Classical Yang-Mills Theory Cascade

Guy Moore and Aleksi Kurkela

arXiv:1207.1663 (Monday)

- Why look at classical Yang-Mills theory?
- Cascade towards UV, scaling of momentum and occupancy
- Approach to a *scaling solution*
- Infrared effects: screening and magnetic screening
- How would condensates behave?

What I really want to study: Quantum YM theory at $\alpha_{\rm s} = 0.3$ with intense-field inhomogeneous expanding initial conditions

What I want to study: Classical YM theory + quantum fluctuations with intense-field inhomo. expanding init. condit.

What I would like to study: Classical YM theory with intense-field expanding initial conditions

What I will study for now: Classical YM, intense-field but non-expanding.

What does classical YM do?

Most theories seek equilibrium.

Classical field thy. in continuum has no equilibrium.

Unlimited UV phase space. Equipartition: energy should move into UV *forever*

Start with $f \sim \frac{1}{g^2 N_c}$ for $p \lesssim Q$, f small for $p \gg Q$.

Typical momentum scale $p_{\max}$ grows, typical occupancy $\tilde{f}$ shrinks, with time

Let's *rigorously define* my scales Q and $p_{\rm max}$:

$$\varepsilon = 2(N_{\rm c}^2 - 1) \int \frac{k^2 dk}{2\pi^2} k\, f(k) \quad \text{and} \quad \varepsilon \sim \frac{Q^4}{g^2 N_{\rm c}}\,, \quad f \sim \frac{1}{g^2 N_{\rm c}}$$

so we define

$$\varepsilon = \frac{2(N_{\rm c}^2 - 1)}{2\pi^2 N_{\rm c} g^2} Q^4 \quad \text{or} \quad Q^4 \equiv \frac{2\pi^2 N_{\rm c} g^2 \varepsilon}{2(N_{\rm c}^2 - 1)}$$

so that, to the extent f is well defined,

$$Q^4 = \int k^3 (g^2 N_{\rm c}\, f(k)) dk$$

Also define "typical momentum scale now":

$$p^2_{\rm max} \equiv \frac{\langle (\nabla \times \mathbf{B})^2 \rangle}{\frac{1}{2} \langle \mathbf{E}^2 + \mathbf{B}^2 \rangle} \qquad p^2_{\rm max} \simeq \frac{\int k^5\, f(k)\, dk}{\int k^3\, f(k)\, dk}$$

Dynamics: Expect collision rate Γ order $\Gamma t \sim 1$.

Estimate $\Gamma \sim g^4 f^2 p_{\rm max}$. Two expressions:

$$g^4 f^2 p_{\rm max} t \sim 1 \,, \qquad p^4_{\rm max} g^2 f \sim Q^4 \text{ time independent}$$

Solving,

$$p_{\rm max} \sim Q (Qt)^{\frac{1}{7}} \,, \qquad f \sim \frac{1}{g^2 N_{\rm c}} (Qt)^{\frac{-4}{7}}$$

see Kurkela and GM arXiv:1107:5050, Blaizot *et al.* arXiv:1107:5296

What about particle number? $\Gamma_{\text{number chg}} \sim g^4 f^2 p_{\rm max}$.
Number change could keep up – or there might be condensates??

Questions we want to ask

Do we observe expected $p_{\rm max} \simeq Q(Qt)^{\frac{1}{7}}$ scaling?

Does $f(p,t)$ approach *scaling solution*?

$$f(p,t) = (Qt)^{\frac{-4}{7}} \tilde{f}(p(Qt)^{\frac{-1}{7}}) \quad \text{Time-independent}$$

