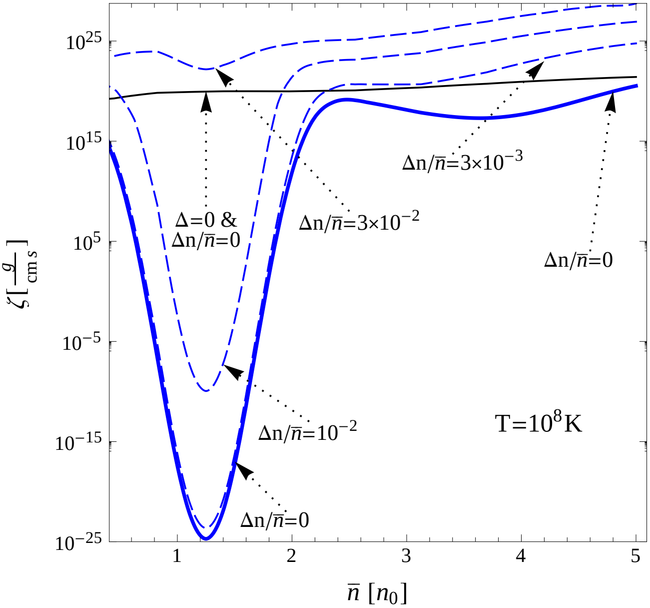



Nuclear superfluid, $T = 10^8\,\mathrm{K}$





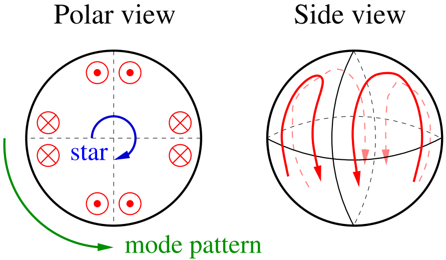

Unpaired matter, $T = 10^6\,\mathrm{K}$




Nuclear superfluid,
$T = 10^8\,\mathrm{K}$

# Signature: r-mode-induced spindown

An r-mode is a mainly quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.



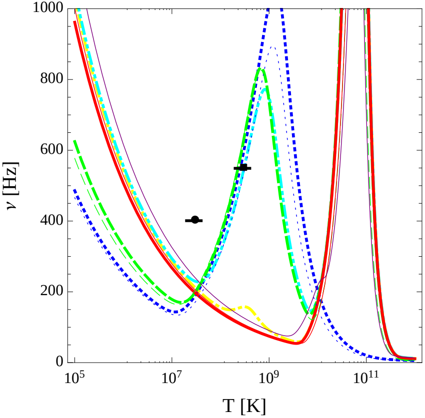

The unstable $r$-mode can spin the star down very quickly, exactly how fast depends on the amplitude at which it saturates.

(Andersson gr-qc/9706075; Friedman and Morsink gr-qc/9706073; Lindblom astro-ph/0101136).

So if we see a star spinning quickly, we can infer that the interior viscosity must be high enough to damp the $r$-modes.

# Constraints from r-modes: current results

Regions above curves are forbidden $\Leftarrow$ viscosity is too low to damp $r$-modes.




Neutron star, nuclear matter
Hybrid star, medium quark matter core
Hybrid star, large quark matter core
Quark star

LMXB data: Aql X-1 (square),
SAX J1808.4-3658 (circle).

Damping by crust is not included.

Alford, Mahmoodifar, Schwenzer `arXiv:1012.4883`

# What is bulk viscosity?

Energy consumed in a compression cycle:

$$\begin{aligned} V(t) &= \bar{V} + \delta V \sin(\omega t) \\ p(t) &= \bar{p} + \delta p \sin(\omega t + \phi) \end{aligned}$$

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\zeta}{\tau}\int_0^\tau (\mathrm{div}\,\vec{v})^2 dt = \frac{\zeta}{2}\omega^2 \frac{\delta V^2}{\bar{V}^2} = -\frac{1}{\tau \bar{V}}\int_0^\tau p(t)\frac{dV}{dt}dt$$

# What is bulk viscosity?

Energy consumed in a compression cycle:

$$
\begin{aligned}
V(t) &= \bar{V} + \delta V \sin(\omega t) \\
p(t) &= \bar{p} + \delta p \sin(\omega t + \phi)
\end{aligned}
$$

