The Chiral transition in a magnetic background: Finite density effects and the functional renormalization group.

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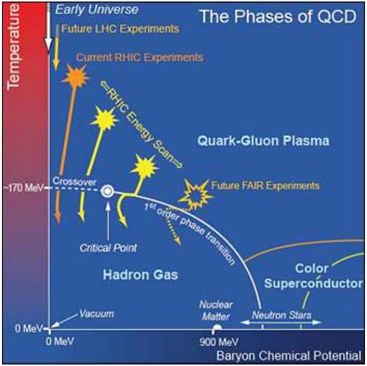
Introduction

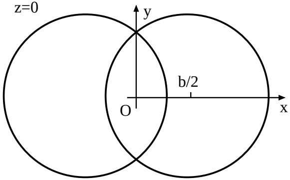
Phase diagram of QCD

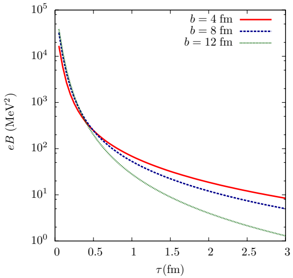


Introduction

Hadronic matter in strong magnetic fields:

- Non-central heavy-ion collisions [2]



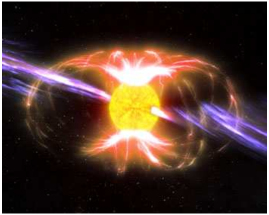


[2] Kharzeev, McLerran, and Warringa, Nucl. Phys. **A** 803 (2008) 227, Skokov, Illarianov, and Toneev, Int. J. Mod. Phys. **A** 24 (2009) 5925.

Introduction

Hadronic matter in strong magnetic fields:

- Magnetars $B = 10^{10}$ T [3]



- Electroweak phase transition $B = 10^{19}$ T [4]

[3] Duncan and Thompson

[4] Vachaspati, Enqvist and Olesen.

Lattice, effective theories, and models

Lattice, effective theories, and models

- Lattice (Endrődi et al, D'Elia et al , Bali et al.)
- Bag model (Fraga and Palhares, talk by Fraga)
- Chiral perturbation theory (Agasian and Shushpanov, Agasian and Fedorov, JOA...)
- (Polyakov-loop extended) Nambu-Jona-Lasinio model (Mizher, Chernodub, and Fraga, Fukushima, Ruggieri and Gatto, Kashiwa...)
- (Polyakov loop extended) Linear sigma model (with quarks) (Skokov, JOA+Tranberg)
- Holographic models (Preis, Rebhan, and Schmitt+poster by Preis)

Some important questions:

- Critical temperature as a function of B
  - Chiral condensate increases with B at $T = 0$
- Splitting of chiral and deconfinement transition
  - Can be addressed by adding the Polyakov loop
- Phase diagram in a magnetic background
  - Does the magnetic field change the order of the transition? Position of critical endpoint?

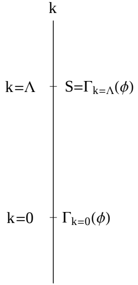
Lagrangian and symmetries

$$\mathcal{L} = \bar{\psi}\left[\gamma_\mu\partial_\mu - \mu\gamma_4 + g(\sigma - i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\right]\psi + \frac{1}{2}\left[(\partial_\mu\sigma)^2 + (\partial_\mu\boldsymbol{\pi})^2\right] + \frac{1}{2}m^2\left[\sigma^2 + \boldsymbol{\pi}^2\right] + \frac{\lambda}{24}\left[\sigma^2 + \boldsymbol{\pi}^2\right]^2 - h\sigma \,,$$

- $O(4)$-symmetry broken to $O(3)$ by chiral condensate. Three Goldstone bosons (pions).
- Magnetic field breaks $SU(2)$ isopin symmetry. Only the neutral pion is a Goldstone mode.

Functional renormalization group

- Flow equation for the effective average action $\Gamma_k[\phi]$ that interpolates between the classical action S for $k = \Lambda$ and full quantum effective action $\Gamma_0[\phi]$ for $k = 0$ [5].



[5] Wetterich, *Nucl. Phys. B* **352** (1991) 529. Schaefer and Wambach, Skokov...

