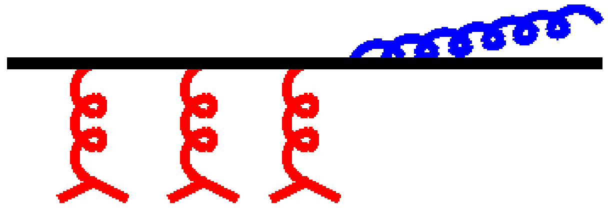
Coupling dependence of “jet”stopping in strongly-coupled gauge theories

Peter Arnold, Philip Szepietowski, and Diana Vaman

arXiV:1203.6658



$$Q_\perp \sim (\hat{q}E)^{1/4}$$

typical transverse momentum transfer during formation time

How stopping length scales with energy (massless case)

weak coupling: $\alpha_s \sim \alpha_s$ small — $\ell_{stop} \propto E^{1/2}$ (up to logs)

mixed coupling: α_s BIG, α_s small — $\ell_{stop} \propto E^{1/2}$ (believed); $\ell_{stop} \sim \alpha_s^{-1}(E/\hat{q})^{1/2}$

all strong coupling: $\alpha_s = \alpha_s$ BIG ($\mathcal{N}=4$ SYM, etc.) — $\ell_{stop} \propto E^{1/3}$

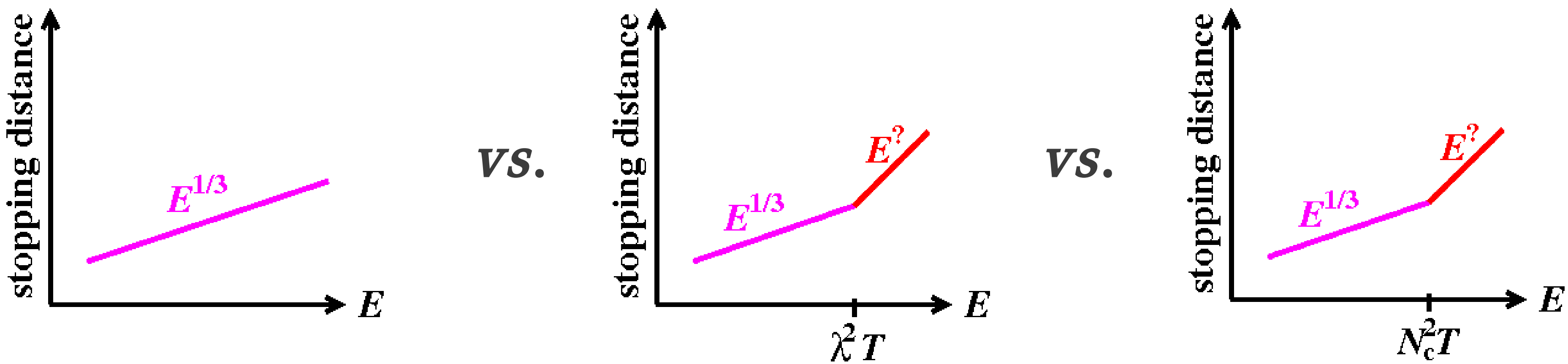
Interesting: Exponent in $\ell_{stop} \propto E^{\nu}$ can depend on α_s.

Caveat: "$\alpha_s = \alpha_s$ BIG" result $\ell_{stop} \propto E^{1/3}$ has only been derived for $N_c = \infty$ and $\lambda = N_c\alpha = \infty$.

What could we learn by also studying N_c and λ BIG but $< \infty$?

Answer: Is the high-energy behavior <u>really</u> $E^{1/3}$?

e.g.



<u>This talk</u>: $N_c = \infty$ but large $\lambda < \infty$.

Measuring the stopping distance

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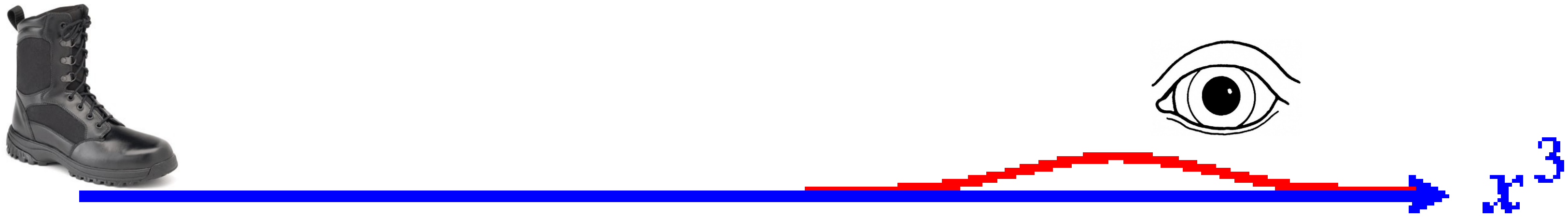
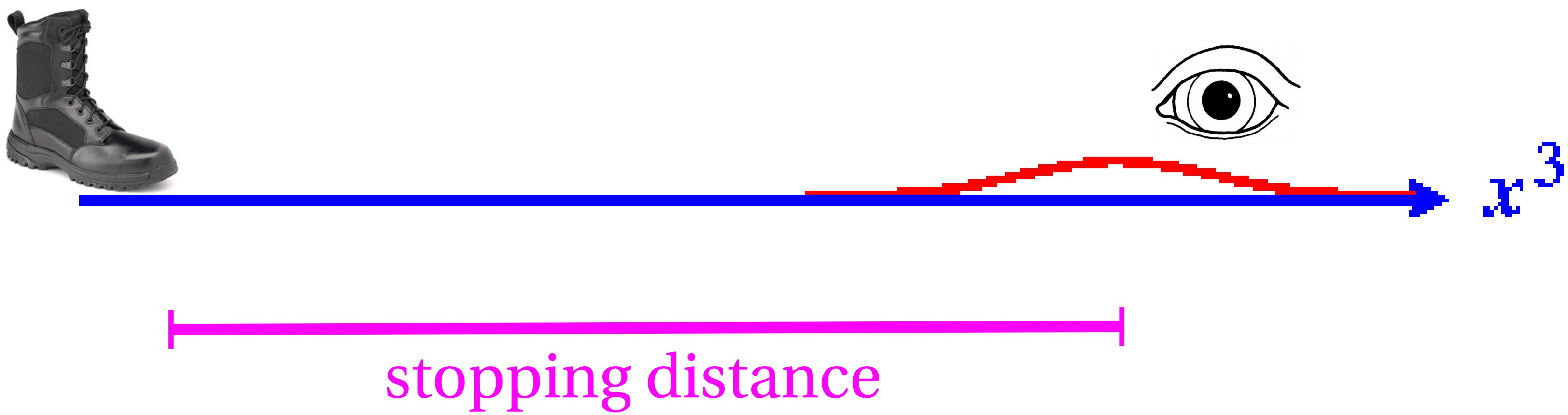
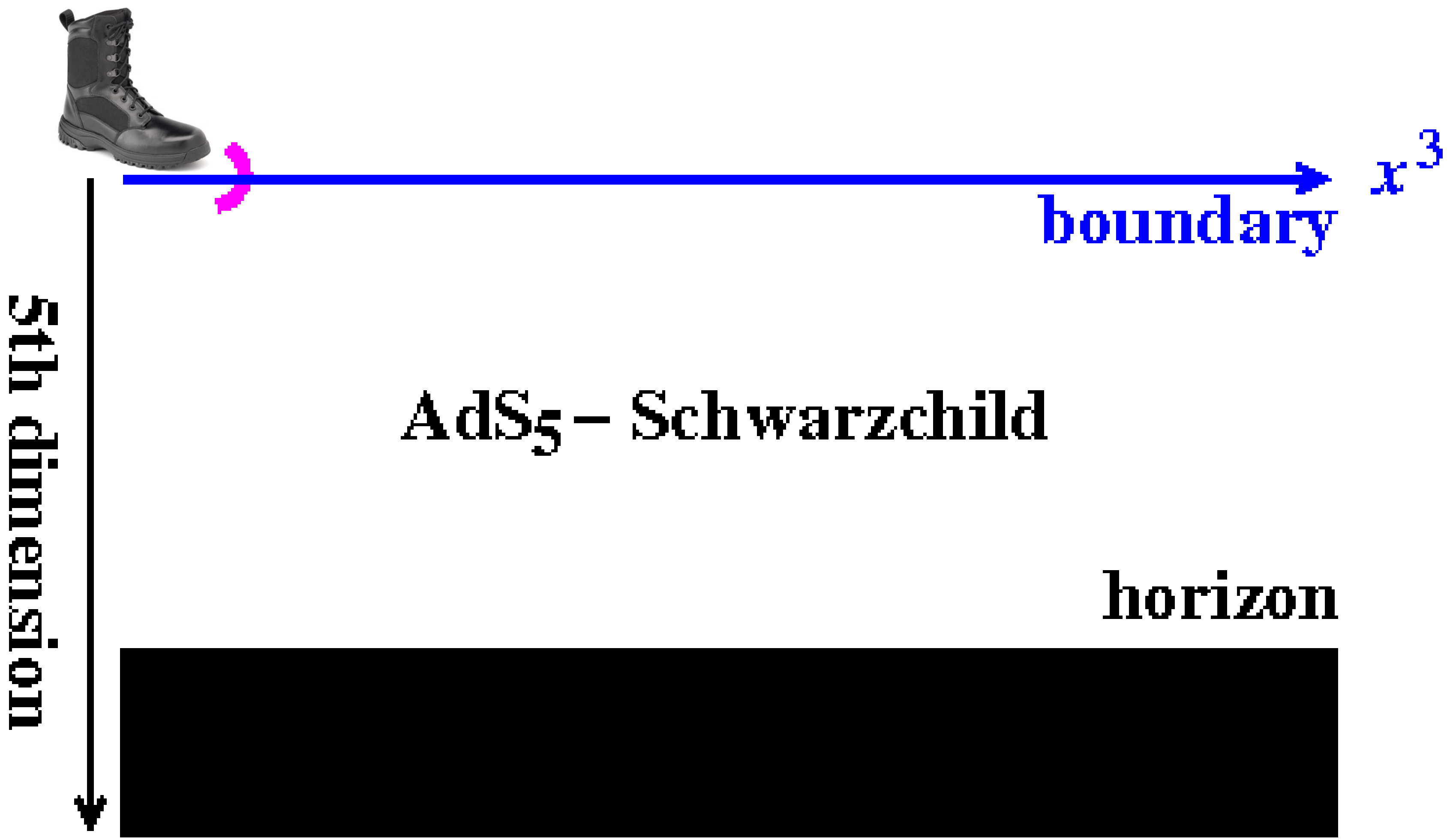


