Holography for thermalization:
transition to hydrodynamics and its features

Michał P. Heller
University of Amsterdam, The Netherlands
&
National Centre for Nuclear Research, Poland (on leave)

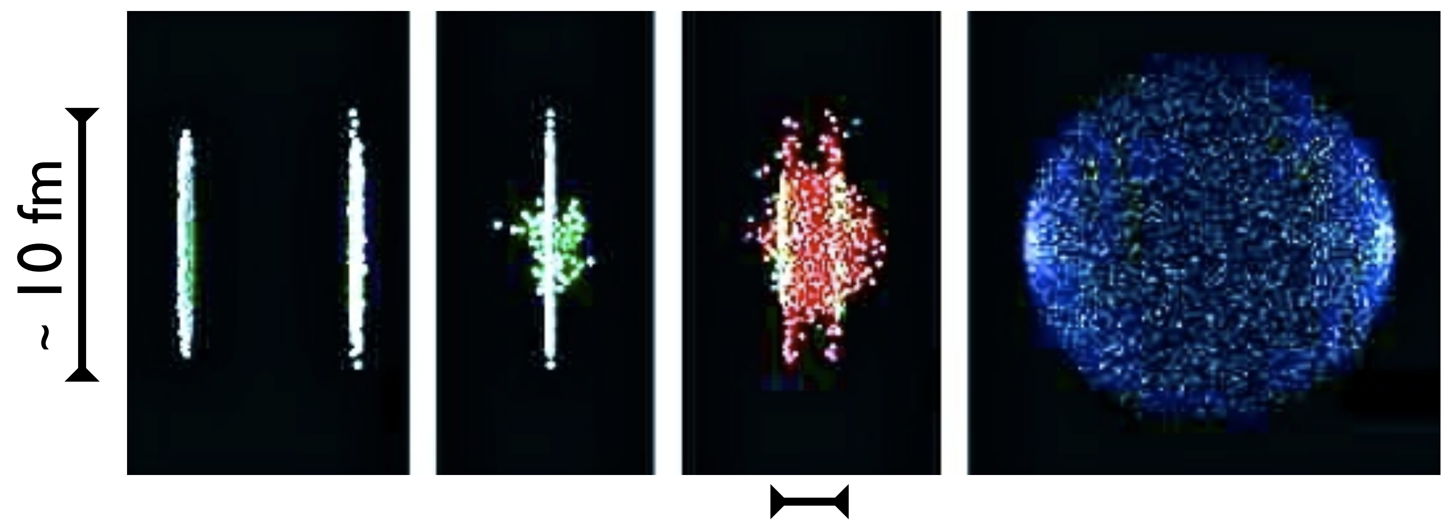
based on
1103.3452 [hep-th] MPH, R. A. Janik & P. Witaszczyk
1203.0755 [hep-th] MPH, R. A. Janik & P. Witaszczyk
& on-going work with D. Mateos, W. van der Schee, M. Spaliński and D. Trancanelli

Motivation: fast hydronization at RHIC

Heinz [nucl-th/0407067]

There are overwhelming evidences that relativistic heavy ion collision program at RHIC (now also at the LHC) created strongly coupled quark-gluon plasma (sQGP)

Successful description of experimental data is based on hydrodynamic simulations of an almost perfect fluid of $\eta/s = \mathrm{O}\,(1/4\pi)$ starting on very early (< 1 fm/c)



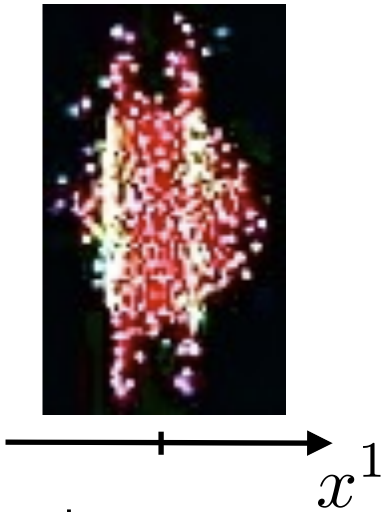
thermalized after < 1 fm/c

This very fast thermalization or rather hydronization (understood here as time after the collision when the stress tensor is described by hydrodynamics) is a puzzle

AdS/CFT provides comparably short hydronization times, which leads to questions

- why?
- how does the thermalization process occurs at strong coupling and what are its features?

Model: boost-invariant flow [Bjorken 1982]



The simplest, yet phenomenologically interesting field theory dynamics is the **boost-invariant flow** with **no transverse expansion.**

|| relevant for central rapidity region

|| no elliptic flow (~ central collision)

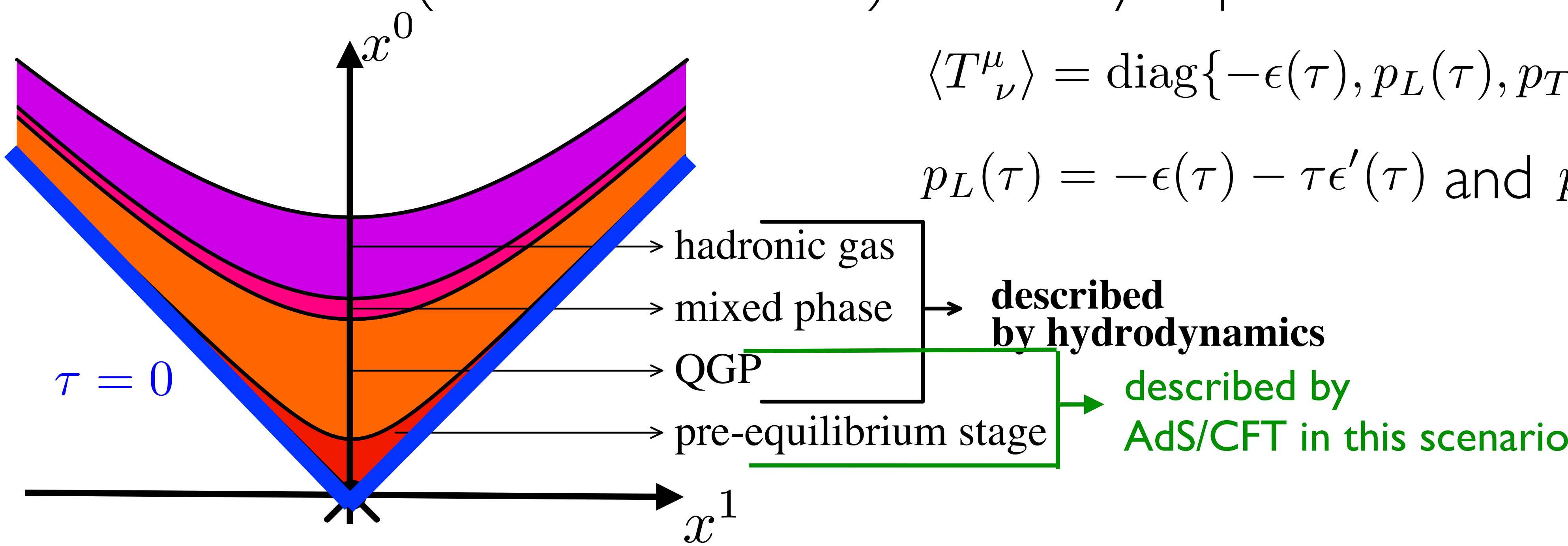
In Bjorken scenario dynamics depends only on proper time $\tau = \sqrt{(x^0)^2 - (x^1)^2}$

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_1^2 + dx_2^2$$

and stress tensor (in conformal case) is entirely expressed in terms of energy density

$\langle T^\mu{}_\nu \rangle = \mathrm{diag}\{-\epsilon(\tau), p_L(\tau), p_T(\tau), p_T(\tau)\}$ with

$p_L(\tau) = -\epsilon(\tau) - \tau\epsilon'(\tau)$ and $p_T(\tau) = \epsilon(\tau) + \frac{1}{2}\tau\epsilon'(\tau)$



We are interested in setting strongly coupled non-equilibrium initial states at $\tau = 0$ (and also at $\tau > 0$) and tracking their unforced relaxation towards hydrodynamics.