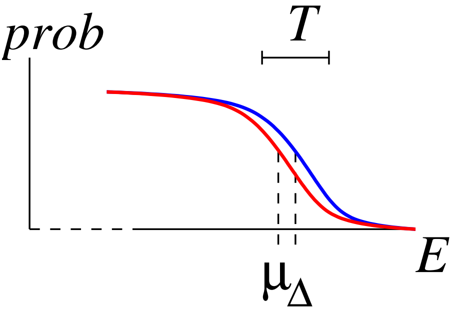
$$
\left\langle \frac{dE}{dt} \right\rangle = -\frac{\zeta}{\tau}\int_0^\tau (\mathrm{div}\,\vec{v})^2 dt = \frac{\zeta}{2}\omega^2 \frac{\delta V^2}{\bar{V}^2} = -\frac{1}{\tau \bar{V}}\int_0^\tau p(t)\frac{dV}{dt}dt
$$

- Bulk viscosity arises from re-equilibration processes.
- If some quantity goes out of equilibrium on compression, and re-equilibrates on a timescale comparable to $\tau$, then pressure gets out of phase with volume.
- The driving force then does net work in each cycle.
- There is an exact analogy with $V$ and $Q$ in an $R$-$C$ circuit.

# Subthermal vs Suprathermal

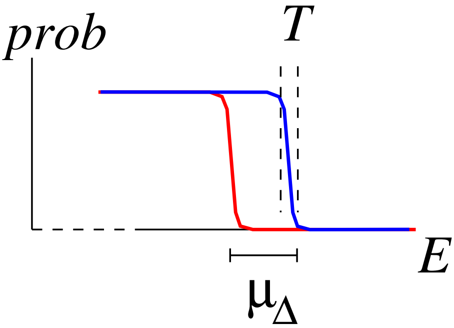
Subthermal

$\mu_\Delta \ll T \ll \mu$




Suprathermal

$T \ll \mu_\Delta \ll \mu$




Madsen, Phys. Rev. D46,3290 (1992); Reisenegger, Bonacic, `astro-ph/0303454`

# Calculating bulk viscosity

- Compression at frequency $\omega$.
  Density of conserved charge oscillates as $n(t) = \bar{n} + \delta n \sin(\omega t)$
- One quantity "$\Delta$" goes out of equilibrium
  E.g.: proton fraction (isospin), strangeness, ...
  In equilibrium, $\mu_\Delta = 0$.
- EoS is characterized by susceptibilities $B, C$.

$$\zeta = -\frac{1}{\pi}\frac{\bar{n}}{\delta n}\frac{C}{B}\int_0^\tau \mu_\Delta(t)\cos(\omega t)dt$$

Bulk visc arises from component of $\mu_\Delta$ that lags behind the forcing oscillation by a phase of 90°; $\mu_\Delta(t)$ is given by

$$\frac{d\mu_\Delta}{dt} = \underbrace{C\frac{\delta n}{\bar{n}}\omega\cos(\omega t)}_{\text{forcing osc.}} - \underbrace{\Gamma(\mu_\Delta, T)}_{\text{equilibration}}$$

# Suprathermal and subthermal bulk viscosity

Subthermal: assume $\mu_\Delta \ll T$,

$$\Gamma(\mu_\Delta, T) = \tilde{\Gamma} T^{2N} \frac{\mu_\Delta}{T}$$

Differential eqn is linear, so $\mu_\Delta \propto \delta n$, yielding

$$\zeta_{\mathrm{sub}} = \frac{C^2}{B} \frac{\gamma_{\mathrm{eff}}}{\omega^2 + {\gamma_{\mathrm{eff}}}^2} \qquad (\gamma_{\mathrm{eff}} \equiv B\tilde{\Gamma} T^{2N})$$

# Suprathermal and subthermal bulk viscosity

Subthermal: assume $\mu_\Delta \ll T$,

$$\Gamma(\mu_\Delta, T) = \tilde{\Gamma}\, T^{2N}\, \frac{\mu_\Delta}{T}$$

Differential eqn is linear, so $\mu_\Delta \propto \delta n$, yielding

$$\zeta_{\rm sub} = \frac{C^2}{B}\frac{\gamma_{\rm eff}}{\omega^2 + {\gamma_{\rm eff}}^2} \qquad (\gamma_{\rm eff} \equiv B\tilde{\Gamma}\, T^{2N})$$

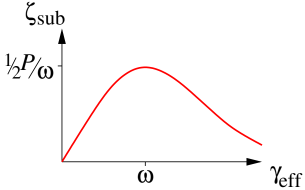
Suprathermal: allow $\mu_\Delta \gtrsim T$ (always assuming $\delta n \ll \bar{n}$).
Differential eqn is nonlinear, $\mu_\Delta(t)$ may not be harmonic.

$$\boxed{\frac{d\mu_\Delta}{dt} = C\omega\frac{\delta n}{\bar{n}}\cos(\omega t) - \gamma_{\rm eff}\mu_\Delta\left(1 + \chi_1\frac{{\mu_\Delta}^2}{T^2} + \cdots + \chi_N\frac{{\mu_\Delta}^{2N}}{T^{2N}}\right)}$$

Has to be solved numerically.