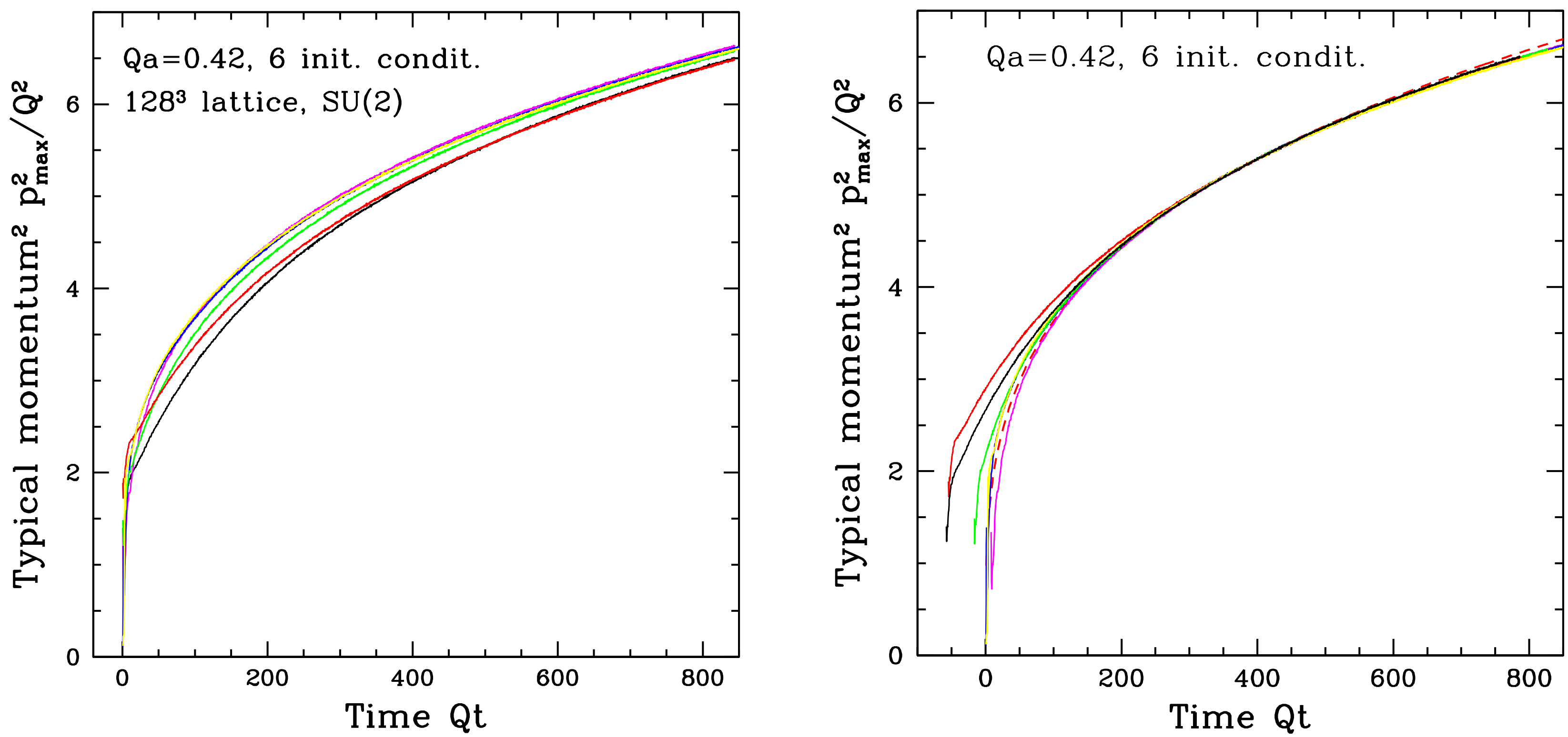
Behavior in infrared: $f \propto p^{-1}$, $f \propto p^{-\alpha}$ (4/3 or 3/2 or...)

Berges Schlichting Sexty

or is there a condensate? (larger IR occupancies than just power IR scaling)

If so, is it electric (plasmons) or magnetic?