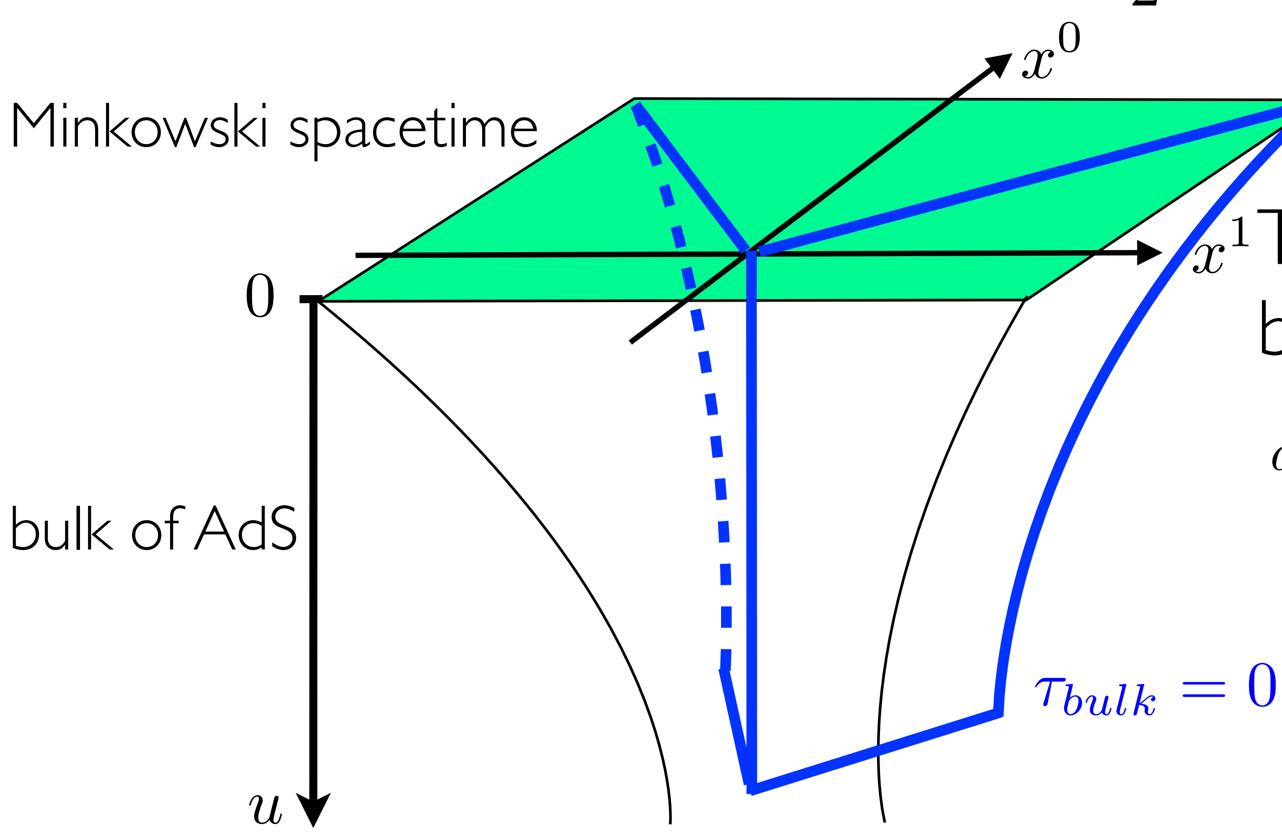
Tool: AdS/CFT correspondence

Maldacena [hep-th/9711200]
review: Mc Greevy 0909.0518 [hep-th]

From applicational perspective AdS/CFT is a tool for computing correlation functions in certain strongly coupled gauge theories, such as $\mathcal{N} = 4$ SYM at large N_c and λ

In its simplest instance AdS/CFT maps the dynamics of the stress tensor of a holographic CFT$_{1+3}$ into (1+4)-dimensional AdS geometry being a solution of

$$R_{ab} - \frac{1}{2} R g_{ab} - \frac{6}{L^2} g_{ab} = 0$$



The stress tensor is encoded in the near-boundary (small u) expansion of geometry

$$ds^2 = \frac{1}{4u^2} du^2 + \frac{1}{u} \left\{ \eta_{\mu\nu} + \frac{2\pi^2}{N_c^2} \langle T_{\mu\nu} \rangle u^2 + \ldots \right\} dx^\mu dx^\nu$$

Skenderis et al. [hep-th/0002230]

Of interest are geometries interpolating between far-from-equilibrium and hydrodynamic forms of the dual stress tensor at some initial and final time.

Such geometries describe black hole equilibration processes in AdS spacetimes

Initial state and the choice of bulk coordinantes

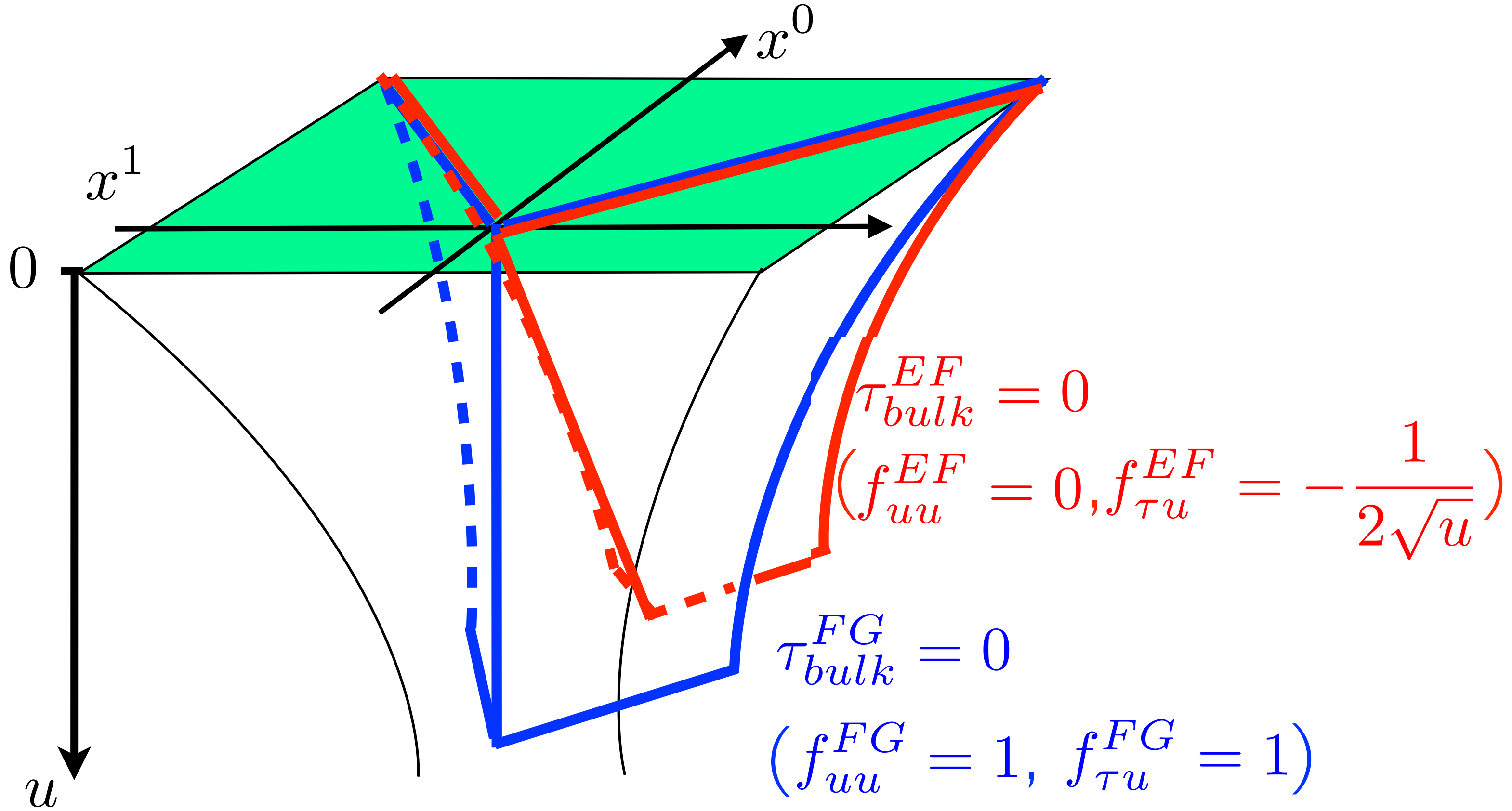
Initial states are solutions of gravitational constraints on chosen $\tau_{bulk} = 0$ hypersurface

Symmetries of a stress tensor dictate metric ansatz

$$ds^2 = \frac{1}{u}\Big\{ \underbrace{\frac{1}{4u} f_{uu} du^2 + 2 f_{\tau u} d\tau du - f_{\tau\tau} d\tau^2}_{} + \tau^2 f_{yy} dy^2 + f_{\perp\perp} dx_\perp^2 \Big\}$$

Diffeomorphism freedom = one is free to choose 2 functions out of $f_{uu}, f_{\tau u}$ and $f_{\tau\tau}$ leaving 3 dynamical warp factors

Different choices cover different patches of spacetime and lead to different foliations by constant time hypersurfaces (in particular, different bulk initial time hypersurface)



