Jet broadening and a gauge invariant jet quenching parameter with Soft-Collinear Effective Theory

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Outline

Jet quenching phenomenology and $\hat{q}$

What is jet quenching?

What is a jet?

- A jet is a narrow cone of hadrons with small invariant mass. They originate from a high energy parton that fragments into this hadrons.

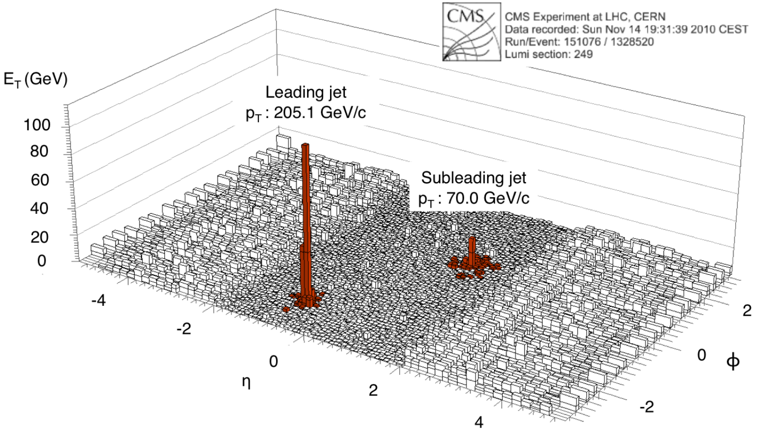
What is to quench?

- To quench is to suppress (in this context).

What is jet quenching?

- Jets lose energy while traversing a medium. This phenomena is what is called jet quenching.

Jet quenching in heavy-ion collisions



CMS collaboration, PRC 84, 024906 (2011)

Processes for jet quenching

If the jet is a light quark

- Bremsstrahlung

If the jet is a gluon

- Bremsstrahlung
- Pair production of $q\bar{q}$

Perturbative example:gluon bremsstrahlung

$$\frac{d\Gamma_{p\to gp}}{dx} = \frac{\alpha_s \mu_\perp^2 P_{p\to gp}(x)}{4\pi\sqrt{2}x(1-x)E},$$

P. Arnold and W. Xiao, PRD 78, 125008 (2008)
where $P_{p\to gp}(x)$ is the vacuum splitting function and

$$\mu_\perp^2 = (8x(1-x)E[\frac{1}{2}C_A + (C_s - \frac{1}{2}C_A)x^2 + \frac{1}{2}C_A(1-x)^2]\hat{q}(Q_0))^{1/2}$$

The medium information in this approach is encoded in $\hat{q}$.

What is $\hat{q}$?

$P(k_\perp)$ is the probability that an initial parton with momentum $(0, Q, 0)$ (light-cone coordinates) transforms into a parton with momentum $(\frac{k_\perp^2}{2Q}, Q, k_\perp)$ after going through an unit of longitude in the medium.

$$\hat{q}(\Lambda) = \frac{1}{L}\int^{\Lambda} \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp)$$

It is only related with jet broadening but it is needed as an input parameter in jet quenching computations.

$\hat{q}$ from Wilson lines

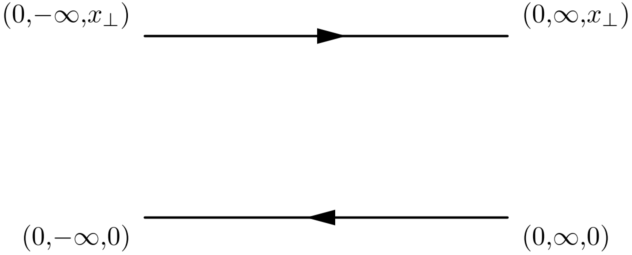
$$P(k_\perp) = \int d^2x_\perp e^{-ik_\perp x_\perp} \mathcal{W}_\mathcal{R}(x_\perp)$$

$$\mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d(\mathcal{R})} \left\langle Tr \left[W^\dagger_\mathcal{R}[0, x_\perp] W_\mathcal{R}[0,0] \right] \right\rangle$$

$$W_\mathcal{R}[y^+, y_\perp] = \mathcal{P}\left\{ exp \left[ig \int_{-\infty}^{\infty} dy^- A^+(y^+, y^-, y_\perp) \right] \right\}$$