Lattice study. gauge invar. measurables: $p^2_{\max}/Q^2$:



6 very different initial conditions converge, obey

$p_{\max} \sim Q(Qt)^{\frac{1}{7}}$

Occupancies? Fix to Coulomb gauge. Perturbatively,

$$\int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle A_a^i(x) A_b^j(0)\rangle = \frac{\delta_{ab}\mathcal{P}_{\mathrm{T}}^{ij}(\mathbf{p})}{|\mathbf{p}|} f(p)\,,$$
$$\int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle E_a^i(x) E_b^j(0)\rangle = \Big(\delta_{ab}\mathcal{P}_{\mathrm{T}}^{ij}(\mathbf{p})\, |\mathbf{p}|\Big) f(p)$$

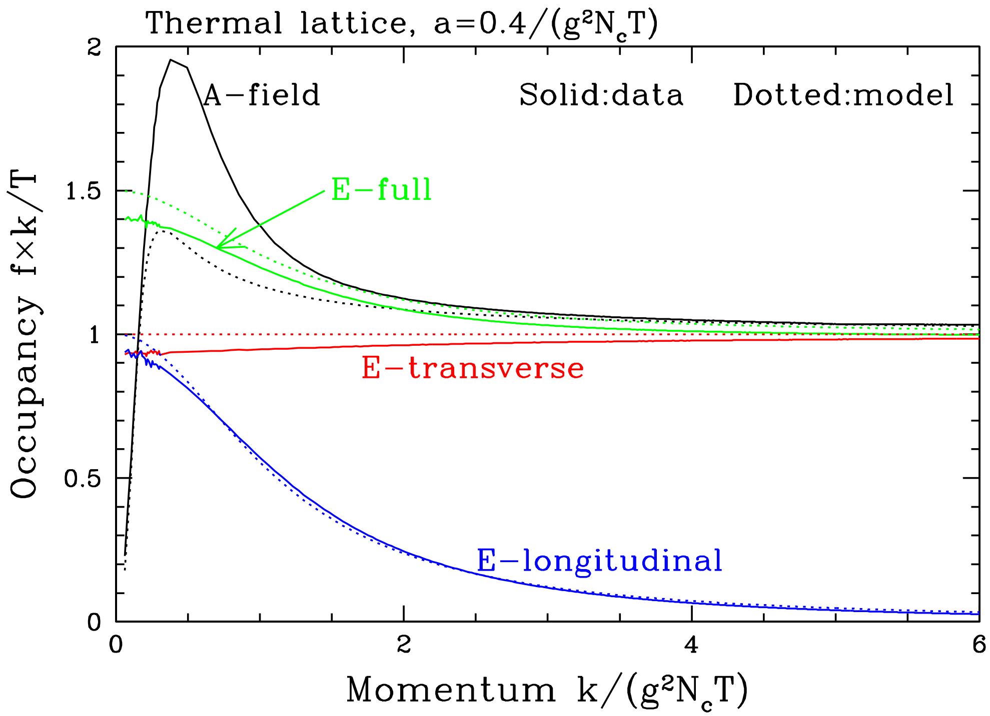
(with $\mathcal{P}_{\mathrm{T}}^{ij} = \delta^{ij} - \hat{p}^i \hat{p}^j$) Then we could simply define:

$$f_A(\mathbf{p}) = \frac{\delta_{ij}\delta_{ab}}{2(N_c^2-1)} |\mathbf{p}| \int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle A_a^i(x) A_b^j(0)\rangle_{\mathrm{coul}}\,,$$
$$f_E(\mathbf{p}) = \frac{\delta_{ij}\delta_{ab}}{2(N_c^2-1)|\mathbf{p}|} \int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle E_a^i(x) E_b^j(0)\rangle_{\mathrm{coul}}\,.$$

Two estimates of occupancy: $\mathbf{A}$-field and $\mathbf{E}$-field.

Trust, but verify

Equilibrium behavior for these “occupancies” 256^3 SU(2)



f_A: peak (fake?) and fall $f \leq 6/(g^2 N_c)$ (magnetic screening?)

f_E: rise in IR (Longitudinal occupancy!)

We made an assumption

We assumed $\langle \mathbf{E}\mathbf{E}(\mathbf{k})\rangle$ remains transverse!