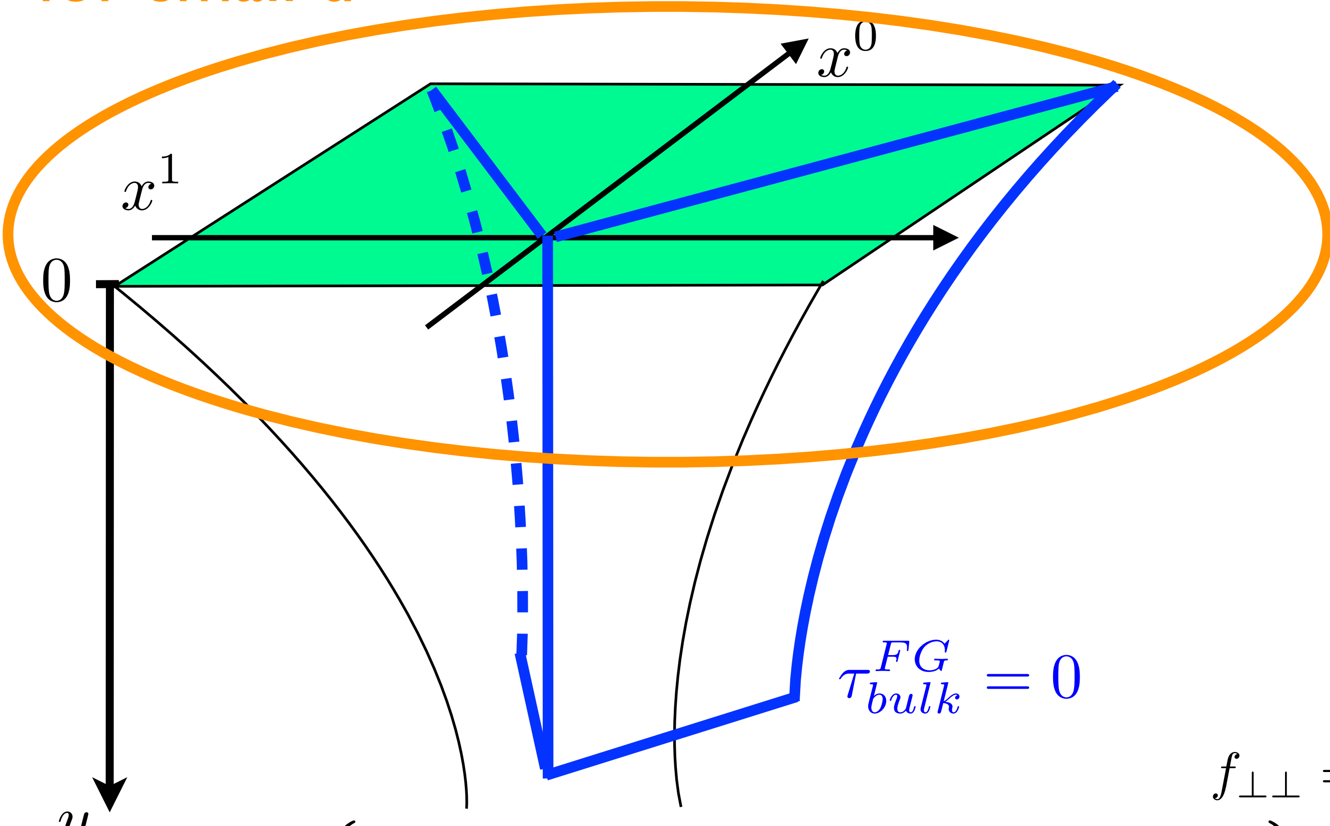
In 0906.4423 [hep-th] we chose $f_{uu}^{FG} = 1$, $f_{\tau u}^{FG} = 0$, and looked at constraint equations at $\tau_{bulk}^{FG} = 0$

Obtained warp factors served as initial data for numerical simulations in 1103.3452 [hep-th] and 1203.0755 [hep-th]

How the bulk geometry encodes dual initial states

Assuming $\langle T^{\mu}{}_{\nu}\rangle = \mathrm{diag}\{-\epsilon(\tau), p_L(\tau), p_T(\tau), p_T(\tau)\}$ one can solve $R_{ab} - \frac{1}{2}R g_{ab} - \frac{6}{L^2} g_{ab} = 0$

for small u



$$ds^2 = \frac{1}{u}\left\{\frac{1}{4u}du^2 - f_{\tau\tau}d\tau^2 + \tau^2 f_{yy}dy^2 + f_{\perp\perp}dx_\perp^2\right\}$$

e.g.

$$f_{\perp\perp} = 1 + (\epsilon + \frac{1}{2}\tau\epsilon')u^2 + (\boxed{\frac{1}{8\tau}\epsilon'} + \frac{5}{24}\epsilon'' + \frac{1}{24}\tau\epsilon''')u^3 + \ldots$$

Let's assume the most general regular $\epsilon(\tau)$

$$\epsilon(\tau)\Big|_{\tau\approx 0} = \epsilon_0 + \epsilon_1\tau + \epsilon_2\tau^2 + \ldots$$

The geometry gets singular

$$f_{\perp\perp} = 1 + (\epsilon_0 + \frac{3}{2}\epsilon_1\tau + 2\epsilon_2\tau^2 + \ldots)u^2 + (\boxed{\frac{1}{8\tau}\epsilon_1} + \frac{2}{3}\epsilon_2\ldots)u^3 + \ldots$$

unless $\frac{d^{2k+1}\epsilon(\tau)}{d\tau^{2k+1}}\Big|_{\tau=0} = 0$ for all k

Lessons and holographic „predictions"

1) The energy density at early times *needs* to have the form $\epsilon(\tau)\Big|_{\tau\approx 0} = \epsilon_0 + \epsilon_2\tau^2 + \epsilon_4\tau^4 + \ldots$

$$\longrightarrow \langle T^{\mu}{}_{\nu}\rangle\Big|_{\tau=0} = \mathrm{diag}\{-\epsilon(0), -\epsilon(0), \epsilon(0), \epsilon(0)\}$$

2) There is a 1:1 map between derivs of stress tensor and small-u bulk metric

e.g. $f_{\perp\perp} = 1 + \epsilon_0 u^2 + \frac{2}{3}\epsilon_2 u^3 + \ldots$ etc

Beuf, MPH, Janik & Peschanski 0906.4423

The idea behind solving initial value problem

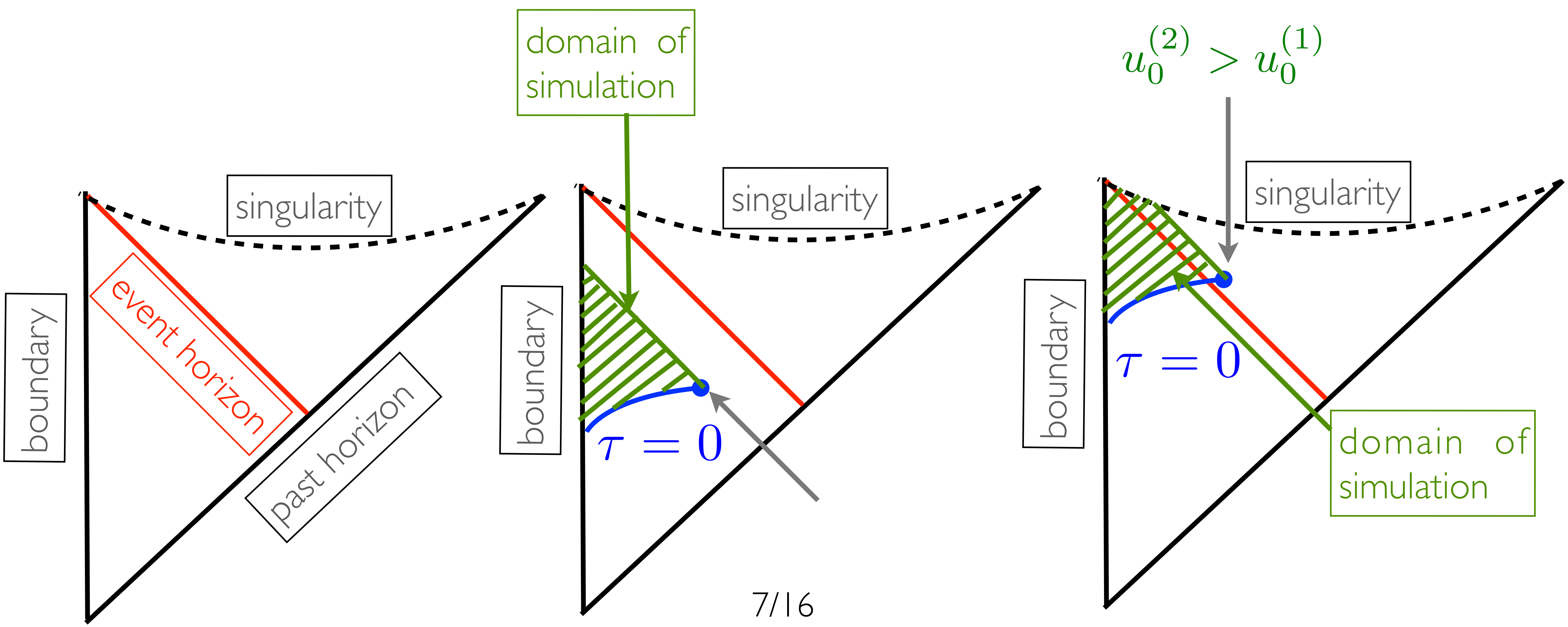
Because of $\epsilon(\tau)\Big|_{\tau\approx 0} = \epsilon_0 + \epsilon_2\tau^2 + \epsilon_4\tau^4 + \ldots$ time derivs of $f_{\tau\tau}$, f_{yy} & $f_{\perp\perp}$ vanish at $\tau = 0$

Then two constraints allows to solve for $f_{\tau\tau}$ and f_{yy} at $\tau = 0$ in terms of $f_{\perp\perp}(\tau = 0, u)$

Typical example of $f_{\perp\perp}(\tau = 0, u)$ is $cosh(u/u_0)^2$. Early time power series is not enough.