Measuring the stopping distance



Strong Coupling using AdS/CFT

BIG $\alpha_s = \alpha_s$: Large-N_c $\mathcal{N}$=4 SYM, etc. with $N_c\alpha_s \to \infty$



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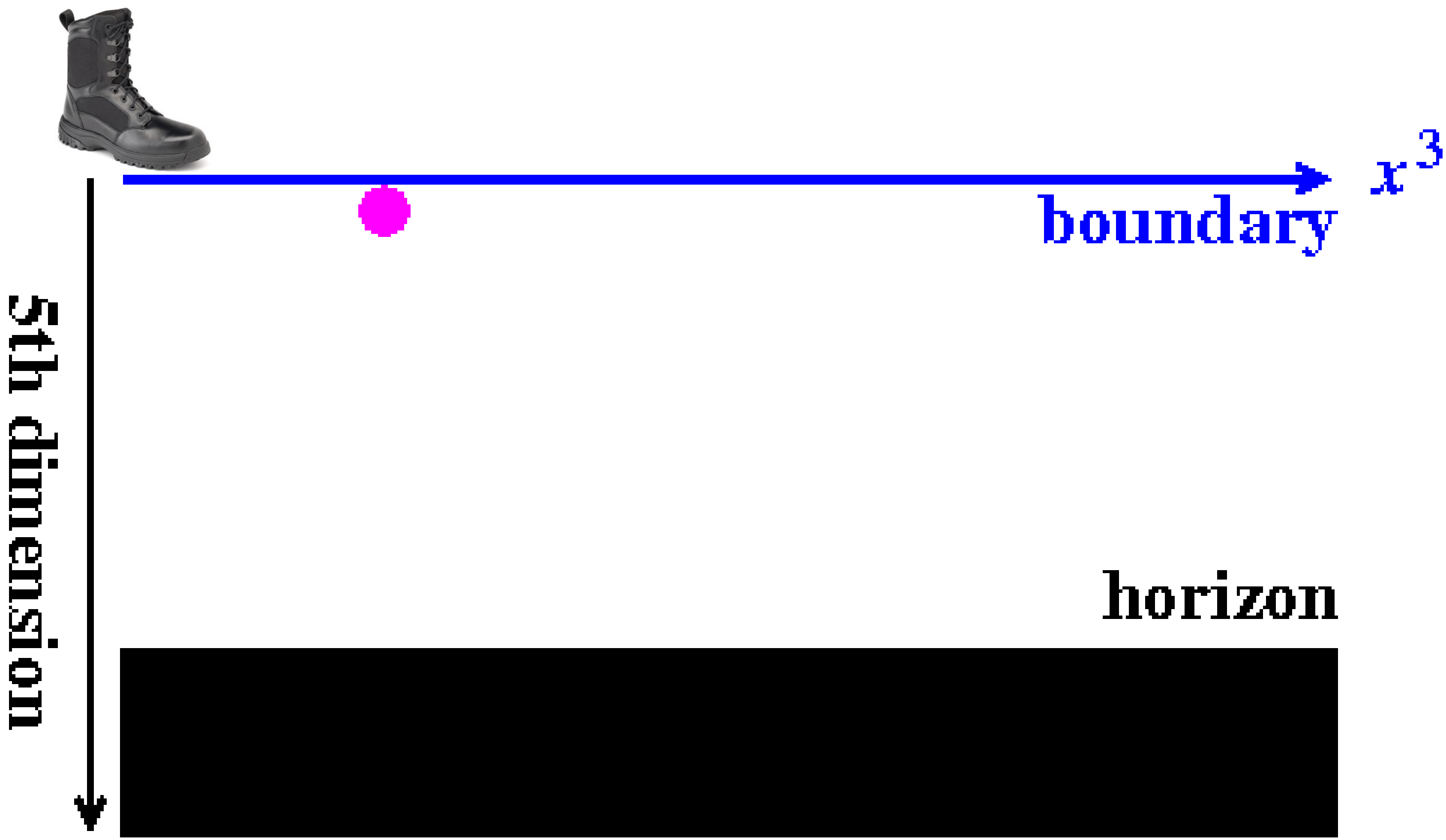

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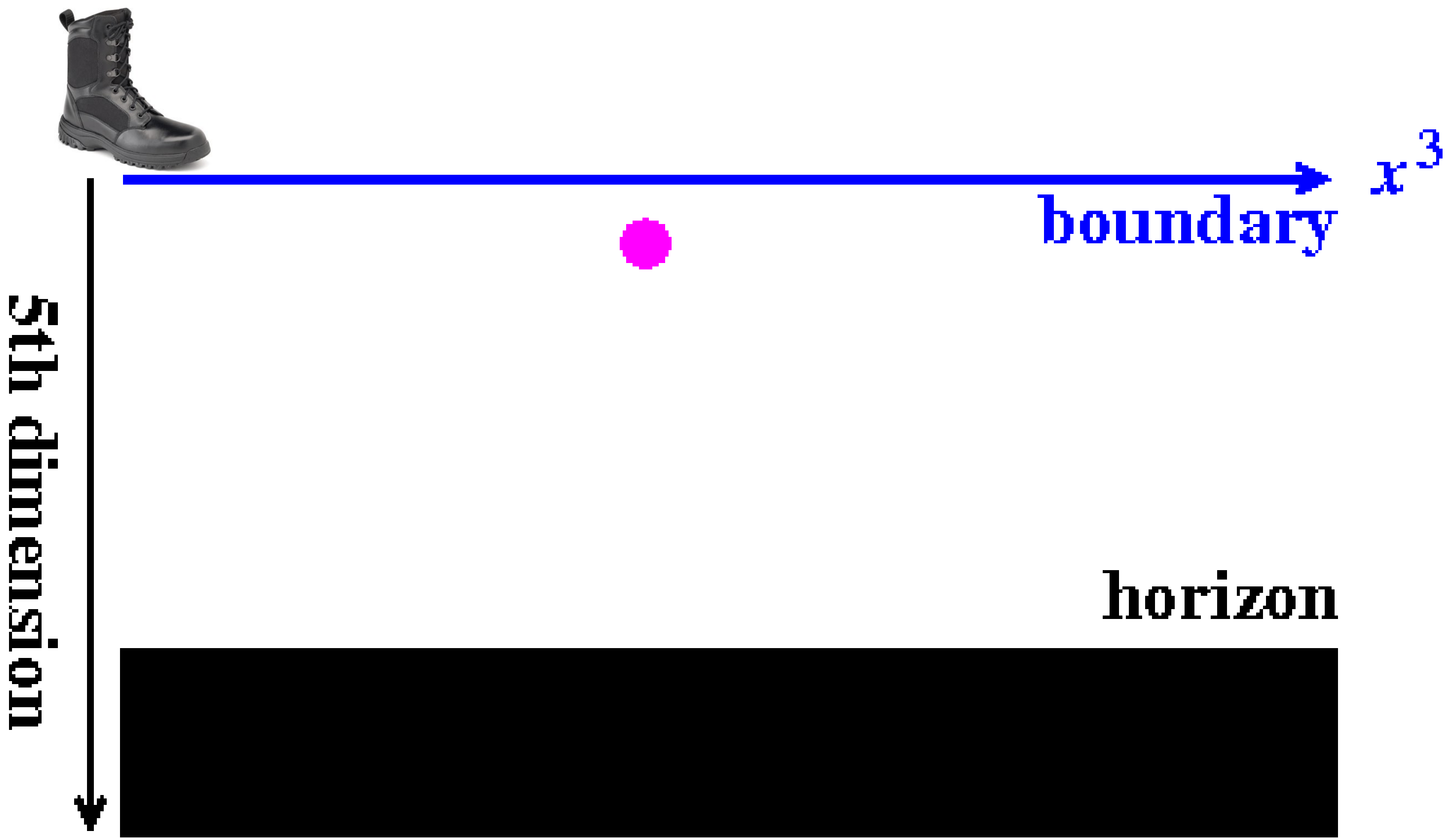