Baier et al. (1997), Zakharov (1996), Casalderrey-Solana and Salgado(2007)

$\hat{q}$ from Wilson lines



Very nice result. Information on the jet factors out. The problem reduces to compute a thermal average of a QCD operator. Similar to static potential.

But it is not gauge invariant. In particular, in the light-cone gauge $A^+ = 0$ it gives no broadening at all.

$\hat{q}$ on the lattice

- By slightly deforming the light-cone Wilson lines one can "Euclideanize" $\hat{q}$ such that all operators are separated by space-like distances. S. Caron-Huot, PRD 79, 065039 (2009).
- Recent framework for the computation of $\hat{q}$ using lattice data. A. Majumder, arXiv:1202.5295

Both of these works assume some way to do close the loop to make it gauge invariant, but they do not derive it.

Soft-collinear effective theory

What are the energy scales that appear in the problem?

- The parton originating the jet has an energy of order Q.
- The particles forming the medium have a typical momentum of order T.
- Depending on the accuracy one may also take into account Λ_{QCD}.

These scales are well separated $Q \gg T \gg \Lambda_{QCD}$.
We can define $\lambda = \frac{T}{Q}$ and expand the result as a series in this parameter.
The fact that λ is small can also induce unexpected enhancements.
Landau-Pomeranchuck-Migdal effect

Soft-collinear effective theory

SCET is an effective theory for degrees of freedom with virtuality much smaller than Q^2. Now on we consider the case in which the jet originates from a collinear light-quark

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i \not{n} (\bar{n} \cdot D) \xi_{\bar{n}} + \bar{\xi}_{\bar{n}} i \not{D}_{\perp} \frac{1}{2in \cdot D} i \not{D}_{\perp} \not{n} \xi_{\bar{n}}$$

where $n = \frac{1}{\sqrt{2}}(1,0,0,1)$ and $\bar{n} = \frac{1}{\sqrt{2}}(1,0,0,-1)$.
In SCET each term in the Lagrangian has a specific power in terms on λ.
Power counting.
C. Bauer, S. Fleming and M. Luke, PRD 63, 014006 (2000)

Separation of gauge fields in SCET

$$A_\mu(x) = A_\mu^{collinear}(x) + A_\mu^{soft}(x) + A_\mu^{ultrasoft}(x) + ...$$

the typical momentum of each of these fields is different

- Collinear $Q(\lambda^2, 1, \lambda)$.
- Soft $Q(\lambda, \lambda, \lambda)$.
- Ultrasoft $Q(\lambda^2, \lambda^2, \lambda^2)$.

In SCET a scaling in powers of λ for the different components of the A_μ field can be defined. Each component can contribute differently.

Glauber gluons

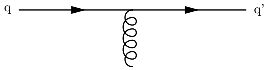
$$A_\mu(x) = A_\mu^{collinear}(x) + A_\mu^{Glauber}(x) + A_\mu^{soft}(x) + A_\mu^{ultrasoft}(x) + \ldots$$

previous slide is what is normally considered in SCET. For problems like jet broadening also Glauber region has to be considered

- Glauber $Q(\lambda^2, \lambda^2, \lambda)$.