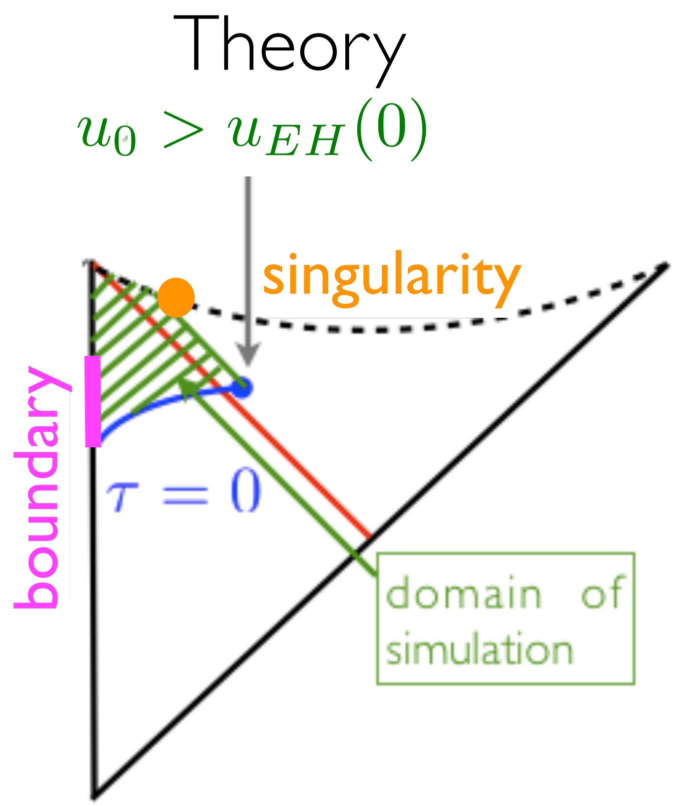
Neither are the Fefferman-Graham coordinates!!!

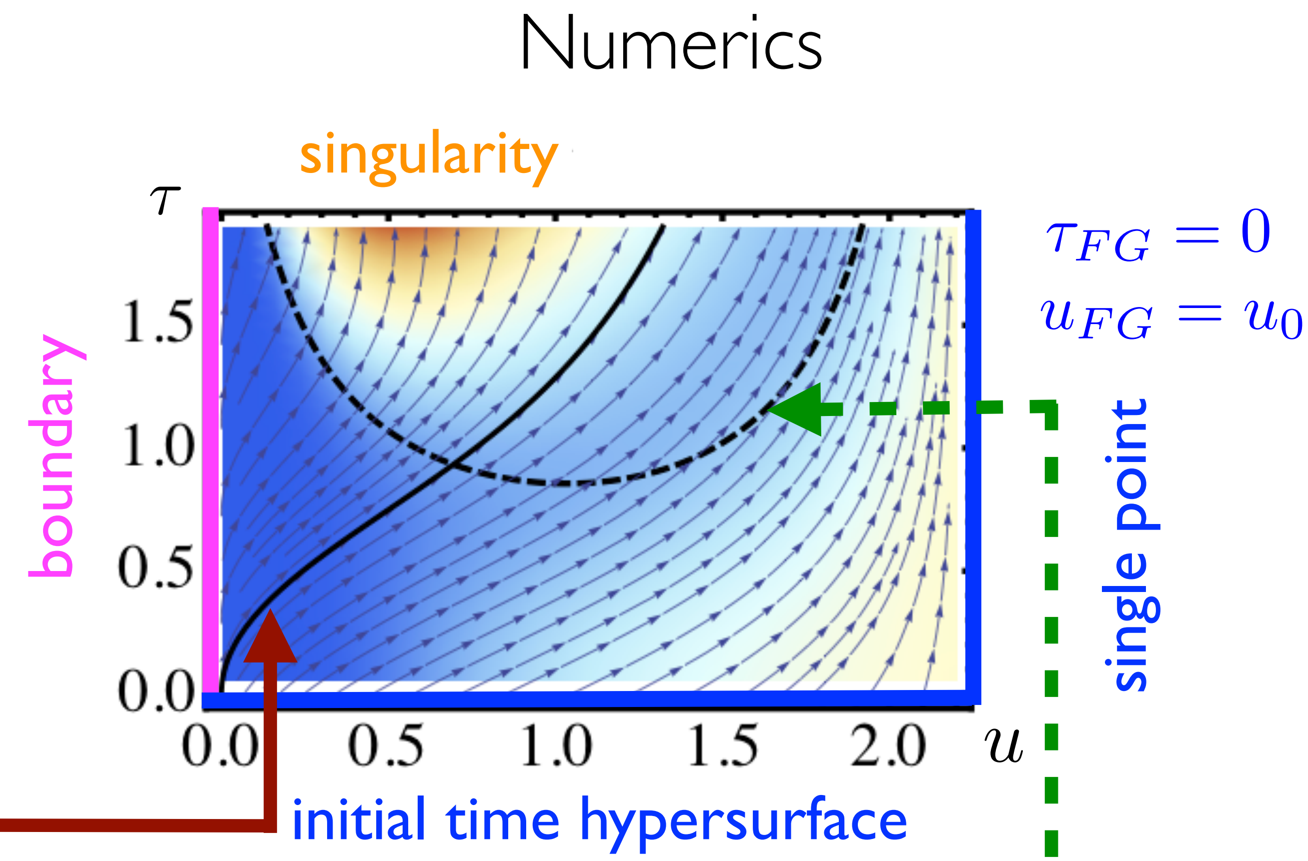
For this reason we considered another coordinate patch, which coincides with FG one at $\tau^{FG}_{bulk} = 0$, but otherwise it is different: $f^{N}_{\tau\tau} \sim (u_0 - u)^2$ instead of $f^{FG}_{uu} = 1$



Non-equilibrium entropy

Rangamani et al. 0902.4696 [hep-th]
Booth, MPH, Spalinski 0910.0748 [hep-th]





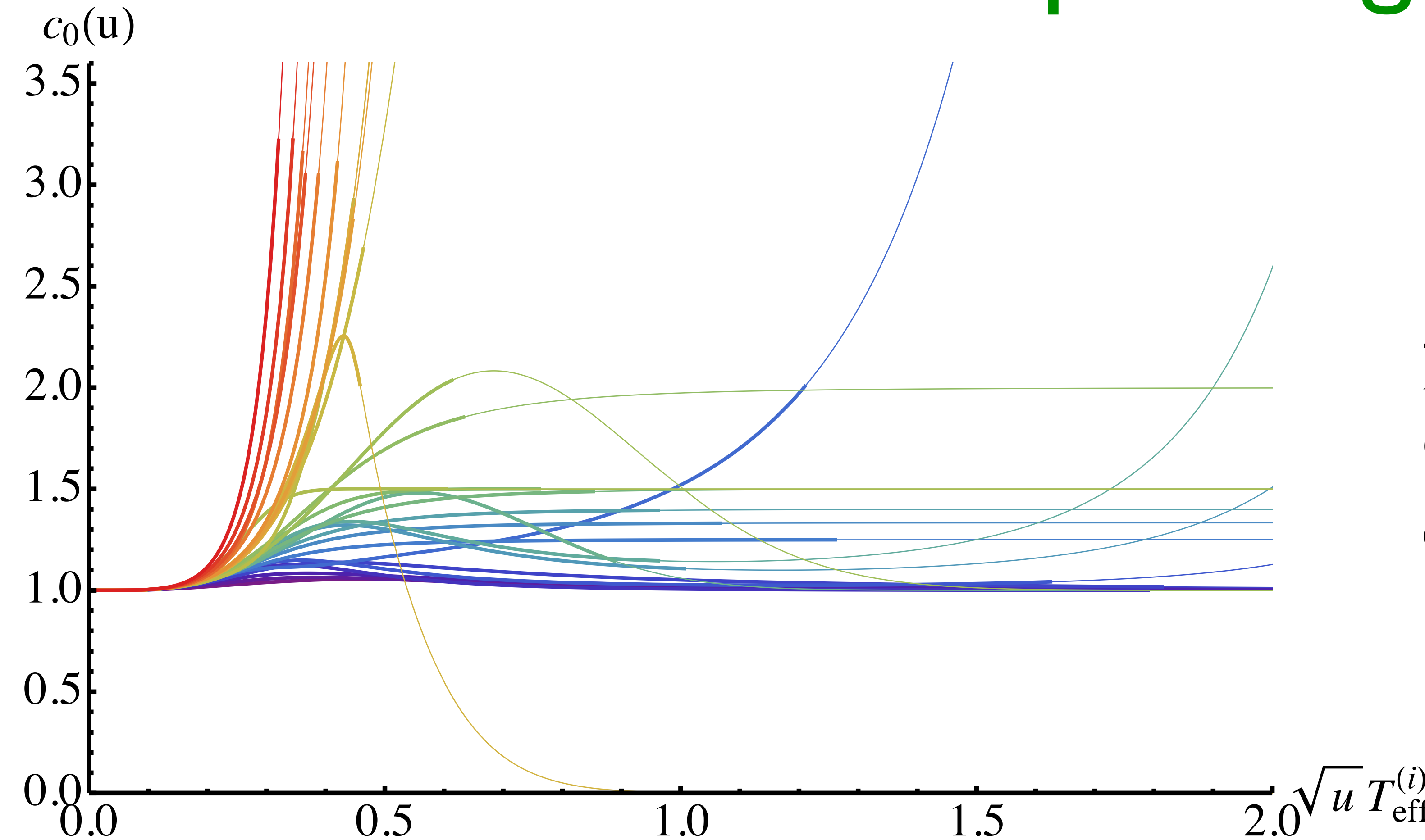
Beyond equilibrium event horizon is not the right notion of entropy.