# The subthermal bulk viscosity

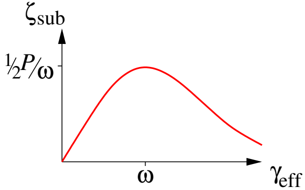
$$\zeta_{\rm sub}(\omega, T) = P\frac{\gamma_{\rm eff}(T)}{\gamma_{\rm eff}(T)^2 + \omega^2}$$




- $\zeta_{\rm sub}$ is *independent of driving amplitude*.
- Prefactor $P = C^2/B$ is a combination of susceptibilities.
- $\gamma_{\rm eff}$ is the effective re-equilibration rate per particle.

# The subthermal bulk viscosity

$$\zeta_{\text{sub}}(\omega, T) = P \frac{\gamma_{\text{eff}}(T)}{\gamma_{\text{eff}}(T)^2 + \omega^2}$$




- $\zeta_{\text{sub}}$ is *independent of driving amplitude*.
- Prefactor $P = C^2/B$ is a combination of susceptibilities.
- $\gamma_{\text{eff}}$ is the effective re-equilibration rate per particle.
- As $\gamma_{\text{eff}} \to 0$, $\zeta \to 0$. No equilibration.
- As $\gamma_{\text{eff}} \to \infty$, $\zeta \to 0$. Infinitely fast equilibration.

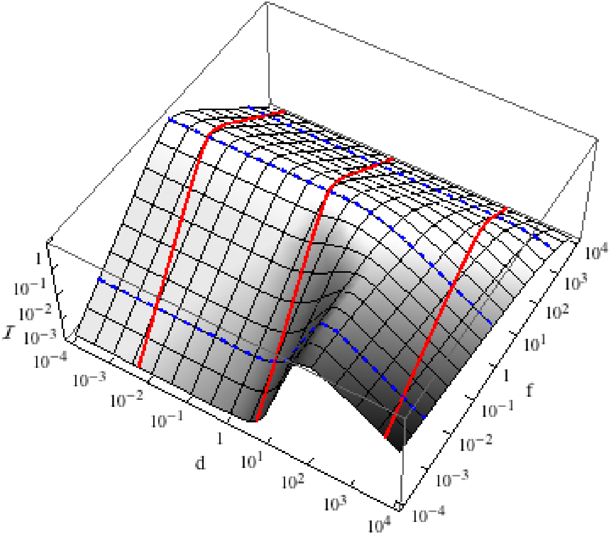
# The general bulk viscosity

To include the suprathermal regime, we have to solve the diffeq for $\mu_\Delta(t)$ numerically.

$$\zeta = \frac{C^2}{2\omega B}\mathcal{I}(d,f)$$

$$f = \gamma_{\rm eff}/\omega$$

$$d = (C/T)\,\delta n/\bar{n}$$

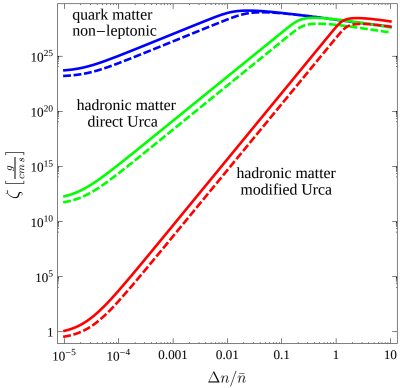



Nuclear matter,
modified Urca

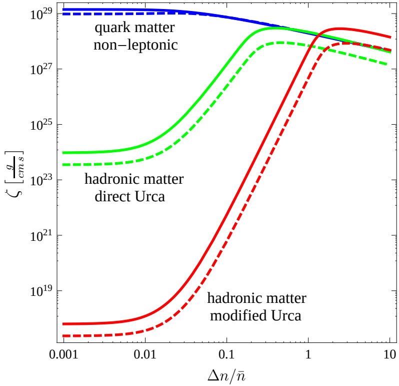
# Suprathermal enhancement of bulk viscosity

(ms pulsar)

