Shear viscosity in a superfluid cold Fermi gas at unitarity

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Outline

• The system under study

• Some basic facts about superfluid systems: $U(1)$ SSB and the NGB

• Shear viscosity due to superfluid phonons
Fermions at unitarity

- Non-relativistic fermions at finite density, subject to two body (zero range) contact interactions

- $s$-wave amplitude saturates the unitarity limit

\[
f_0(k) = \frac{1}{ik - \frac{1}{a_0} + \frac{1}{2}r_0 k^2}
\]

\[
|f_0|^2 \leq \frac{1}{k^2} \quad r_0 k_F^2 \ll 1, \quad \frac{1}{k_F a} \to 0
\]

At unitarity the system is scale invariant!
Experimental success

- Ultracold: Fermi T in the microKelvin range
- Feshbach resonance techniques allow to tune $k_F a$ at will, and reach the unitarity limit
- We are particularly interested in the measure of transport coefficients at unitarity
Fig. 3 Estimated ratio of the shear viscosity to the entropy density.
Superfluidity

Quantum phenomenon associated to the appearance of a quantum condensate = SSB of a global $U(1)$

\[ \psi = |\psi|e^{-i\varphi} \quad \langle \psi\psi \rangle = |\langle \psi\psi \rangle|e^{-2i\varphi} \]

\[ \mathbf{j} = -\frac{i}{2m} (\psi^\dagger\nabla\psi - \nabla\psi^\dagger\psi) \]

\[ \mathbf{j} = \rho_s \mathbf{v}_s \quad \mathbf{v}_s = \frac{\nabla \phi}{m} \]

Hydrodynamics complicated: two-fluid model

Superfluid component: no dissipation

Normal component: dissipative processes are possible

\[ \rho = \rho_n + \rho_s \quad \mathbf{j} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s \]
Low energy EFT for the phonons

\[ \mathcal{L}_{\text{LO}} = P(X) \quad X = \mu_0 - V(r) - \partial_t \varphi - \frac{(\nabla \varphi)^2}{2m} \]

valid for all superfluids, with the same global symmetries

\[ P = \frac{2^{5/2}}{15\pi^2 \xi^{3/2}} m^{3/2} \mu_0^{5/2} \]

EoS fermi gas at unitarity

\[ \mu_0 = \xi E_F, \quad \xi \sim 0.4 \]
\[
\mathcal{L}_{\text{LO}} = \frac{1}{2} \left( (\partial_t \phi)^2 - v_{ph}^2 (\nabla \phi)^2 \right) - g \left( (\partial_t \phi)^3 - 3 \eta_g \partial_t \phi (\nabla \phi)^2 \right) + \lambda \left( (\partial_t \phi)^4 - \eta_{\lambda,1} (\partial_t \phi)^2 (\nabla \phi)^2 + \eta_{\lambda,2} (\nabla \phi)^4 \right)
\]

\[
g = \frac{1}{6 \sqrt{m \rho} \ c_s} \left( 1 - 2 \frac{\rho}{c_s} \frac{\partial c_s}{\partial \rho} \right), \quad \eta_g = \frac{c_s}{6 \sqrt{m \rho} \ g}
\]

\[
\lambda = \frac{1}{24 \ m \rho \ c_s^2} \left( 1 - 8 \frac{\rho}{c_s} \frac{\partial c_s}{\partial \rho} + 10 \frac{\rho^2}{c_s^2} \left( \frac{\partial c_s}{\partial \rho} \right)^2 - 2 \frac{\rho^2}{c_s} \frac{\partial^2 c_s}{\partial \rho^2} \right),
\]

\[
\eta_{\lambda,2} = \frac{c_s^2}{8 \ m \rho \ \lambda}, \quad \eta_{\lambda,1} = 2 \frac{\eta_{\lambda,2}}{\eta_g}
\]

Escobedo and CM, 2010
Low energy EFT for the phonons

\[
\mathcal{L}_{\text{NLO}} = c_1 m^{1/2} \frac{(\nabla X)^2}{\sqrt{X}} + \frac{c_2}{\sqrt{m}} (\nabla^2 \varphi)^2 \sqrt{X}
\]

\[
E_k = c_s k (1 + \gamma k^2)
\]

\[
\gamma = - \left( c_1 + \frac{3}{2} c_2 \right) \frac{\pi^2 \sqrt{2 \xi}}{k_F^2} \approx 0.18 \frac{1}{k_F^2}
\]

and at higher orders

\[
E_k = c_s k (1 + \psi(k))
\]

\[
\psi(k) = \gamma k^2 + \delta k^4 + \mathcal{O} \left( \frac{k^6}{k_F^6} \right)
\]

Rupak and Schafer, 2009
Shear viscosity due to phonons

\[ \varphi \varphi \leftrightarrow \varphi \varphi \rightarrow \text{large angle collisions} \]

\[ \varphi \leftrightarrow \varphi \varphi \rightarrow \text{small angle collisions, only if } E \text{ curves upward} \]

\[ E_p = c_s p \]

\[ E_p = c_s p (1 + \gamma^2) \]

\[ E_p = c_s p (1 + \gamma p^2 - \delta p^4) \]

\[ \eta \propto \frac{1}{T^5} \]

Rupak & Schafer 2007

\[ \eta \sim \infty \]

\[ \eta \propto \frac{1}{T^5} \]

\[ \eta \propto \frac{1}{T} \]
**Knudsen number**

For $\text{Kn} > 1$ phonons collide more often with the boundary.

Hydro needs $\text{Kn} \ll 1$

$$\psi_{\text{max}} = -\frac{\gamma^2}{4\delta}$$

$$\psi_{\text{max}} = 0.2, 0.3, 0.4$$
Data for the phonons of He4

\[ \eta_{\text{bulk}} = \frac{1}{5} \rho_{\text{ph}} c_s l_{\text{ph}} \]

\[ \eta_{\text{ball}} = \frac{1}{5} \rho_{\text{ph}} c_s a \]

\[ \eta_{\text{eff}} = \left( \eta_{\text{bulk}}^{-1} + \eta_{\text{ball}}^{-1} \right)^{-1} \]

FIG. 3. Temperature dependence of the coefficient of effective viscosity near the transition from the hydrodynamic to the ballistic regime: this work (■); Ref. 11 (△); Ref. 10 (◇), (▽); Ref. 15 (☆). Line 1—calculation using Eq. (22); 2—temperature dependence of the coefficient of viscosity of the normal component.

A. Zadorozhko et al., Low Temperature Physics 35, 100 (2009)
ψ_{max} = 0.3
Conclusions

- Phonons can explain the low T values of the shear viscosity for the unitarity Fermi gas.
- We used EFT to determine the phonon interactions; an anomalous dispersion law and finite size effects are needed to explain the data.