In the gravity dual to boost-invariant flow it seems sensible to associate non-equilibrium entropy with unique translationally-invariant **apparent horizon**

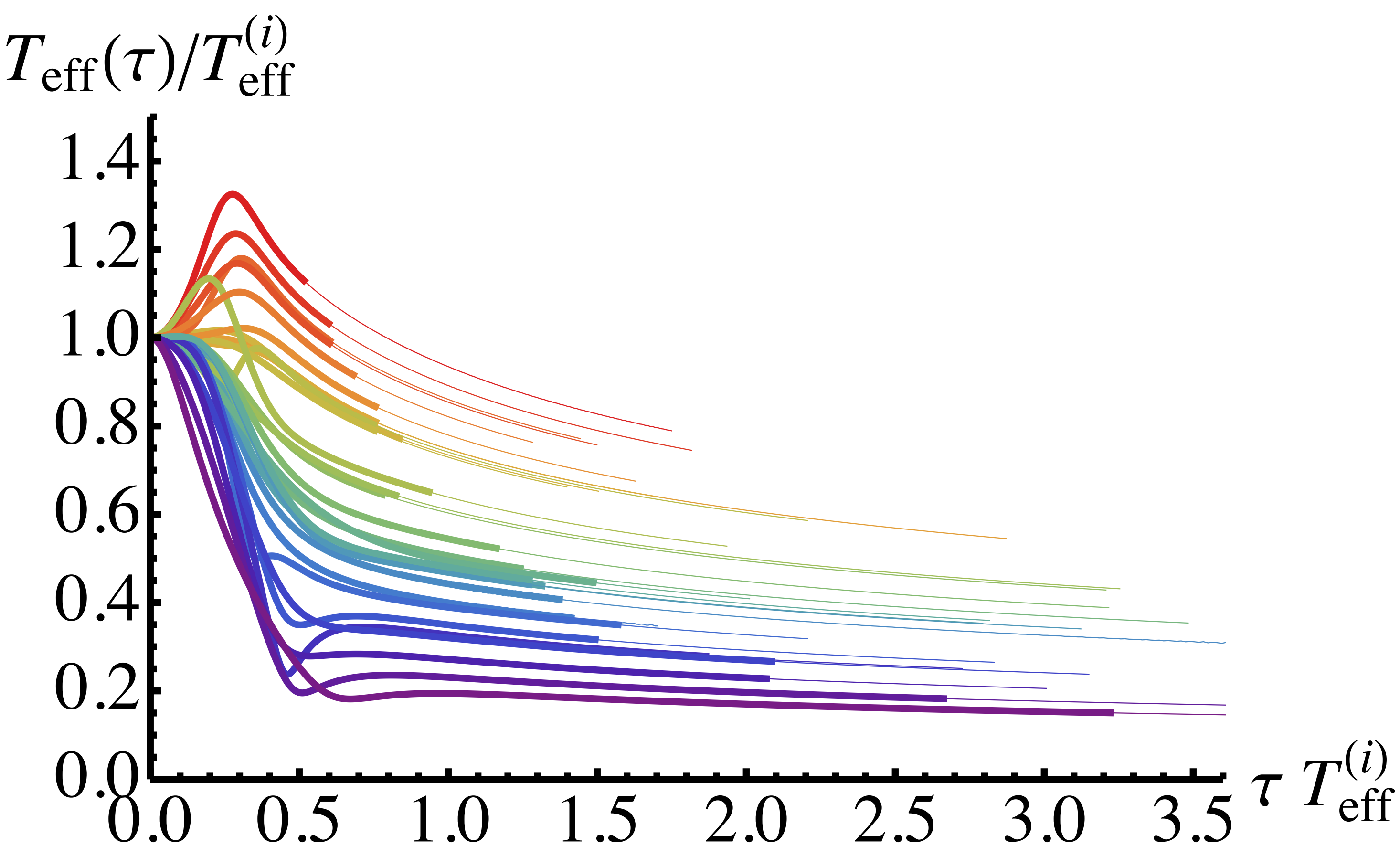
Its area element is associated with points on the boundary lying on the same **ingoing radial null geodesic** (bulk-boundary map)

All considered initial data had a non-zero non-equilibrium entropy at $\tau_{boundary} = 0$, thus **hydronization is not horizon formation, but rather horizon equilibration!**

Initial data and corresponding energy densities



29 Initial warp factors in Fefferman-Graham coordinates with radial cut off enabling seeing thermalization



Normalized effective temperatures

$$\epsilon(\tau) = \frac{3}{8} N_c^2 \pi^2 T_{eff}(\tau)^4$$

as functions of proper time

Boost-invariant hydrodynamics

Hydrodynamics: $\nabla_\mu \langle T^{\mu\nu} \rangle = 0$ and $\langle T^{\mu\nu} \rangle = \{\epsilon(T) + P(T)\}\, u^\mu u^\nu + P(T)\eta^{\mu\nu} + \ldots$

In the conformal hydrodynamics $\ldots$ have gradients of u^μ only

But here due to symmetries $u^\mu \partial_\mu = \partial_\tau$, so its gradients are trivial (Christoffels)

Because of this $\nabla_\mu T^{\mu\nu} = 0$ in the boost-invariant hydro is a 1st order ODE for $\epsilon(\tau)$

We define T_{eff} by $\epsilon(\tau) = \frac{3}{8} N_c^2 \pi^2 T_{eff}(\tau)^4$ and use dimensionless qty $w = \tau T_{eff}$

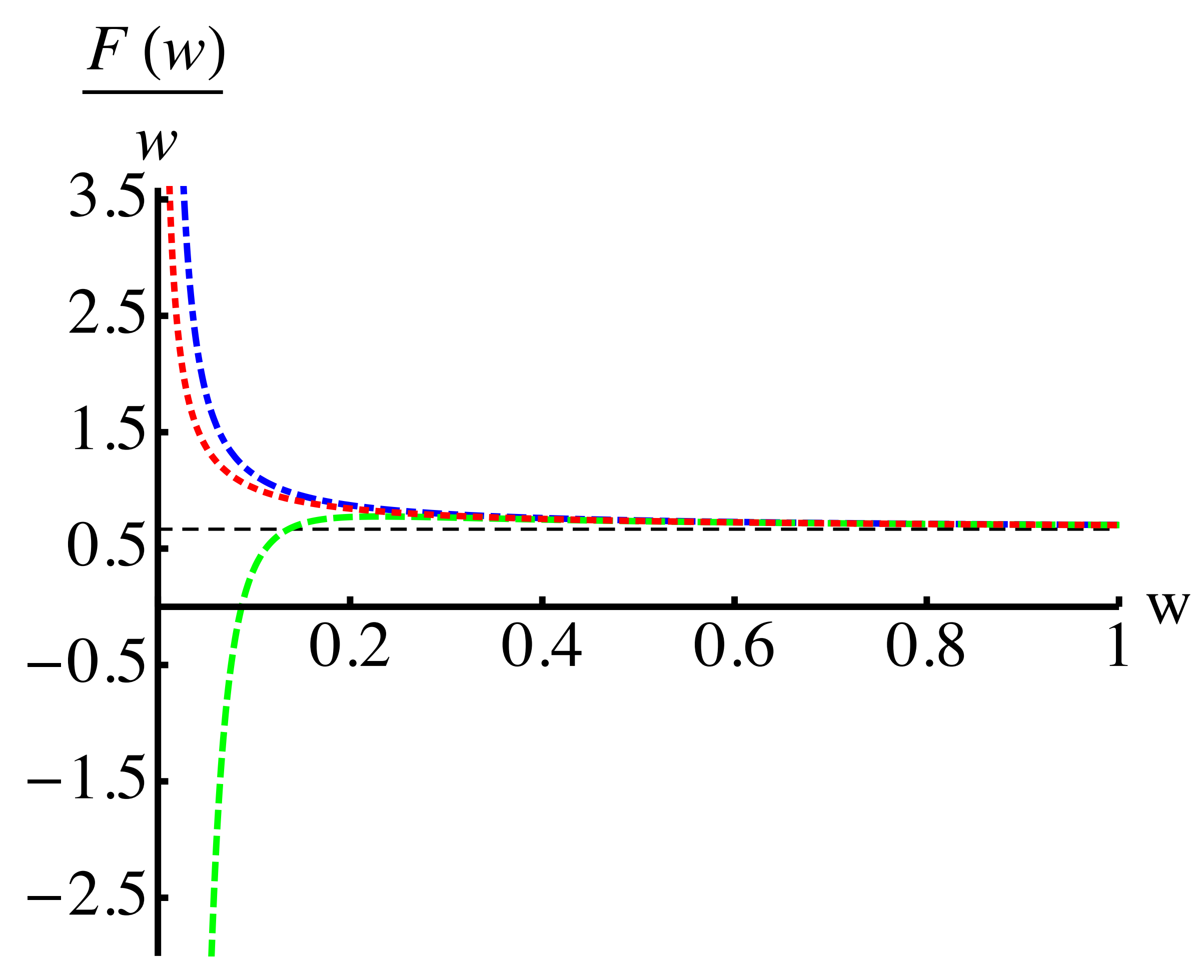
Equations of hydro: $\frac{\tau}{w}\frac{d}{d\tau} w = \frac{F_{hydro}(w)}{w}$

||

perfect fluid

$$\frac{2}{3} + \frac{1}{9\pi w} + \frac{1-\log 2}{27\pi^2 w^2} + \frac{15 - 2\pi^2 - 45\log 2 + 24\log^2 2}{972\pi^3 w^3} + \ldots$$