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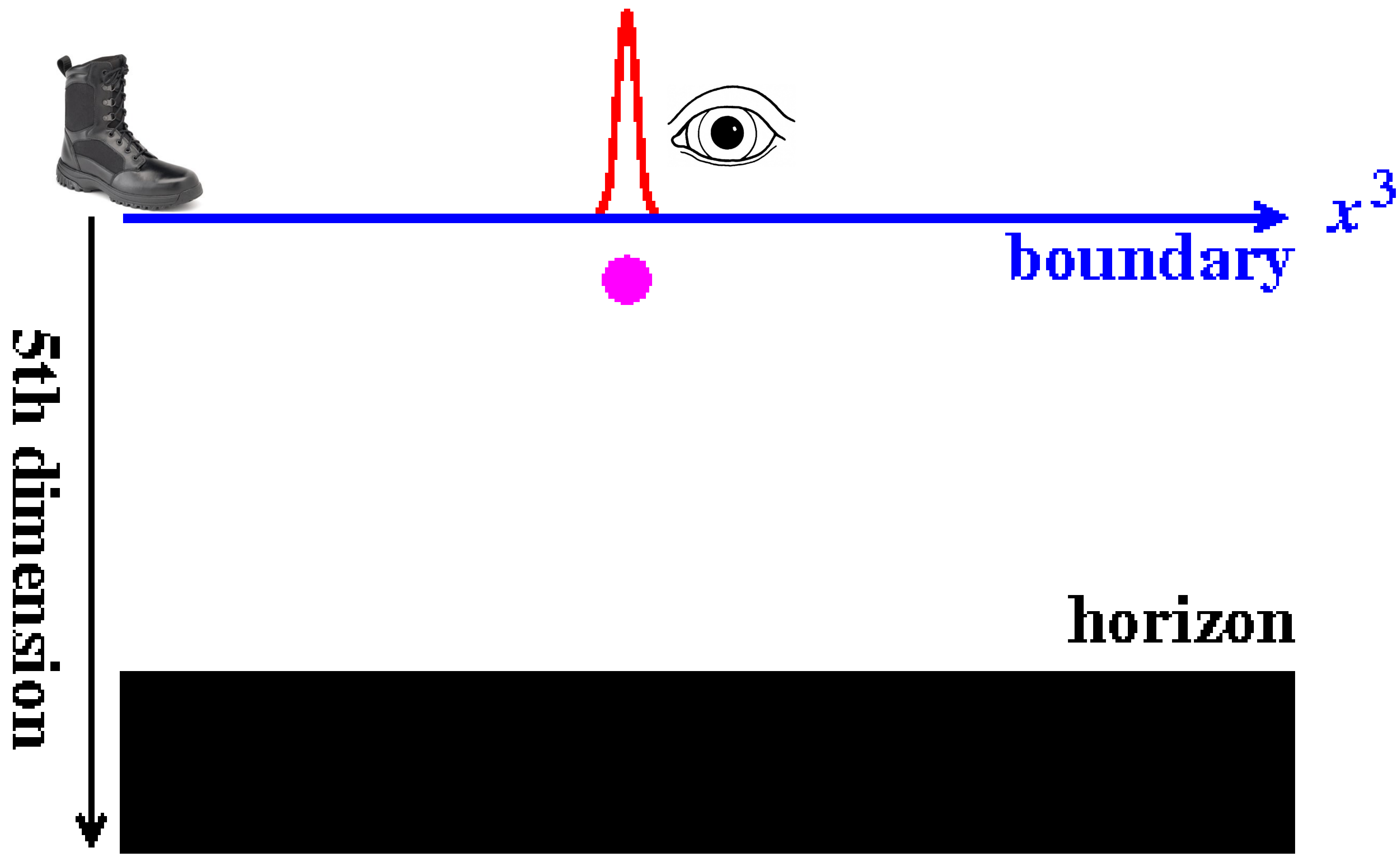

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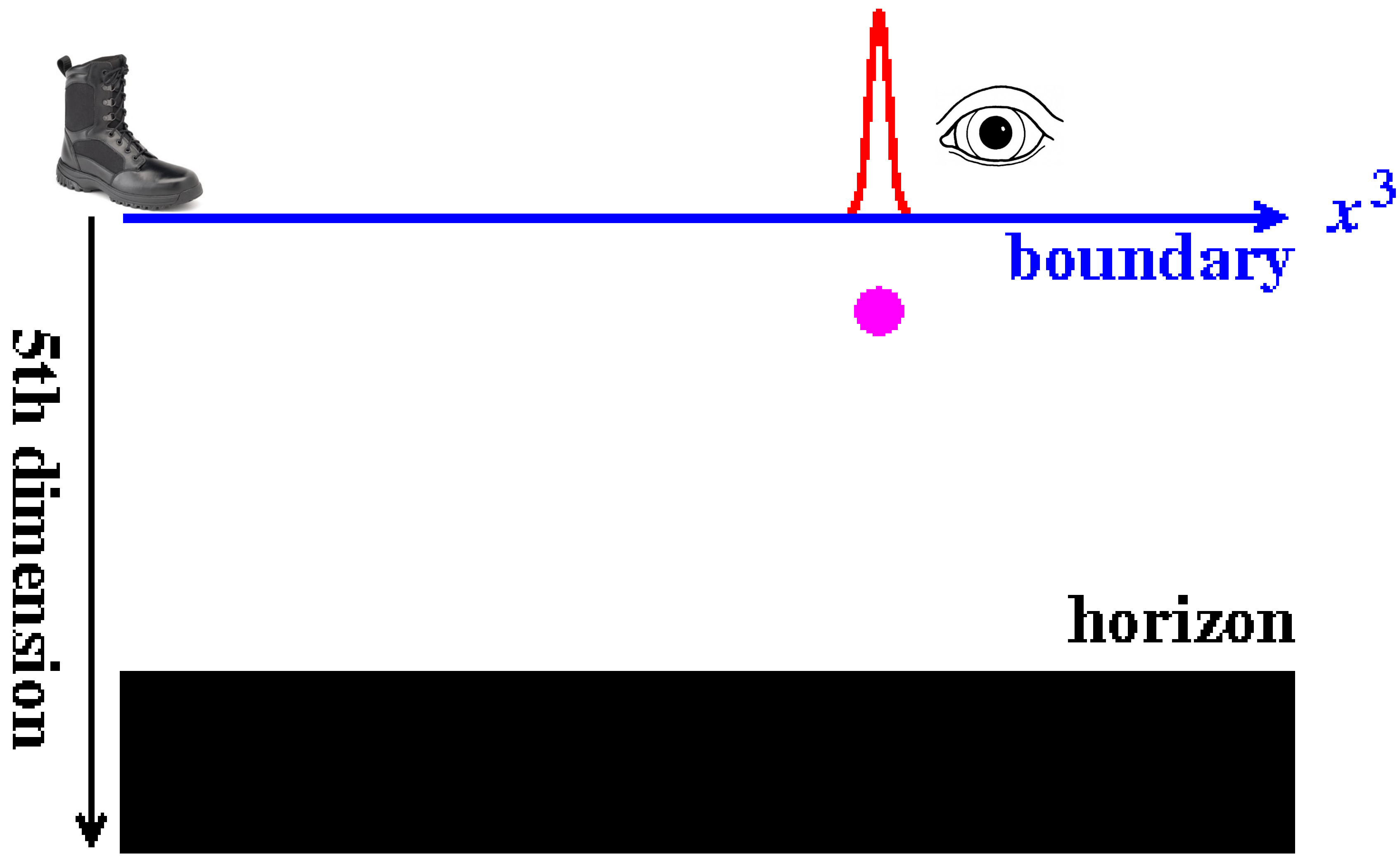