$SCET_G$ is an extended version of SCET suited to treat this gluons. The Lagrangian is the same but the power counting is different.
A. Idilbi and A. Majumder, PRD 80, 054022 (2009)

Glauber gluons



- collinear quark + soft gluon=hard-collinear quark.

$$Q(\lambda^2, 1, \lambda) + Q(\lambda, \lambda, \lambda) = Q(\lambda, 1, \lambda)$$

$\sqrt{s} = Q\sqrt{\lambda} = \sqrt{QT}$

- collinear quark+ Glauber gluon=collinear quark.

$$Q(\lambda^2, 1, \lambda) + Q(\lambda^2, \lambda^2, \lambda) = Q(\lambda^2, 1, \lambda)$$

$\sqrt{s} = Q\lambda = T.$

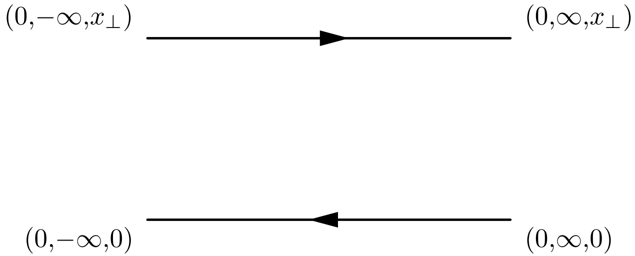
The leading order contribution to broadening will come from Glauber gluons.

$SCET_G$ in a covariant gauge

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i\not{n}(\bar{n}\cdot D)\xi_{\bar{n}} + \bar{\xi}_{\bar{n}} i\not{\partial}_\perp \frac{1}{2in\cdot D} i\not{\partial}_\perp \not{n}\xi_{\bar{n}}$$
$$-ig\bar{\xi}_{\bar{n}} i\not{\partial}_\perp \frac{1}{2in\cdot D} i\not{A}_\perp \not{n}\xi_{\bar{n}} - ig\bar{\xi}_{\bar{n}} i\not{A}_\perp \frac{1}{2in\cdot D} i\not{\partial}_\perp \not{n}\xi_{\bar{n}} - g^2\bar{\xi}_{\bar{n}} i\not{A}_\perp \frac{1}{2in\cdot D} i\not{A}_\perp \not{n}\xi_{\bar{n}}$$

Only red term will contribute at leading order in λ. In other words, to $\mathcal{O}(1)$.

$P(k_\perp)$ computed with $SCET_G$ in a covariant gauge



F. d'Eramo, H. Liu and K. Rajagopal, PRD 84, 065015 (2011).

Jet broadening in the light-cone gauge

$SCET_G$ in the light-cone gauge

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i\not{n}(\bar{n}\cdot\partial)\xi_{\bar{n}} + \bar{\xi}_{\bar{n}} i\not{\partial}_\perp \frac{1}{2in\cdot D} i\not{\partial}_\perp \not{n}\xi_{\bar{n}}$$
$$-ig\bar{\xi}_{\bar{n}} i\not{\partial}_\perp \frac{1}{2in\cdot D} i\not{A}_\perp \not{n}\xi_{\bar{n}} - ig\bar{\xi}_{\bar{n}} i\not{A}_\perp \frac{1}{2in\cdot D} i\not{\partial}_\perp \not{n}\xi_{\bar{n}} - g^2\bar{\xi}_{\bar{n}} i\not{A}_\perp \frac{1}{2in\cdot D} i\not{A}_\perp \not{n}\xi_{\bar{n}}$$

If one assumes that the power counting does not depend on the gauge we have a problem.

No interaction at all with gluons in the LO term.

Differences between a covariant and a no-covariant gauge

Consider as an example

$$D(k_\mu) = \int d^4x e^{ixk} \langle A^\mu(x) A_\mu(0) \rangle$$

In a covariant gauge and with an isotropic medium moving with a velocity v_μ.

$$D(k_\mu) = D(k_\mu k^\mu, v_\mu k^\mu)$$

For Glauber gluons

$$D_G(k_\mu) = D(-k_\perp^2, 0)$$

No explicit dependence of k^+ and k^-.