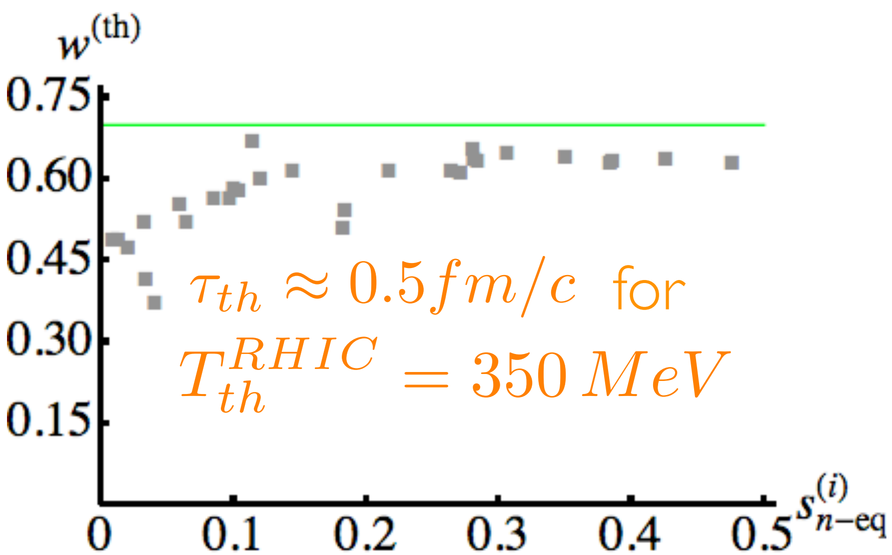
1st 2nd 3rd order hydro

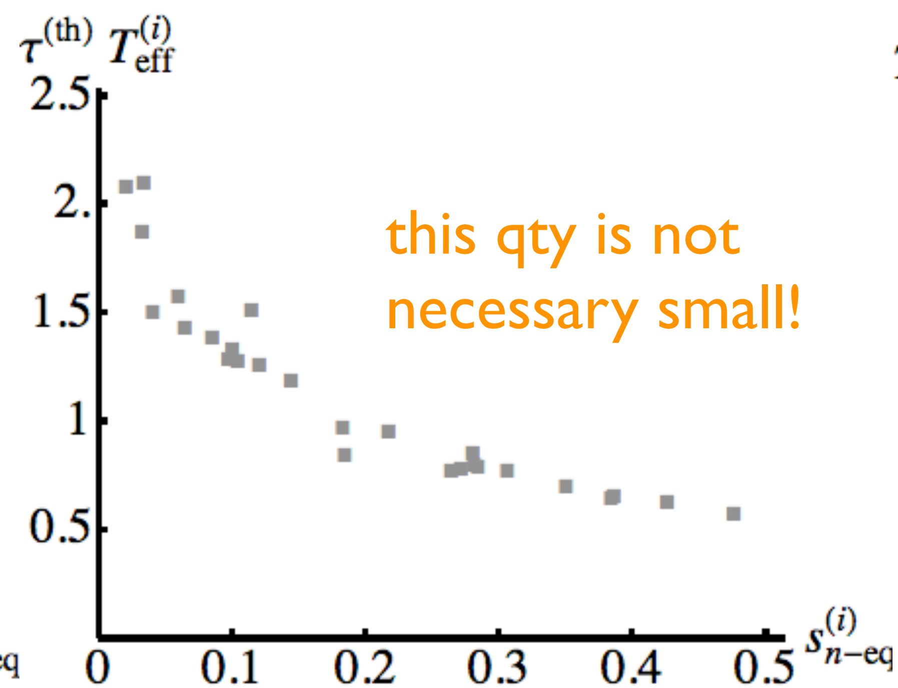


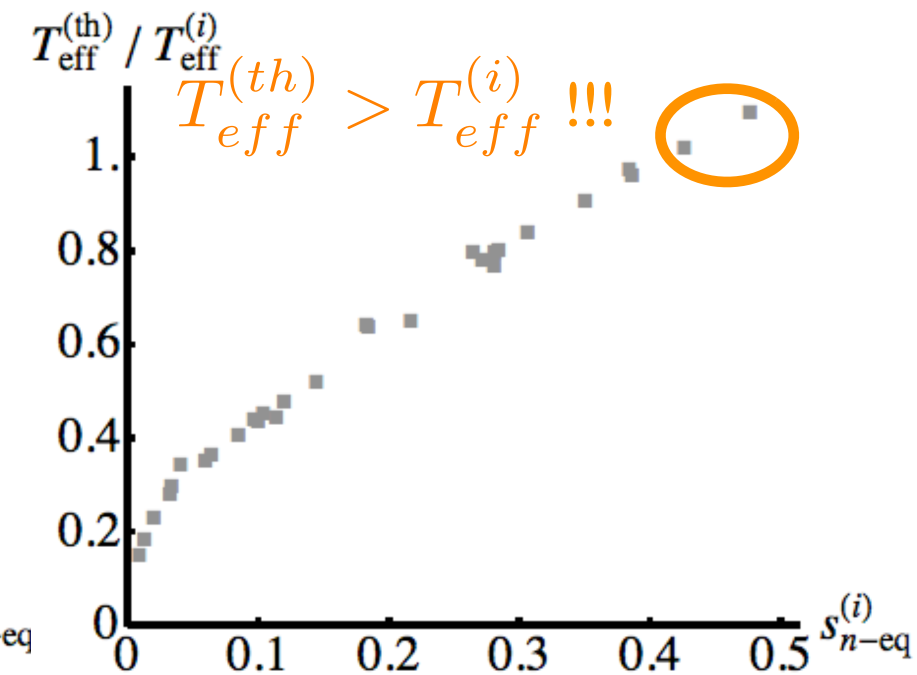
Characteristics of hydronization

We choose $\left\| \frac{\tau \frac{d}{d\tau} w}{F_{hydro}^{3^{rd}\,order}(w)} - 1 \right\| < 0.5\%$ as a criterium for hydronization

Below are the plots of various non-equilibrium characteristics of plasma as a function of dimensionless entropy density defined by $S \cdot T_{eff}(0)^{-2} = N_c^2 \cdot \frac{1}{2}\pi^2 \cdot s$