Strong Coupling using AdS/CFT

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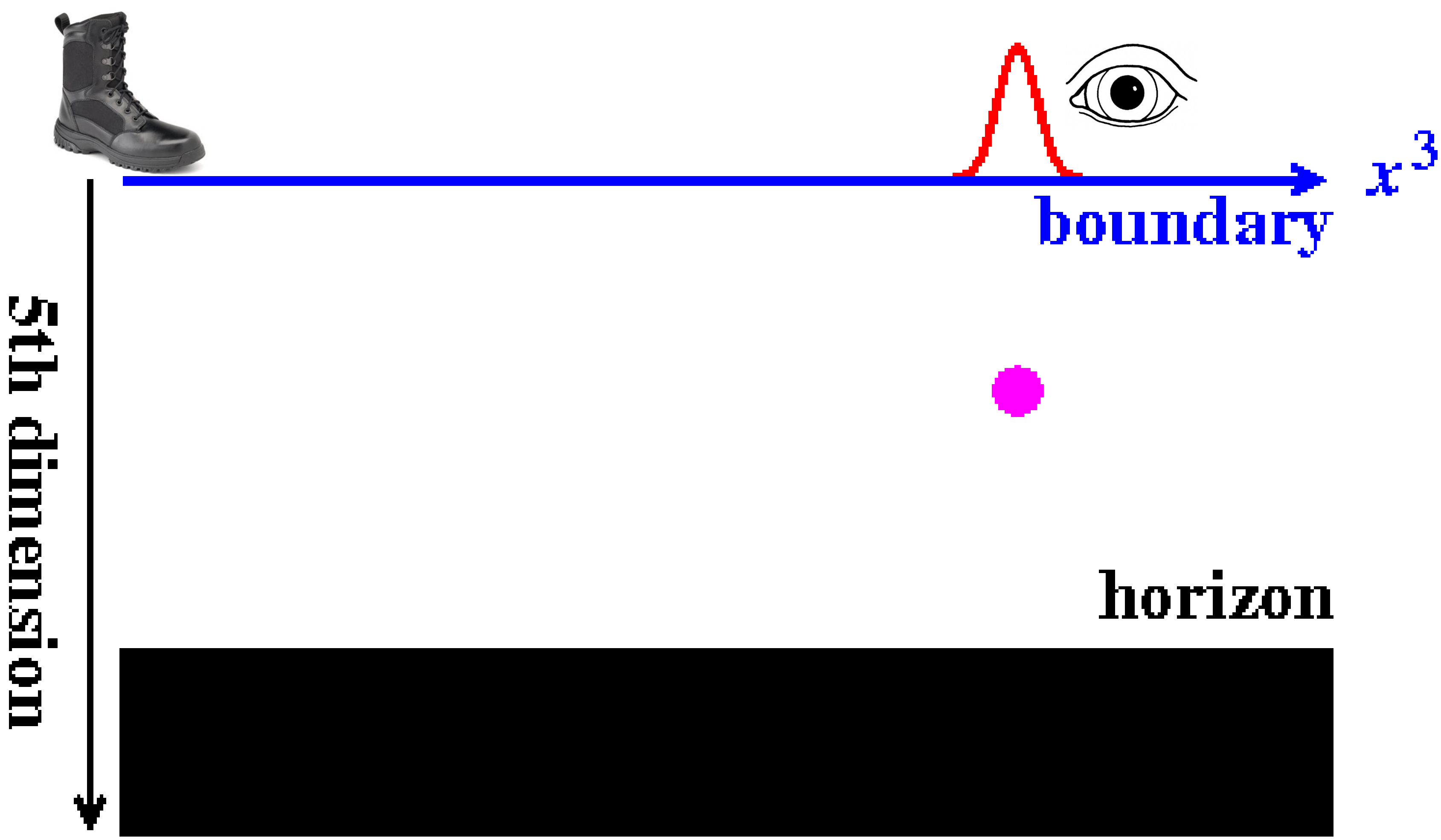

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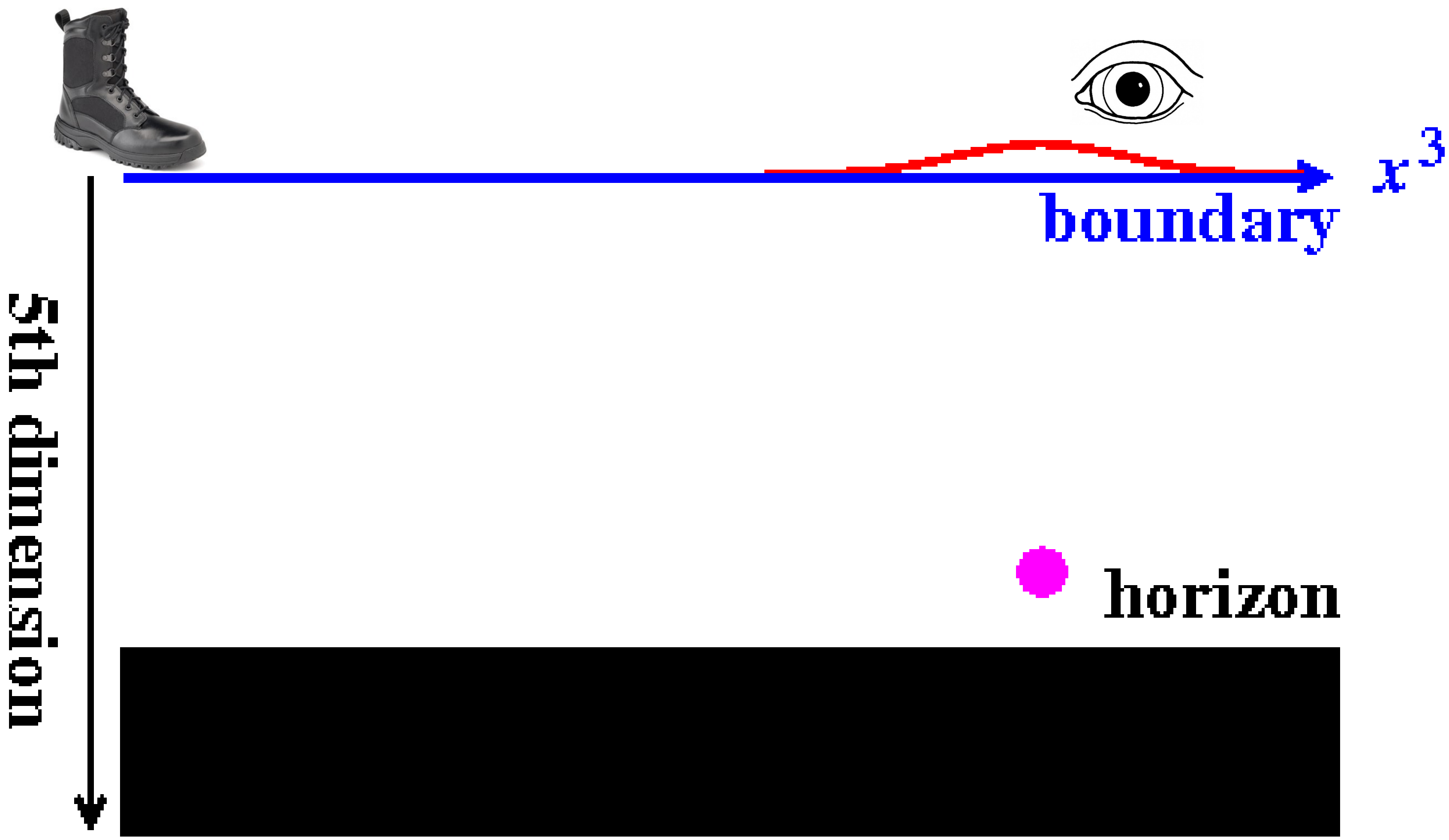