Differences between a covariant and a no-covariant gauge

Consider as an example

$$D(k_\mu) = \int d^4x e^{ixk} \langle A^\mu(x) A_\mu(0) \rangle$$

In the light cone gauge

$$D(k_\mu) = D(k_\mu k^\mu, v_\mu k^\mu, n_\mu k^\mu)$$

For Glauber gluons

$$D_G(k_\mu) = D(-k_\perp^2, 0, k^+)$$

There is a dependence on k^+ because it has poles at $k^+ = 0$.

$SCET_G$ in the light-cone gauge

Consequence

$$A_\perp^{light-cone}(x) \gg A_\perp^{covariant}$$

$$\mathcal{L}_{\bar n} = \bar\xi_{\bar n} i \not{n} (\bar n \cdot \partial) \xi_{\bar n} + \bar\xi_{\bar n} i \not\partial_\perp \frac{1}{2in\cdot D} i \not\partial_\perp \not{n} \xi_{\bar n}$$
$$- ig \bar\xi_{\bar n} i \not\partial_\perp \frac{1}{2in\cdot D} i \not{A}_\perp \not{n} \xi_{\bar n} - ig \bar\xi_{\bar n} i \not{A}_\perp \frac{1}{2in\cdot D} i \not\partial_\perp \not{n} \xi_{\bar n} - g^2 \bar\xi_{\bar n} i \not{A}_\perp \frac{1}{2in\cdot D} i \not{A}_\perp \not{n} \xi_{\bar n}$$

- Covariant gauge, three body vertex that does not depend on the momentum.
- Light-cone gauge, three body vertex thad depends on the momentum + four body vertex.

Glauber gluons in coordinate space

$$D_G(k_\mu) = A(k_\perp) + \frac{B(k_\perp)}{k^+}$$

Power counting tells us that the second term is bigger. When doing the Fourier transform the first term is 0 at $x^- = \pm\infty$ while the second term is not.

$$A_\mu(x_+, x_-, x_\perp) = A_\mu^{\mathrm{cov}}(x_+, x_-, x_\perp)$$
$$+\theta(x_-)A_\mu(x_+, \infty, x_\perp) + \theta(-x_-)A_\mu(x_+, -\infty, x_\perp)$$

The leading contribution will come from the fields at $x^- = \pm\infty$.

Gauge fields at infinity

- At infinity the chromoelectric and chromomagnetic fields must be zero in order to have a finite energy.
- That means that the gauge field at $y^- = \pm\infty$ is a pure gauge.

$$A_\mu(y^+,\infty,y_\perp) = \partial_\mu\phi^+(y^+,y_\perp) \quad A_\mu(y^+,-\infty,y_\perp) = \partial_\mu\phi^-(y^+,y_\perp)$$

These considerations has been also used to make a gauge invariant description of semi-inclusive deep inelastic scattering in Belitsky, Ji and Yuan (2003).

Gauge fields at $y^- = \infty$

$$A_\mu(y^+, \infty, y_\perp) = \partial_\mu \phi^+(y^+, y_\perp)$$

This can be seen as a differential equation for ϕ^+ in the coordinates y^+ and $y_\perp$.

$$\phi^+(y^+, y_\perp) = \int_{b^\mu}^{y^\mu} d\xi^\nu A_\nu(\xi^+, \infty, \xi_\perp)$$

What is b^μ? b^μ must belong to the plane $y^- = \infty$ but we can not say anything more. A change in b^μ gives different ϕ^+ but same A_μ, so it is equivalent to a gauge transformation.