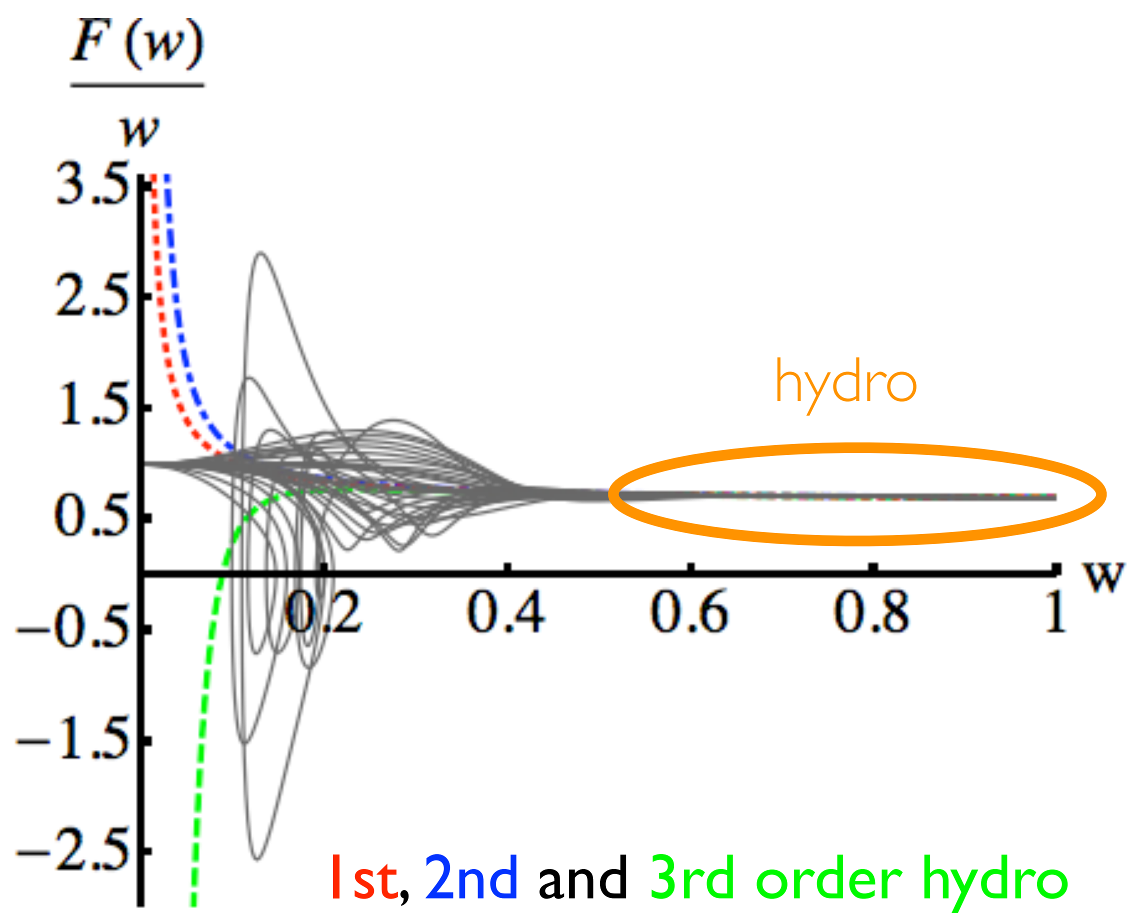


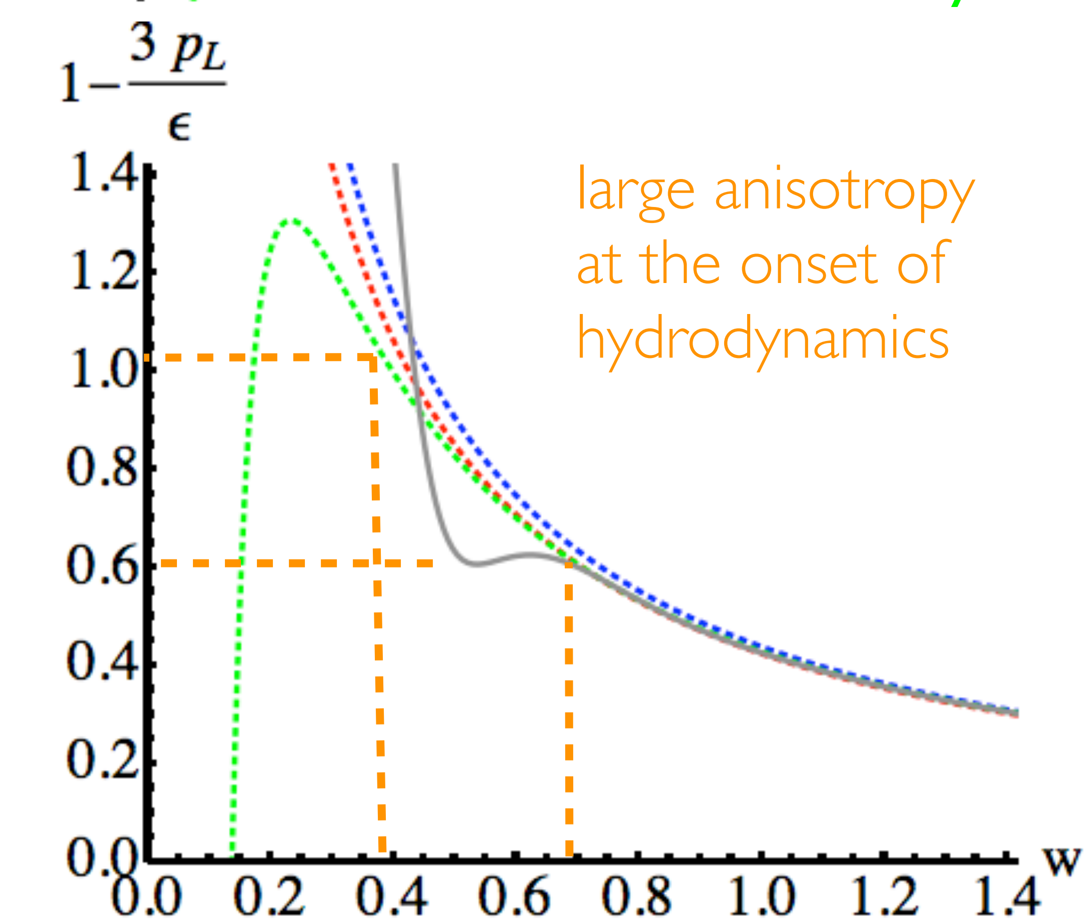




Although initial far-from-equilibrium state is specified by infinitely many numbers (infinite number of derivatives of energy density at $\tau = 0$), **its energy density and non-equilibrium entropy seem to** determine crude features of thermalization!

Hydronization vs thermalization/isotropization





Rewriting equations of hydrodynamics in a form

$$\frac{\tau}{w}\frac{d}{d\tau}w = \frac{F_{hydro}(w)}{w}$$

allows to explicitly see whether non-hydro modes already relaxed when curves coincide!

Note that hydronization time is not given by the convergence radius of the hydro expansion!

Chesler & Yaffe 0906.4426

Lublinsky & Shuryak 0704.1647 and 0905.4069

The single most interesting result is that **hydronization occurs well before isotropization!**

Pressure anisotropy is observed to be between

$$1-\frac{3p_L}{\epsilon}\approx 0.6 \text{ to } 1.0$$

with hydrodynamics already being a valid description of the stress tensor dynamics.

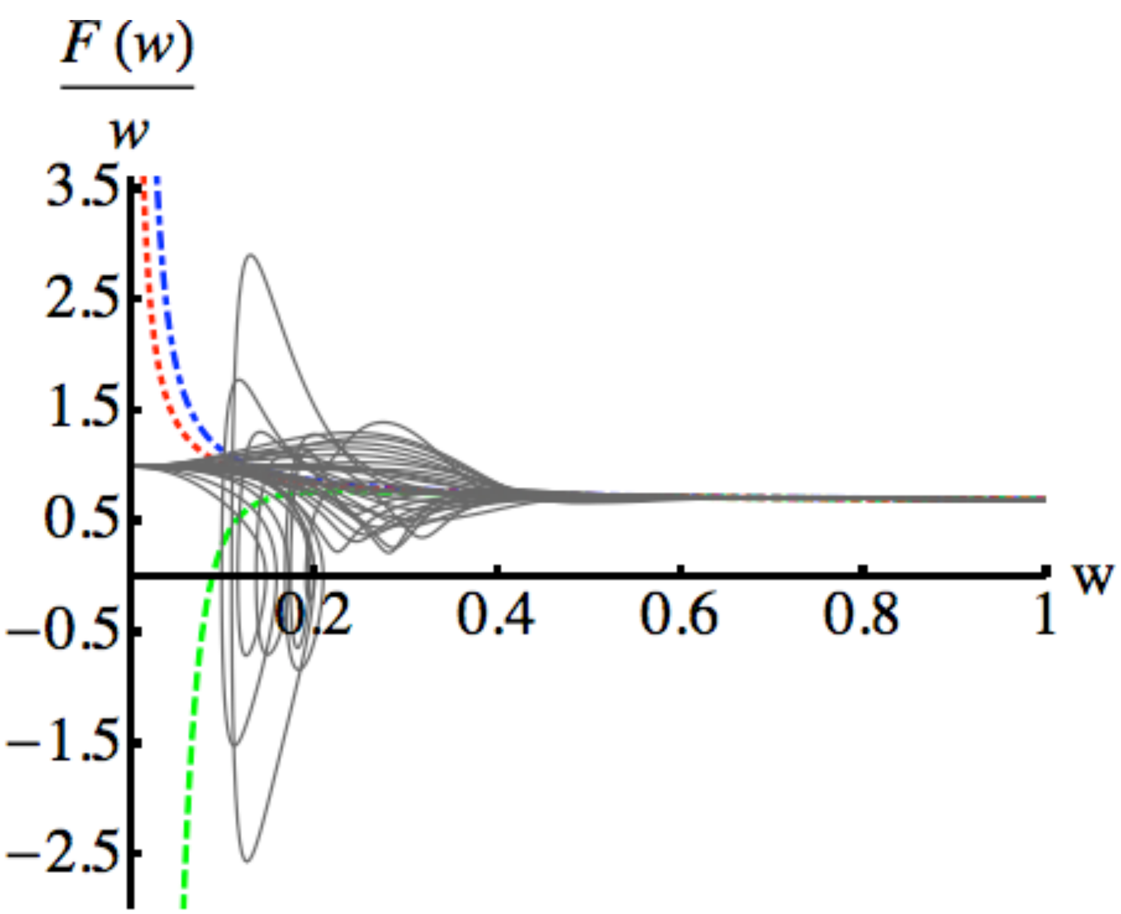
similar findings in Chesler & Yaffe 0906.4426 and 1011.3562