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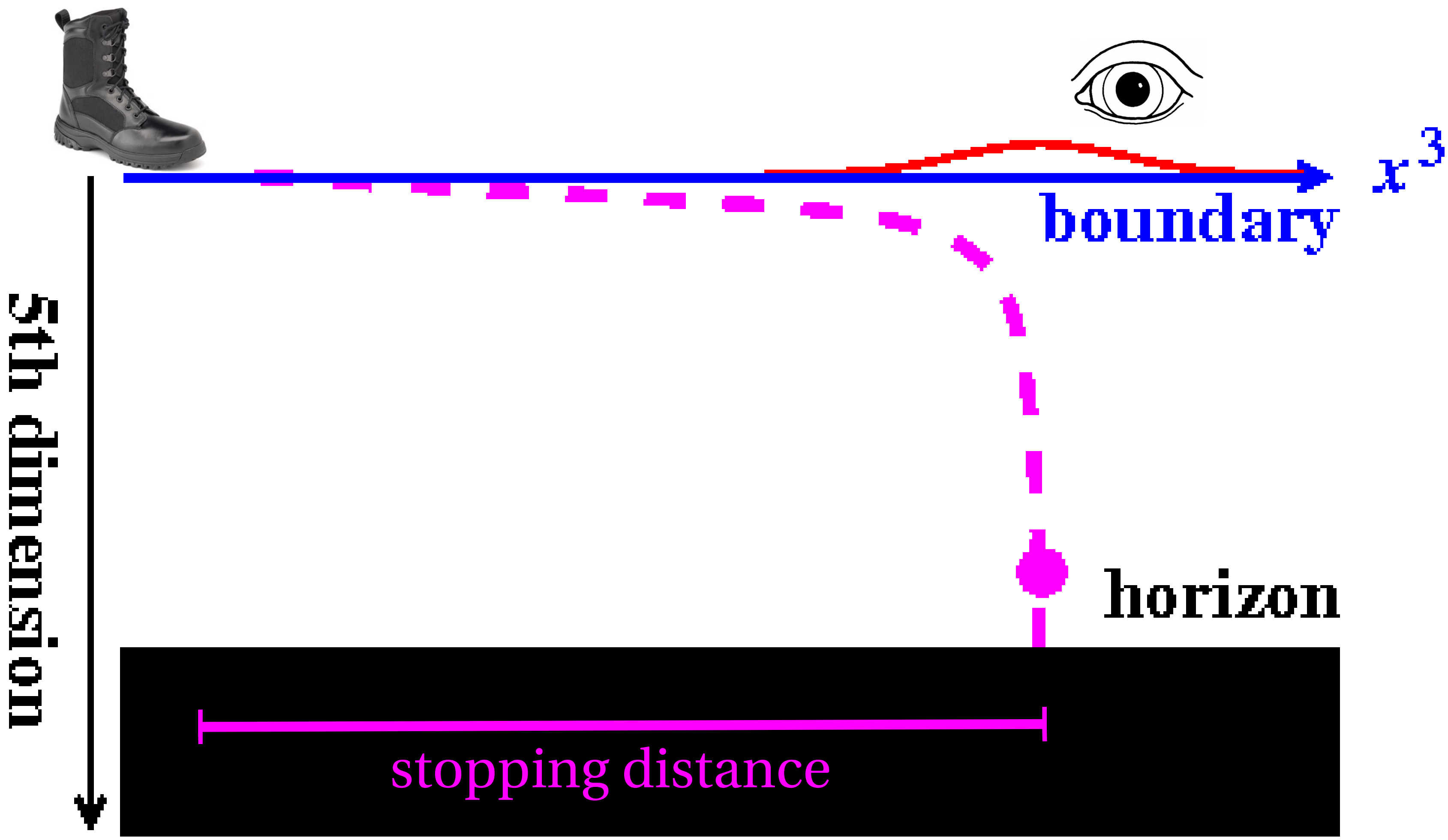
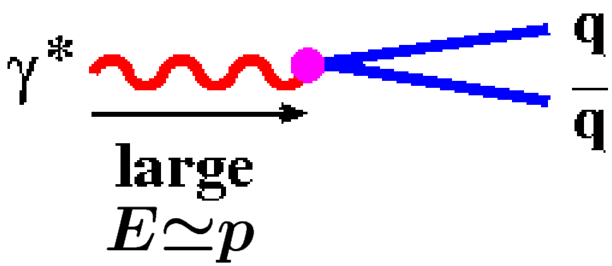
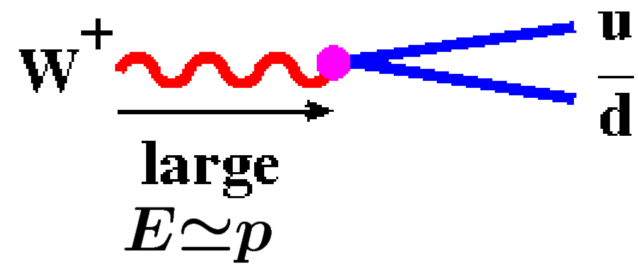


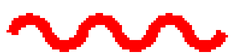
Our Method

In the field theory, think impressionistically of



or



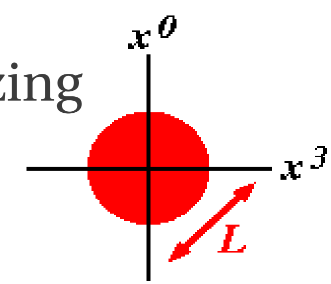
Treat  as a localized external field:

our 

$$\mathcal{L}_{\mathrm{QFT}} \rightarrow \mathcal{L}_{\mathrm{QFT}} + \boxed{\mathcal{O}(x)\,\Lambda_L(x)\,e^{i\bar{k}\cdot x}} \qquad \text{with } \bar{k}^\mu \simeq (E,0,0,E)$$

some source operator
e.g. $j_\mu(x)$

smooth envelope function localizing
source in space and time



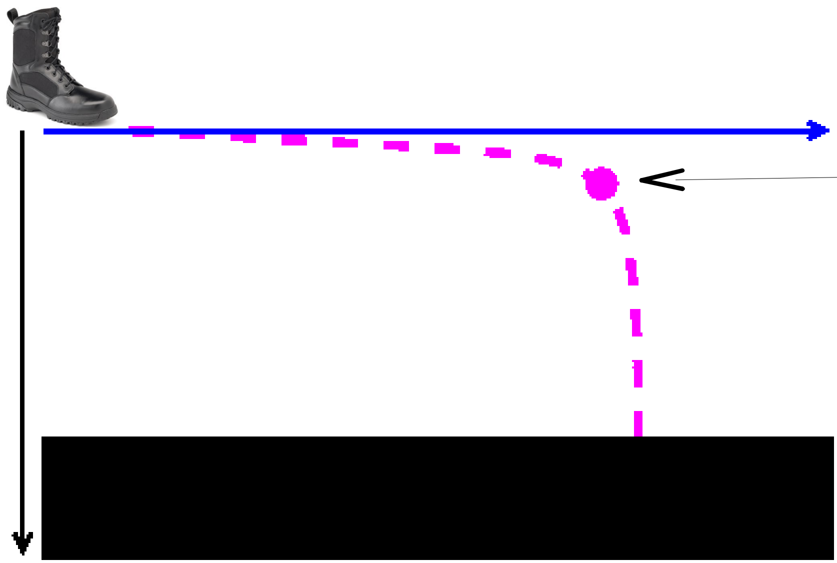
Definition for purposes of this talk:

"jet" = localized, high-p excitation moving through the plasma.

Result

$$\boldsymbol{\ell}_{\mathbf{stop}} \quad \lesssim \quad \boldsymbol{\ell}_{\mathbf{max}} \propto \boldsymbol{E}^{\mathbf{-1/3}}$$

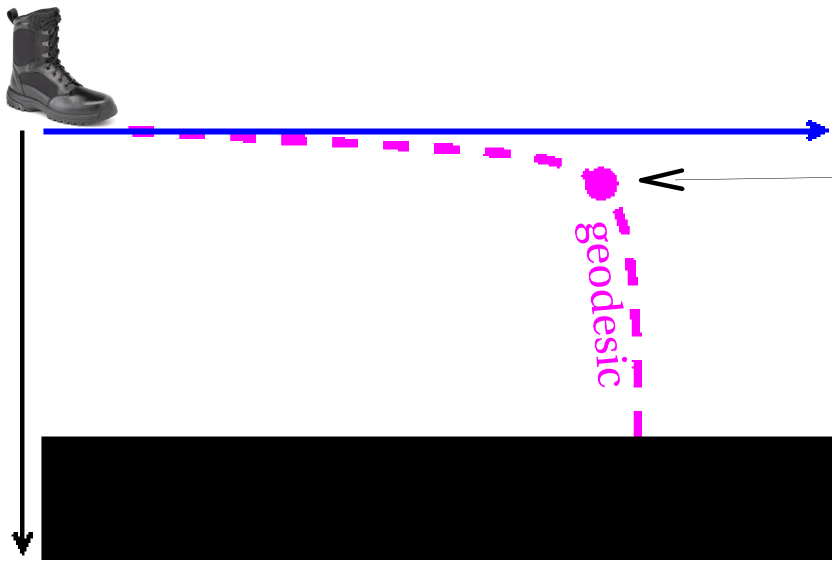
A simplified picture for $\ell_{\mathbf{stop}} \ll \ell_{\mathbf{max}}$



small size

Excitation moves like a 5-dim. particle!

A simplified picture for $\ell_{\rm stop} \ll \ell_{\rm max}$



small size

Excitation moves like a 5-dim. particle!

How soon it falls will depend on the initial "velocity" in the 5th dimension.

Q: What determines $\ell_{\rm stop}$?