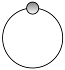
Functional renormalization group

$$S \quad \rightarrow \quad S + \frac{1}{2}\int \frac{d^dq}{(2\pi)^d}\phi(-q)R_k(q)\phi(q) \; ,$$

- $R_k(q)$ satisfies various conditions to implement RG ideas.

$$\partial_k\Gamma_k[\phi] \quad = \quad \frac{1}{2}\mathrm{Tr}\left[\partial_k R_k(q)\left[\Gamma_k^{(2)} + R_k(q)\right]^{-1}\right] \; .$$

$$\partial_k\Gamma_k \quad = \quad \frac{1}{2}$$



Functional renormalization group

- Cannot solve flow equation exactly. Derivative expansion:

$$\Gamma[\phi] = \int_0^\beta d\tau \int d^3x \left\{ \frac{1}{2} Z_k^{(1)} \left[(\nabla\sigma)^2 + (\nabla\boldsymbol{\pi})^2 \right] + \frac{1}{2} Z_k^{(2)} \left[(\partial_0\sigma)^2 + (\partial_0\boldsymbol{\pi})^2 \right] + U_k(\phi) + Z_k^{(3)} \bar{\psi}[\gamma_0\partial_0 - \gamma_4\mu]\psi + Z_k^{(4)} \bar{\psi}\gamma_i\partial_i\psi + g_k\bar{\psi}\left[\sigma - i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right]\psi + \ldots \right\} ,$$

- Local-potential approximation: $Z_k^{(i)} = 1$ yields equation for effective potential $U_k[\phi]$.

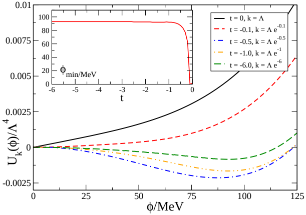
Functional renormalization group

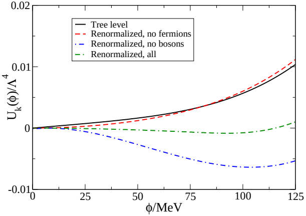
$$\begin{aligned}\partial_k U_k &= \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_{1,k}}\left(1+2n_B(\omega_{1,k}\right)+\frac{1}{\omega_{2,k}}\left(1+2n_B(\omega_{2,k}\right)\right] \\ &+\frac{|qB|}{2\pi^2}\sum_{m=0}^{\infty}\frac{k}{\omega_{1,k}}\sqrt{k^2-p_\perp^2(q,m,0)}\,\theta\left(k^2-p_\perp^2(q,m,0)\right) \\ &\times\left[1+2n_B(\omega_{1,k})\right] \\ &-\frac{N_c}{2\pi^2}\sum_{s,f,m=0}^{\infty}\frac{|q_fB|k}{\omega_{q,k}}\sqrt{k^2-p_\perp^2(q_f,m,s)} \\ &\times\theta\left(k^2-p_\perp^2(q_f,m,s)\right)\left[1-n_F^+(\omega_{q_f,k})-n_F^-(\omega_{q_f,k})\right] ,\end{aligned}$$

[4] Skokov, Phys. Rev. **D** 85 (2012) 034026

Numerical results

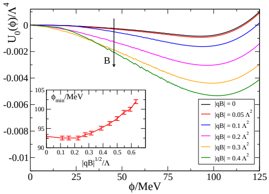
- Renormalization of effective potential
  - Tune bare parameters such that renormalized quantities in the vacuum are correct ($m_\pi = 140\,\mathrm{MeV}, \phi_{\min} = 93\,\mathrm{MeV}$...).

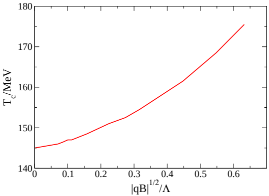




Numerical results

- Magnetic catalysis[6] and $T_c(B)$





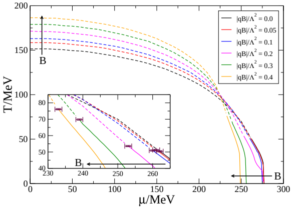
- T_c increases with B. Agreement with most model calculations. In disagreement with lattice at physical point [7]

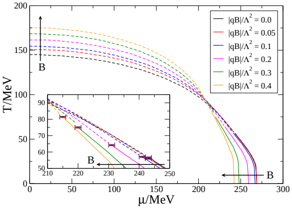
[6] V. Gusynin, V. Miransky, and I. Shovkovy, Phys. Rev.Lett. **73** (1994) 3499, Phys.Lett. **B** 349 (1995) 477...

[7] G. Endrődi *et al*, *JHEP* **1007** (2011) 001, M. D'Elia *et al Phys. Rev. D* **82**, (2010) 051501(R). G. S. Bali *et al* arXiv:1206.4205 [hep-lat].

Numerical results

- Phase diagram





- Inverse catalysis for large μ_B [8]

[8] Inagaki, Kimura, and Murata, Prog. Theor. Phys. **111** (2004) 371; Preis, Rebhan and Schmitt, JHEP **3** (2011) 33+poster by Preis.

Summary and Outlook

- Mapped out phase diagram in a magnetic background.
- Critical temperature increasing with magnetic field B at $\mu_B = 0$ and decreasing for large μ_B
- Understand disagreement between lattice results and model calculations.