It doesn't: $\mathbf{D}\cdot\mathbf{E}=0$, not $\boldsymbol{\nabla}\cdot\mathbf{E}$.
Fluctuations: effective random charge density.

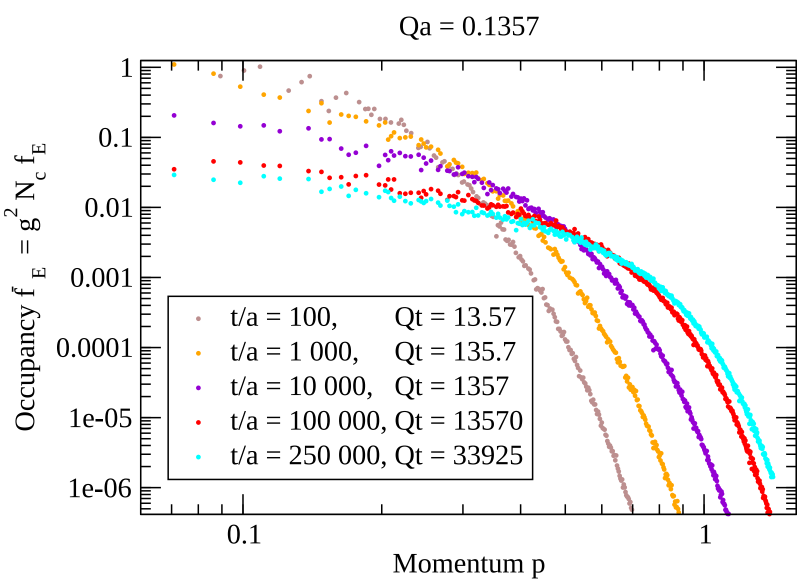
perturbatively but working a bit harder,

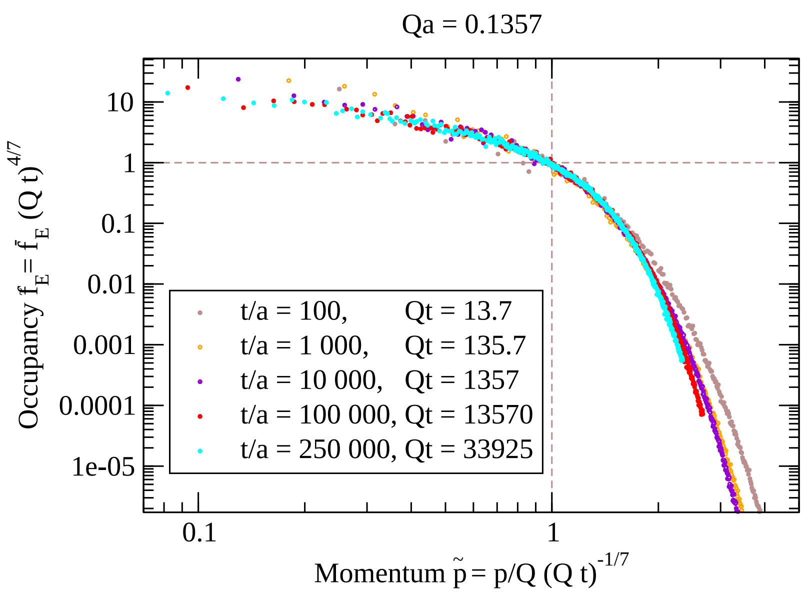
$$\int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle E^i_a(x) E^j_b(0)\rangle_{\rm eq} = \delta_{ab} T \left(\mathcal{P}^{ij}_{\rm T}(\mathbf{p}) + \frac{m_{\rm D}^2}{m_{\rm D}^2 + p^2} \hat{p}^i \hat{p}^j \right)$$

Below scale $m_{\rm D}$, significant *longitudinal* contrib.