$SCET_G$ in the light-cone gauge

Consequence

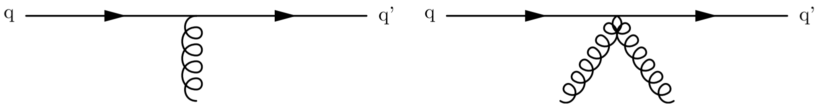
$$A_\perp^{light-cone}(x) \gg A_\perp^{covariant}(x)$$

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}} i\not{n}(\bar{n}\cdot\partial)\xi_{\bar{n}} + \bar{\xi}_{\bar{n}} i\not{\partial}_\perp \frac{1}{2in\cdot D} i\not{\partial}_\perp \not{n}\xi_{\bar{n}}$$
$$-ig\bar{\xi}_{\bar{n}} i\not{\partial}_\perp \frac{1}{2in\cdot D} i\not{A}_\perp \not{n}\xi_{\bar{n}} - ig\bar{\xi}_{\bar{n}} i\not{A}_\perp \frac{1}{2in\cdot D} i\not{\partial}_\perp \not{n}\xi_{\bar{n}} - g^2\bar{\xi}_{\bar{n}} i\not{A}_\perp \frac{1}{2in\cdot D} i\not{A}_\perp \not{n}\xi_{\bar{n}}$$

- Covariant gauge, three body vertex that does not depend on the momentum.
- Light-cone gauge, three body vertex that depends on the momentum + four body vertex, but we only need to consider the field at infinity, where it is a pure gauge.

Vertices

Computation method: compute the term with one insertion of the gauge field, compute the term with two insertions... sum all the series.



For Glauber gluons two insertions of the 3-body vertex contributes to the same order as one insertion of the 4-body vertex. We have to take into account all possible combinations.

Summing all the series

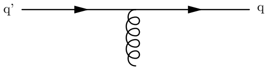
Define $G_n(k_\perp)$ as the Green function for going from $(0, Q, 0) \to (\frac{k_\perp^2}{2Q}, Q, k_\perp)$ with n insertions of the gluon field. By $i\epsilon$ prescription

$$G_n(k_\perp) = \sum_{j=0}^{n} \int \frac{d^4q}{(2\pi)^4} G^+_{n-j}(k_\perp, q) \frac{iQ\not{\bar{n}}}{2Qq^+ - q_\perp^2 + i\epsilon} G^-_j(q)$$

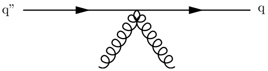
G^+ only includes ϕ^+ and G^- only includes ϕ^-.

Computation of G^-

$$G_n^-(q) = \int \frac{d^4q'}{(2\pi)^4} G_{n-1}^-(q') \cdot$$



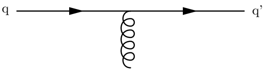
$$+ \int \frac{d^4q'}{(2\pi)^4} \frac{d^4q''}{(2\pi)^4} G_{n-2}^-(q'') \cdot$$



This is an equation that can be solved, for example, by induction method.

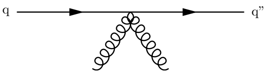
Computation of G^+

$$G_n^+(k_\perp, q) = \int \frac{d^4q'}{(2\pi)^4}$$



$$G_{n-1}^+(k_\perp, q')$$

$$+ \int \frac{d^4q'}{(2\pi)^4} \frac{d^4q''}{(2\pi)^4}$$



$$G_{n-2}^+(k_\perp, q'')$$

$P(k_\perp)$ in the light-cone gauge

$$G_n(k_\perp) = \frac{1}{n!}\int dy^+\, d^2y_\perp e^{iy^+(k^- - Q)} e^{-iy_\perp k_\perp} \times$$
$$\times \bar{\mathcal{T}}(ig\phi^+(y^+, y_\perp) - ig\phi^-(y^+, y_\perp))^n \not{n}$$

from this we can extract $P(k_\perp)$

$$P(k_\perp) = \frac{1}{N_c}\int d^2x_\perp e^{ik_\perp x_\perp} \times$$
$$\times Tr\left[\mathcal{T}\left(e^{-ig\phi^-(0,x_\perp)+ig\phi^+(0,x_\perp)}\right)\bar{\mathcal{T}}\left(e^{ig\phi^-(0,0)-ig\phi^+(0,0)}\right)\right]$$

$P(k_\perp)$ in the light-cone gauge

- Note that we still have not choose the value of b^μ.

$$\phi^+(y^+, y_\perp) = \int_{b^\mu}^{y^\mu} d\xi^\nu A_\nu(\xi^+, \infty, \xi_\perp)$$

- If the condition $A^+ = 0$ is fulfilled with a given b^μ it is also fulfilled with any other because it does not change A_μ.
- We choose values that exploit the symmetries of the result we have obtained. $b^\mu = (0, \pm\infty, 0)$.