Eddington-Finkelstein analysis

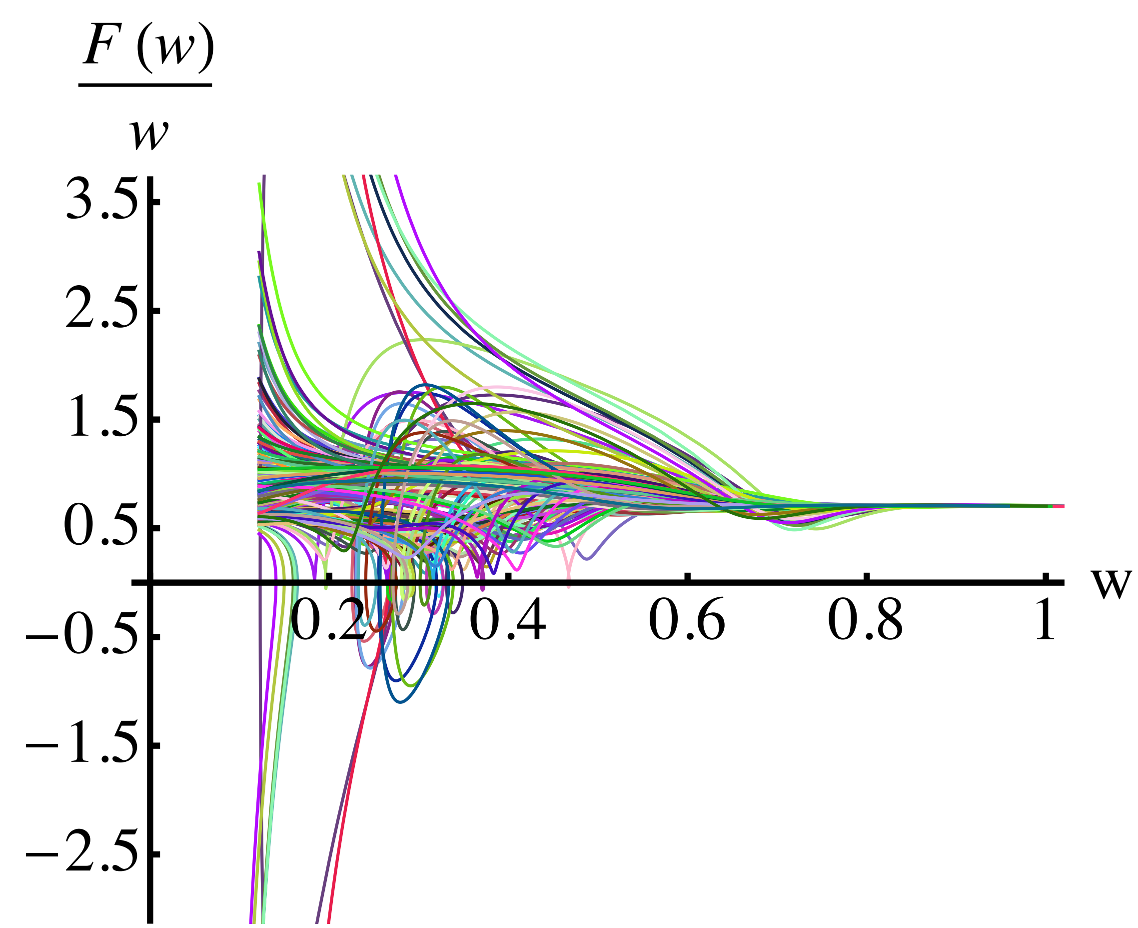
We can solve exactly the same problem in the Eddington-Finkelstein coordinates

$$ds^2 = \frac{1}{u}\left\{-\frac{1}{\sqrt{u}}d\tau du + f_{\tau\tau}d\tau^2 + \tau^2 f_{yy}dy^2 + f_{\perp\perp}dx_\perp^2\right\}$$

and the findings are similar:



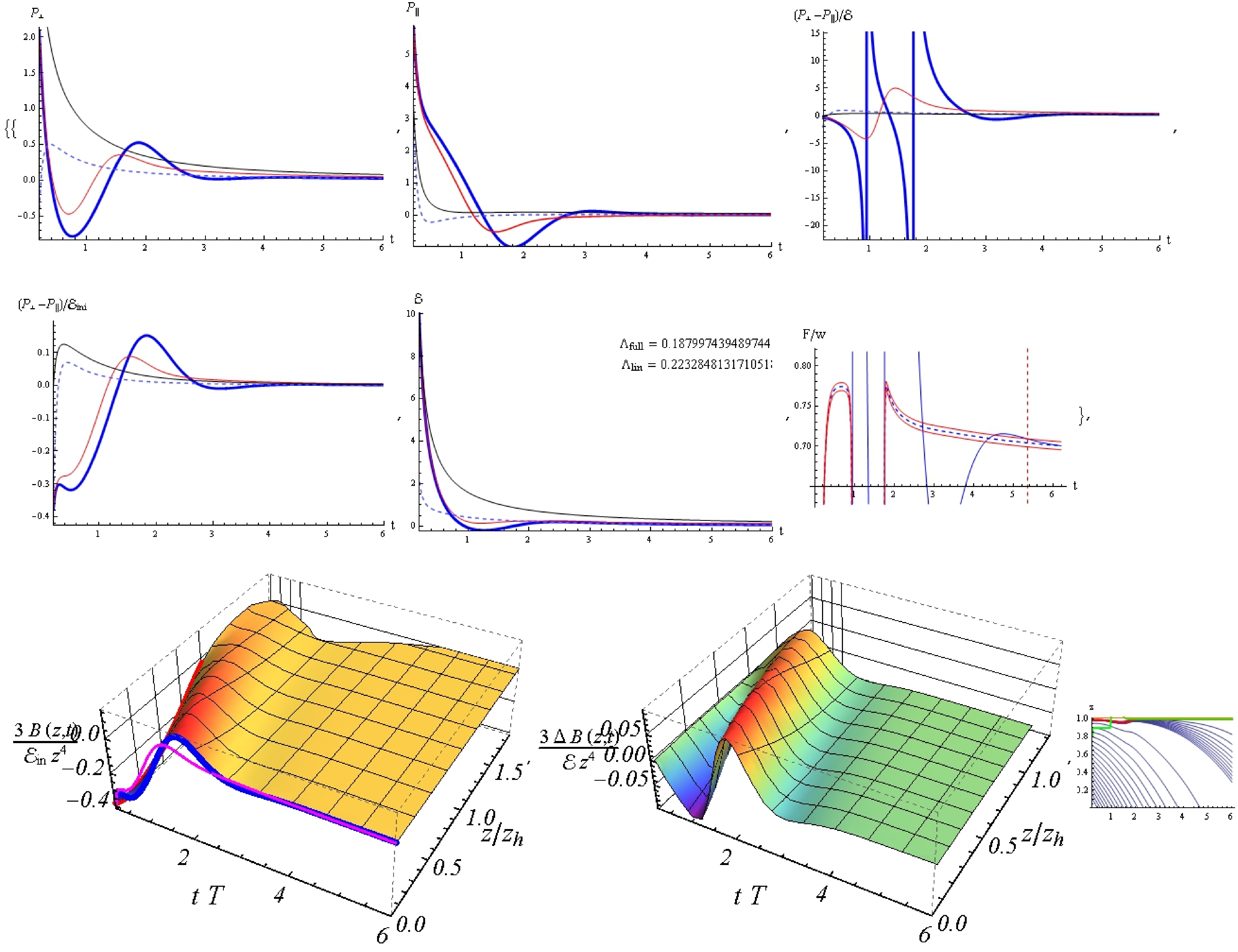
29 data from 1103.3452 [hep-th]



221 data from ongoing work in EF coord

Linearized vs full Einstein's equations

very preliminary results



Summary

§ AdS/CFT seems to naturally lead to short hydronization times

§ Holographic hydronization = bulk black hole equilibration rather than formation

§ Preliminary results suggest that linearized gravity gives a reasonable approximation

§ Plasma can be very anisotropic $(\epsilon - 3p_L)/\epsilon \approx 0.6 \text{ to } 1.0$, yet in the hydro regime

§ Thus hydronization needs to be distinguished from isotropization/thermalization

Open directions

§ How does hydronization proceed in the presence of transverse dynamics?

§ Do anisotropies in hydrodynamic regime leave an imprint on particles produced?

§ Why is the holographic thermalization quick?

Lorentz Center

International Center for workshops in the Sciences

Current Workshop | Overview   Back | Print | Home | Search | Contact

Holographic Thermalization

from 8 Oct 2012 through 12 Oct 2012

Venue: Lorentz Center@Oort

- **Description and aim** of the workshop
- **Registration** form
- **Participants**
- Scientific organizers:
  Jan de Boer (Amsterdam, Netherlands)
  Paul Chesler (Cambridge, USA)
  Ben Craps (Brussels, Belgium)
  Michal P. Heller (Amsterdam, Netherlands)
- Workshop Coordinator: **Sietske Kroon,** Tel: +31 71 5275585

  Organizational Log-in (restricted)

Lorentz fonds

Workshop lectures are open to anyone interested.

[Back]

http://tinyurl.com/HoloTh