A: the 4-virtuality $q^2 \equiv q_\mu \eta^{\mu\nu} q_\nu$ of the source 

Why?

Consider massless 5-dim. particle near the boundary:

$$0 = q_\mu q^\mu + q_5 q^5$$

Bigger -$q_\mu q^\mu$ $\rightarrow$ bigger q^5 $\rightarrow$ falls sooner !

The result:

$$\ell_{\rm stop} \simeq \frac{\Gamma^2(\frac{1}{4})}{\sqrt{4\pi}} \left(\frac{E^2}{-q^2} \right)^{1/4}$$

Higher curvature corrections to gauge-gravity duality

AdS/CFT correspondence:

$\mathcal{N}$=4 SYM ⟺ string theory in $AdS_5 \times S^5$ background

Strong-coupling limit:

$\lambda \to \infty$ ⟺ "low energy" string theory
= supergravity in in $AdS_5 \times S^5$ background
(gravitons + other massless string modes)

$\mathcal{L}_{\mathbf{grav}} \sim R$

Higher curvature corrections to gauge-gravity duality

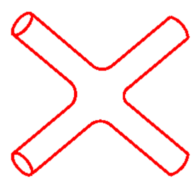
AdS/CFT correspondence:

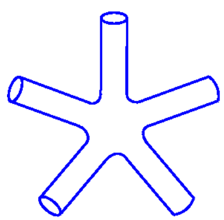
$\mathcal{N}$=4 SYM $\Longleftrightarrow$ string theory in $AdS_5 \times S^5$ background

Strong-coupling limit:

$\lambda \to \infty$ $\Longleftrightarrow$ "low energy" string theory
= supergravity in in $AdS_5 \times S^5$ background
(gravitons + other massless string modes)

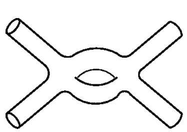
$$\mathcal{L}_{\text{grav}} \sim R \quad + \quad \alpha'^3 R^4 + \alpha'^5 D^4 R^4 + \alpha'^6 D^6 R^4 + \cdots \quad + \quad \alpha'^5 D^2 R^5 + \alpha'^6 D^4 R^5 + \cdots \quad + \quad \cdots$$



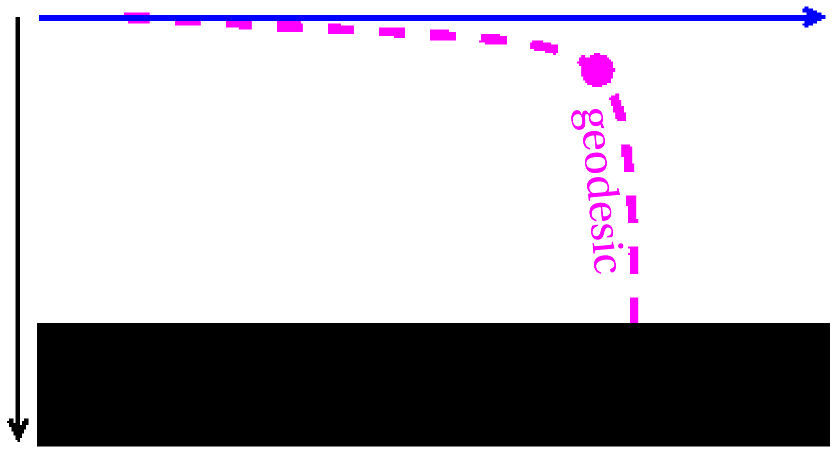


where

$\frac{1}{\sqrt{\lambda}}$ $\Longleftrightarrow$ proportional to $\alpha' \sim \frac{1}{\text{string tension}}$

Note: Loops  are suppressed by $g_{\text{string}} \propto \frac{1}{N_c}$.

How do the higher-derivative corrections affect λ

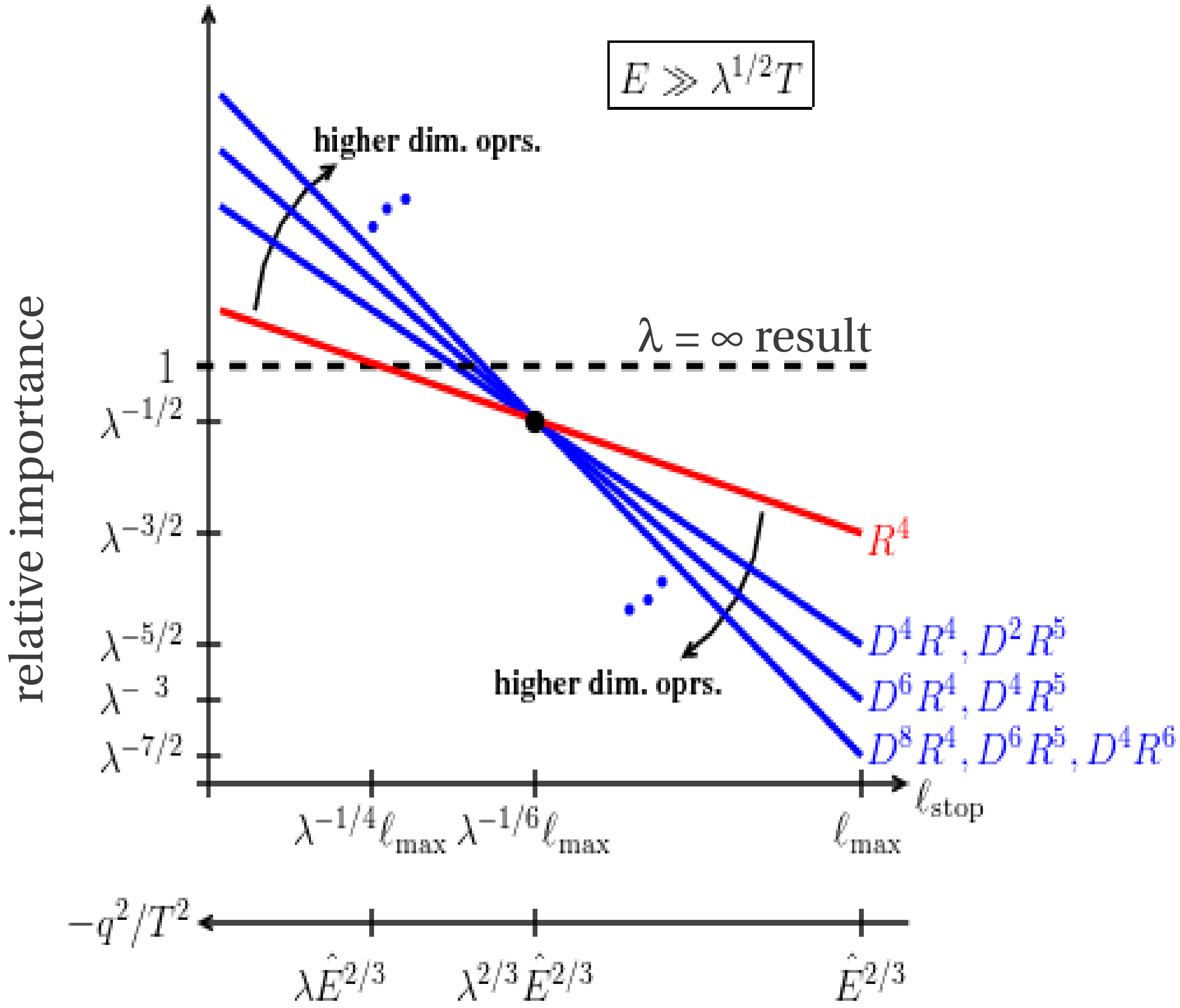


?

(1) Small corrections to AdS_5-Schwarzschild background (independent of E)
→ small corrections to geodesics. TINY

(2) Corrections to equation of motion
→ wave packet no longer follows a geodesic.
(Corrections depend on E.) POTENTIALLY LARGE

Importance to Jet Stopping



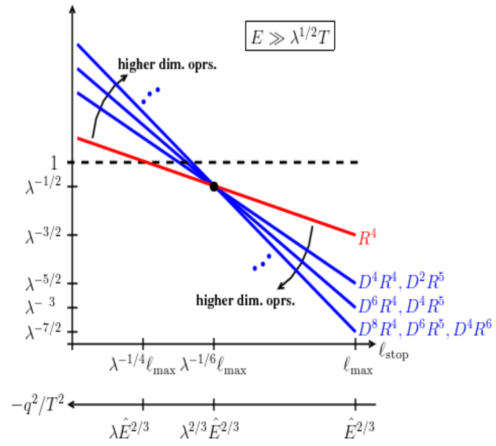
Moral: Expansion in $1/\lambda$ is well-behaved for $\boldsymbol{\lambda^{-1/6}\ell_{max} \ll \ell_{stop} \lesssim \ell_{max}}$

Expansion breaks down for $\boldsymbol{\ell_{stop} \lesssim \lambda^{-1/6}\ell_{max}}$

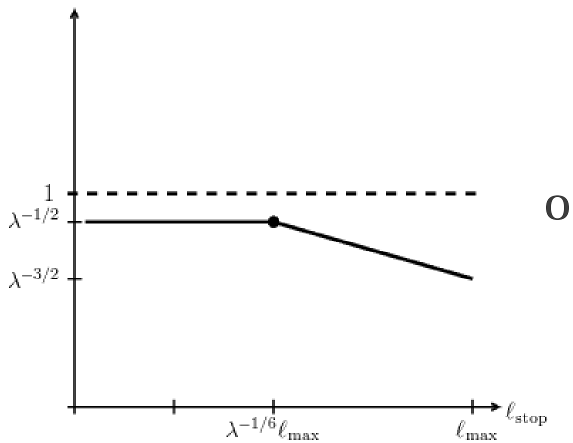
Note: Individual corrections all small ($\lambda^{-1/2}$) where expansion first breaks down.