$P(k_\perp)$ in the light-cone gauge

Jet broadening in a general gauge

General considerations

- Changing b^μ is equivalent to a (specific type) gauge transformation. If we choose a specific value and the final result is gauge invariant the result does not depend of the choice of b^μ also.
- For the interaction with the A^+ field we can use the results on covariant gauge computed in SCET by D'Eramo, Liu and Rajagopal (2010).
- The interaction with $A_\perp$ field is the one that we have just computed.
- Conclusion: we already have everything.

$P(k_\perp)$ in an arbitrary gauge

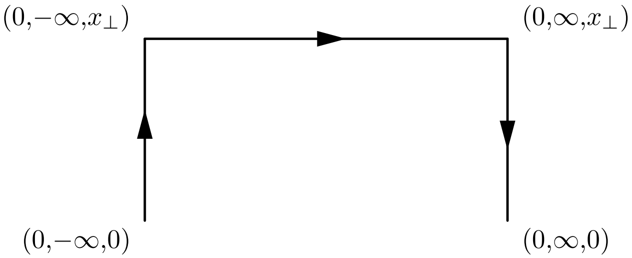
$$P(k_\perp) = \frac{1}{N_c}\langle \int d^2x_\perp e^{ik_\perp x_\perp} Tr\left[W_A(x_\perp) W_B\right]\rangle$$

$$W_A(x_\perp) = \mathcal{P} e^{-ig\int_{C_1} d\xi_\mu A^\mu(\xi)}$$

$$W_B = \bar{\mathcal{P}} e^{ig\int_{C_2} d\xi_\mu A^\mu(\xi)}$$

$P(k_\perp)$ in an arbitrary gauge

$$W_A(x_\perp) = \mathcal{P}e^{-ig\int_{C_1} d\xi_\mu A^\mu(\xi)}$$

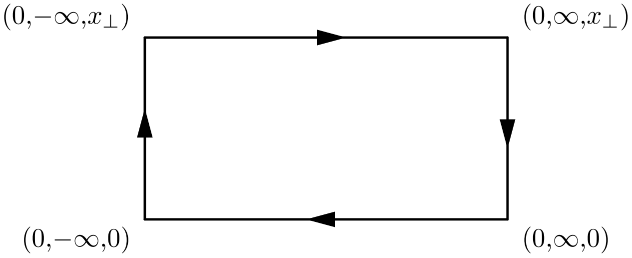


$P(k_\perp)$ in an arbitrary gauge

$$W_B = \bar{\mathcal{P}} e^{ig\int_{C_2} d\xi_\mu A^\mu(\xi)}$$

$$(0,-\infty,0) \longrightarrow (0,\infty,0)$$

$P(k_\perp)$ in an arbitrary gauge



It is gauge invariant. Similar to the static potential in the sence that is the expected value of a Wilson loop.
But light-cone Wilson lines and it is not time-ordered.

Conclusions

Conclusions

- A gauge invariant definition of $P(k_\perp)$ and hence $\hat{q}$ can be obtained using SCET.
- Transverse lines at $y^- = \pm\infty$ appear in order to construct a closed Wilson loop.
- This shows that the computation using $SCET_G$ is consistent. It also puts on more solid grounds previous works who assumed that the transverse lines were there.
- Maybe it opens the way to a computation of $\hat{q}$ on the lattice.