$T = 10^6$ K




$T = 10^9$ K



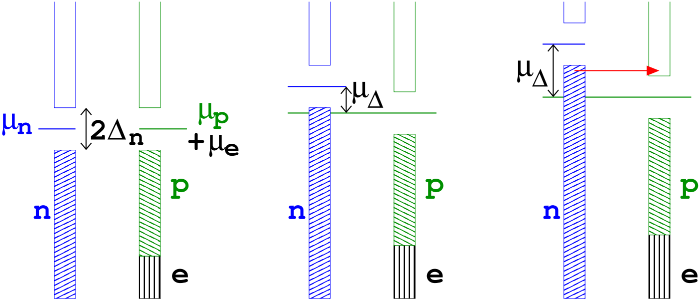

- Bulk visc rises very steeply in suprathermal regime
- Max reached at $\delta n/n \sim 0.1$; max value indp of temperature
- Suprathermal enhancement is greater at low $T$ and for matter where $\zeta$ goes as higher power of $\mu_\Delta$.

# Consequences of suprathermal enhancement

- Superthermal bulk viscosity can stop r-mode growth, but only at very high amplitude $\alpha \sim 1$ ($\delta n/\bar{n} \sim 0.03$).
  Other mechanisms may saturate r-mode first, e.g. mode-coupling at $\alpha \sim 10^{-4}$ (Bondarescu, Teukolsky, Wasserman, `arXiv:0809.3448`)
- Superthermal enhancement of bulk viscosity and neutrino emission may affect heating and cooling of stars undergoing r-mode spindown
- Response of stars to other high-amplitude compressions will also be affected.

# Suprathermal enhancement in a superfluid ("gap-bridging")

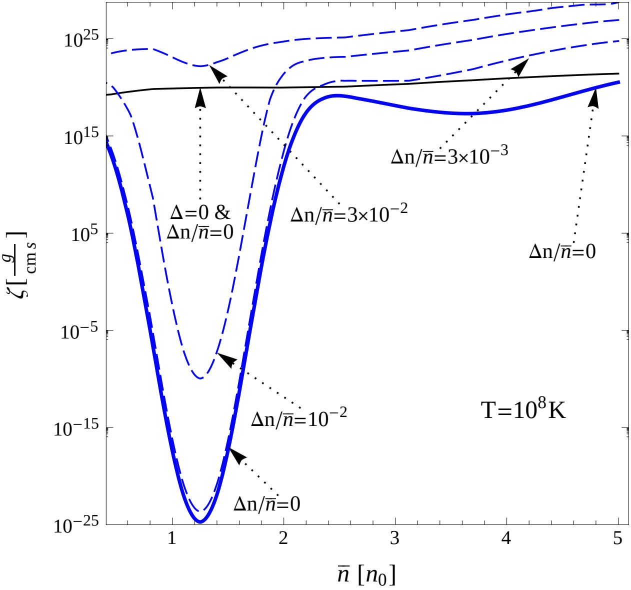
If density amplitude is high enough, $\mu_\Delta$ can be large enough to open up phase space above the gap, overcoming $\exp(-\Delta/T)$ suppression.




Beta equilibrium $\mu_\Delta = 0$ | Compression $\mu_\Delta < \Delta_n + \Delta_p$ | Gap–bridging compression $\mu_\Delta > \Delta_n + \Delta_p$

$\mu_\Delta = C\frac{\delta n}{\bar{n}}$ For nuclear matter (APR EoS), $C \sim 20$ to 150 MeV
Oscillation with $\delta n/\bar{n} \sim 0.01$ can overcome $\Delta \sim 1$ MeV.

# Gap-bridging enhancement of bulk viscosity




Illustrative example:

direct Urca allowed

*s*-wave pairing for *p* and *n*

$\Delta_p$ peak of 1 MeV at $n = 1.3n_0$

$\Delta_n$ peak of 0.12 MeV at $n = 3.7n_0$ [Cas A]

There is similar enhancement for neutrino emissivity.

# Future directions

Transport:

- Study suprathermal enhancement in other phases, e.g. Hyperonic nuclear matter, neutron star crust
- Gap-bridging: apply to realistic case, modified Urca, $^3P_2$ neutron pairing, etc.
- Investigate effect of multiple equilibrating quantities

Astrophysics:

- What is the saturation amplitude of an r-mode?
- Evolution of r-mode spindown, trajectory in $(T, \Omega)$ space (requires assumed r-mode saturation amplitude and a cooling model)
- Semi-analytic expressions for freq of star spun down by r-modes, insensitive to microscopic details and sat. ampl. (Alford & Schwenzer, forthcoming)
- Complications with r-modes: layered stars, role of crust, etc
- Apply to other modes, e.g. pulsations, f-modes (which emit grav waves), violent accretion events