Open Question

Does sum of corrections



add up to



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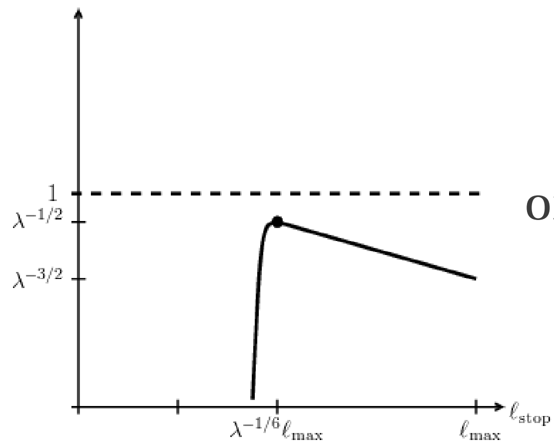

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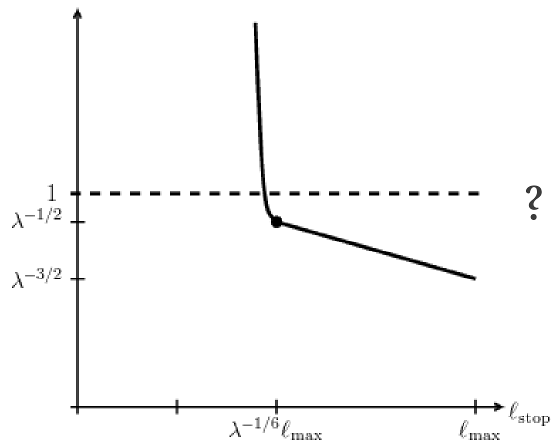


?

$\lambda = \infty$ results are okay everywhere!

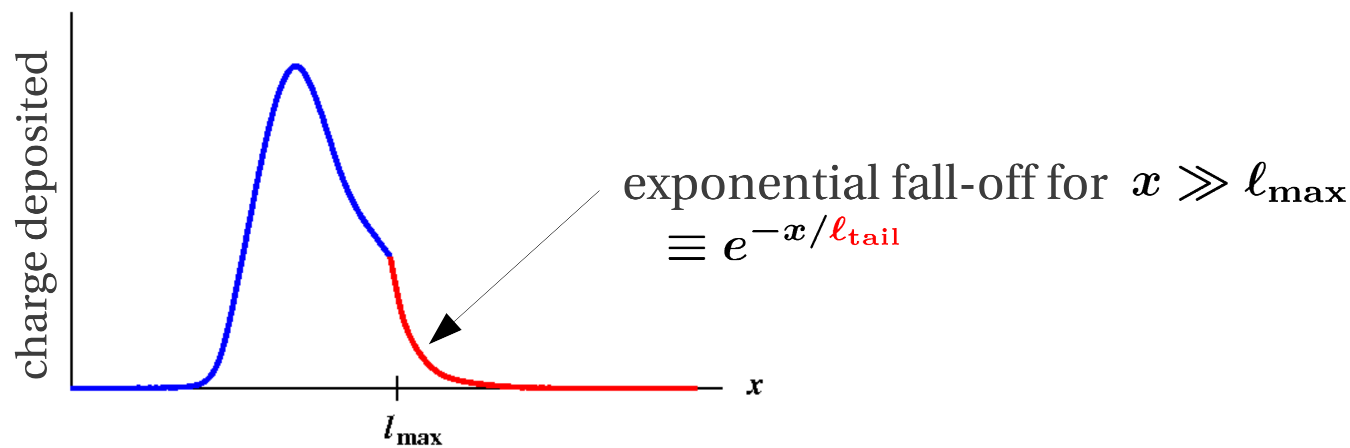
$\lambda = \infty$ works only for $\boldsymbol{\ell_{\rm stop} \gtrsim \lambda^{-1/6}\ell_{\rm max}}$

Moral: Fate of $\lambda = \infty$ results uncertain for $\boldsymbol{\ell_{\rm stop} \lesssim \lambda^{-1/6}\ell_{\rm max}}$

How big are corrections to ℓ_{max}?

Ill-posed: exact definition of ℓ_{max} scale is fuzzy.

Suppose you try to get make a “jet” go far.



Calculate ℓ_{tail} as a proxy for ℓ_{max}.

Example:  = decay of a high-momentum, slightly off-shell graviton

$$\ell_{\text{tail}} = \frac{0.3259\, E^{1/3}}{(2\pi T)^{4/3}} \left[1 + \frac{47.162}{\lambda^{3/2}} + \cdots \right]$$

Form of this result highlights an outstanding mystery...

An outstanding mystery

How does $\ell_{\max} \sim E^{f(\lambda)}$ interpolate from $f(\infty) = \frac{1}{3}$ (strong coupling)
to $f(0) = \frac{1}{2}$ (weak coupling)?

Naively, we might guess something like

$$f(\lambda) = \frac{1}{3} + \frac{\#}{\lambda^{3/2}} + \frac{\#}{\lambda^{5/2}} + \cdots$$

which would give

$$\ell_{\max} \sim E^{f(\lambda)} \sim E^{1/3}\left[1 + \frac{\#}{\lambda^{3/2}} \ln E + \cdots\right]$$

But there was no “$\ln E$” in the result for our proxy ℓ_{tail} !

Partial Bibliography

$\ell_{\max} \propto E^{1/2}$ (weak coupling)

BDMPS-Z: Baier, Dokshitzer, Mueller, Peigne, Schiff '96 / Zakharov '96

Calculation of q (hat) at strong coupling as a way to study α_s BIG but α_s small

Liu, Rajagopal, Wiedemann '06

$\ell_{\max} \propto E^{1/3}$

Gubser, Gulotta, Pufu, Rocha '08
Hatta, Iancu, Mueller '08
Chesler, Jensen, Karsch, Yaffe '08

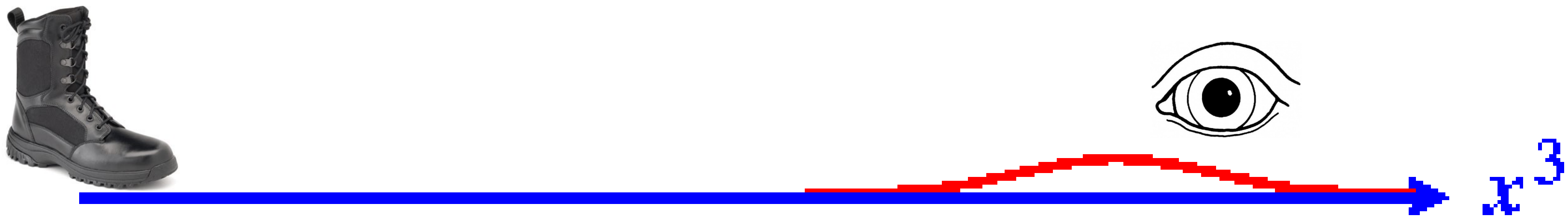
Formulating stopping distance in terms of late-time hydrodynamic response

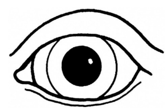
Chesler, Jensen, Karsch '08

AdS/CFT, the higher-curvature expansion, etc.

Various famous people.

Backup

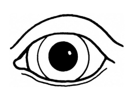
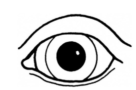


The response  is measured by a **<u>3-point correlator</u>**.

A crude way to understand this:

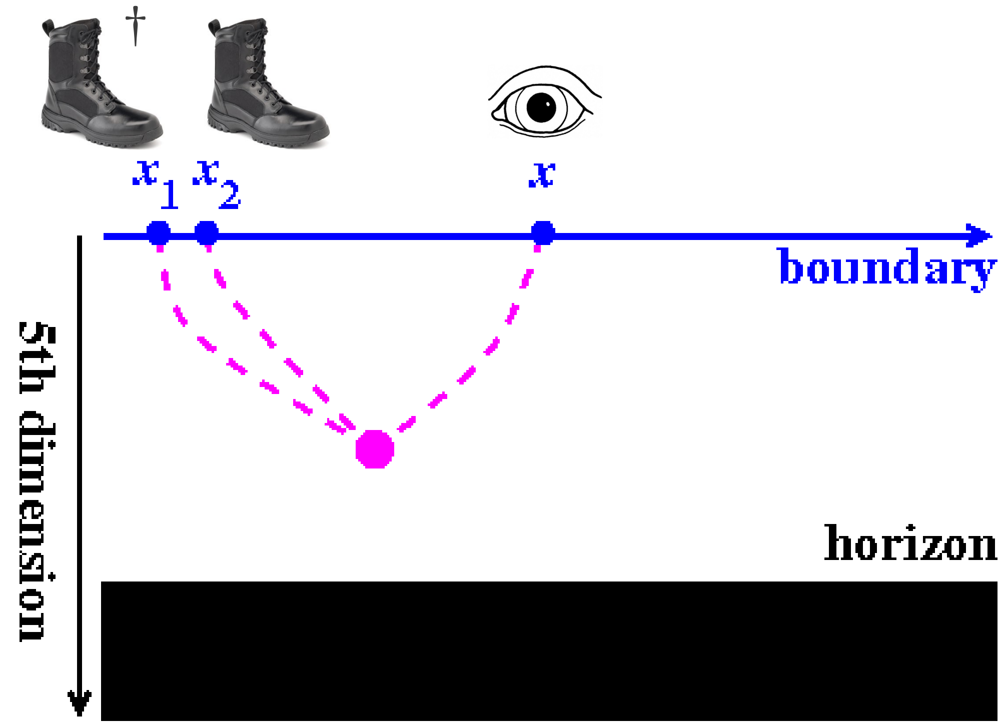
$$|\mathbf{jet}\rangle \quad = \qquad |\mathbf{plasma}\rangle .$$

So we want

$\langle \mathbf{jet}|$  $|\mathbf{jet}\rangle \;=\; \langle \quad {}^{\dagger}$  $\quad \rangle .$

For *finite-temperature* AdS/CFT calculations:

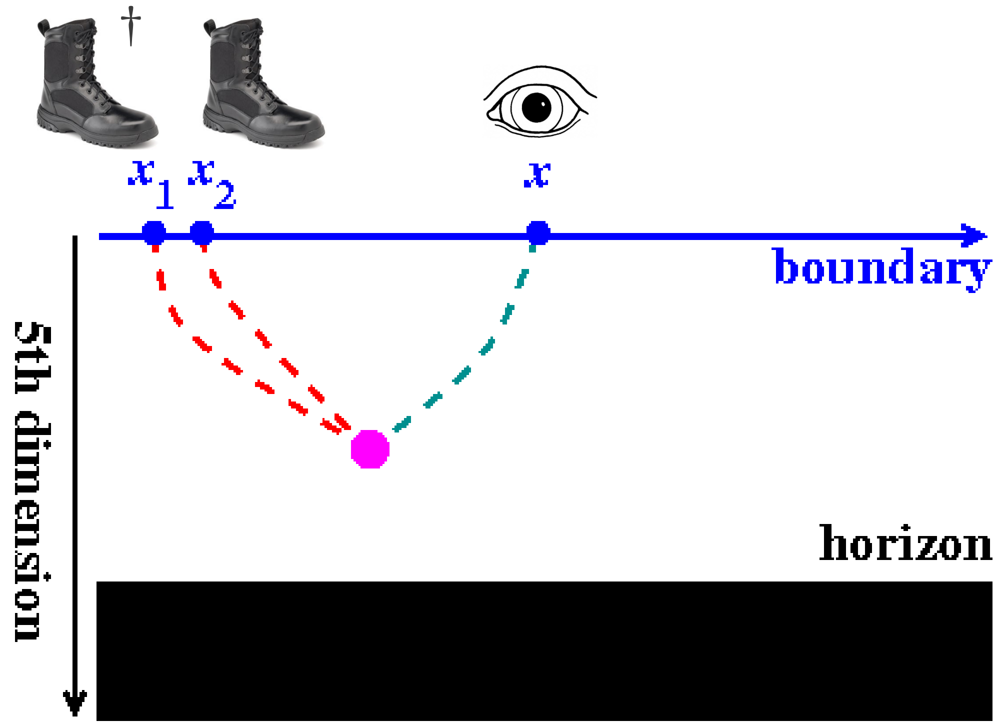
- lots in literature on computing 2-point correlators
- almost nothing on 3-point correlators



● = 5-dim. SUGRA vertex

– – – = a *Heun* function →
hard to make any analytic or numeric progress!

Fortunately, in our problem,

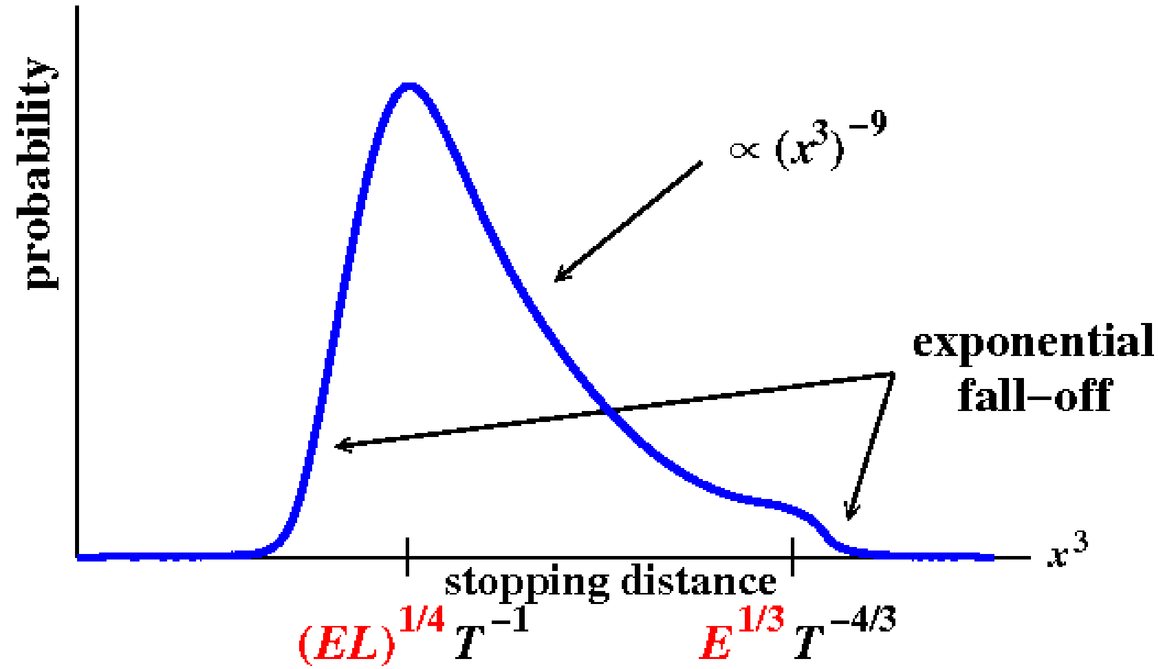


high-energy source →
high-k approximation (WKB / geometric optics)

want to observe late-time diffusion →
low-k approximation

Can do calculation!

Our Result



The farthest a jet will ever go is indeed $\propto E^{1/3}$.

But almost all jets will instead stop sooner at $\propto (EL)^{1/4}$

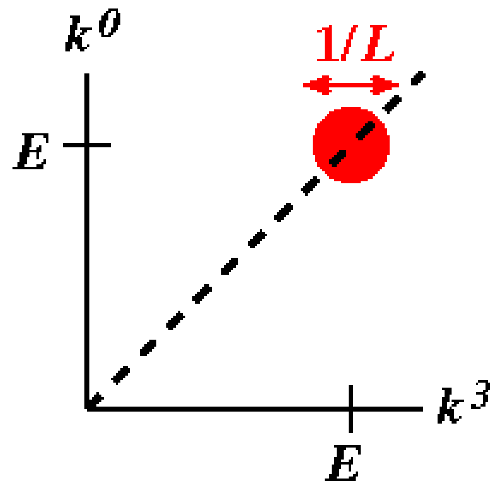
where L is the size of the space-time region in which the jet was initially created.

Q: What does the size L of the source have to do with it?

A: It determines how off-shell the source is.

 $\propto$ $\Lambda_L(x)\, e^{i\bar{k}\cdot x}$ with $\bar{k}^\mu = (E, 0, 0, E)$

implies that  has Fourier components



Typical stopping distance $(EL)^{1/4}$ really means $(E^2/q^2)^{1/4}$
where

q^2 = typical virtuality of the source