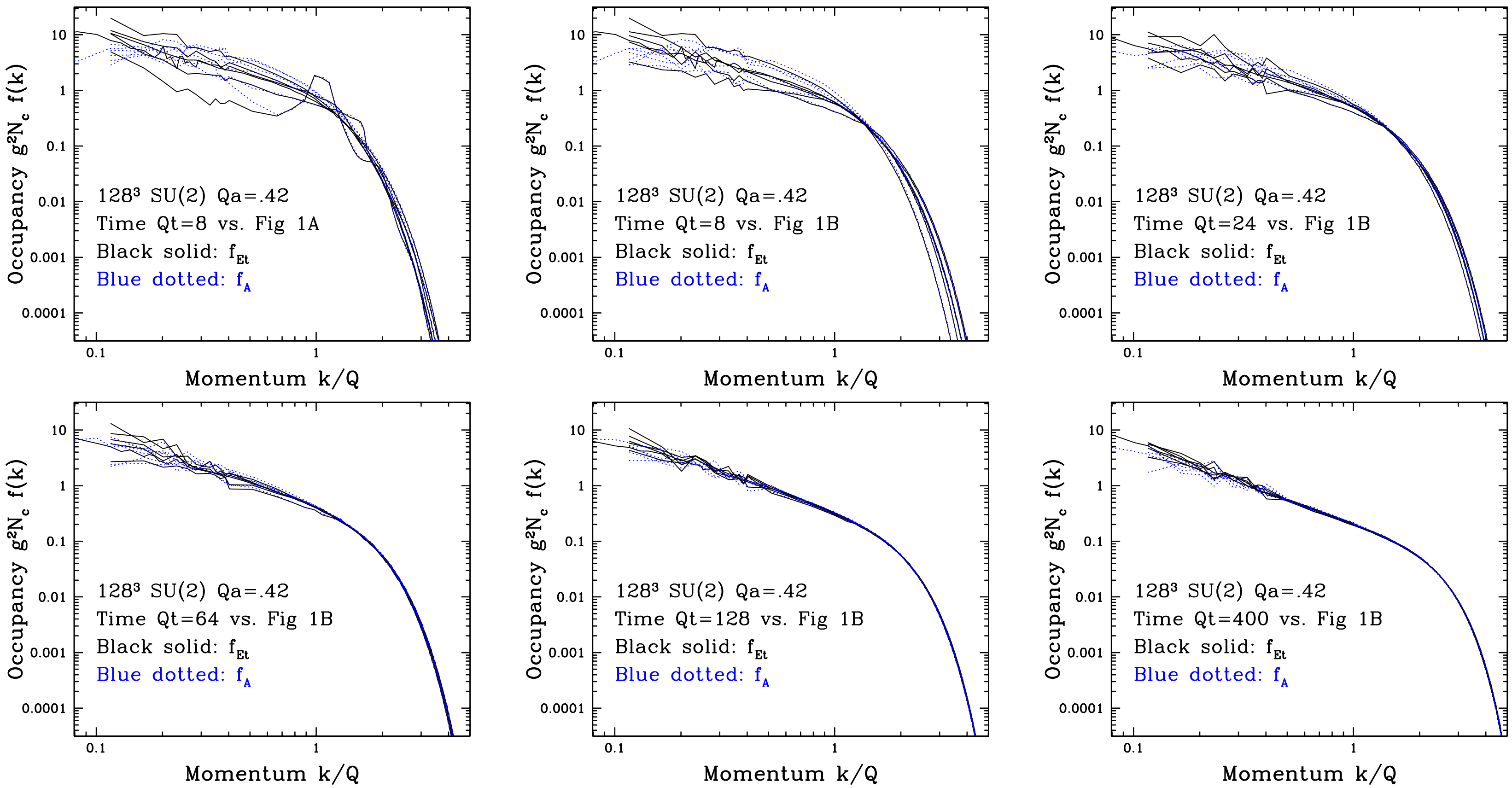
Best solution: separate into $f_{E_t}(k)$, $f_{E_l}(k)$, believe f_{E_t}.
(Works in equilibrium, at least....)

Scaling works!



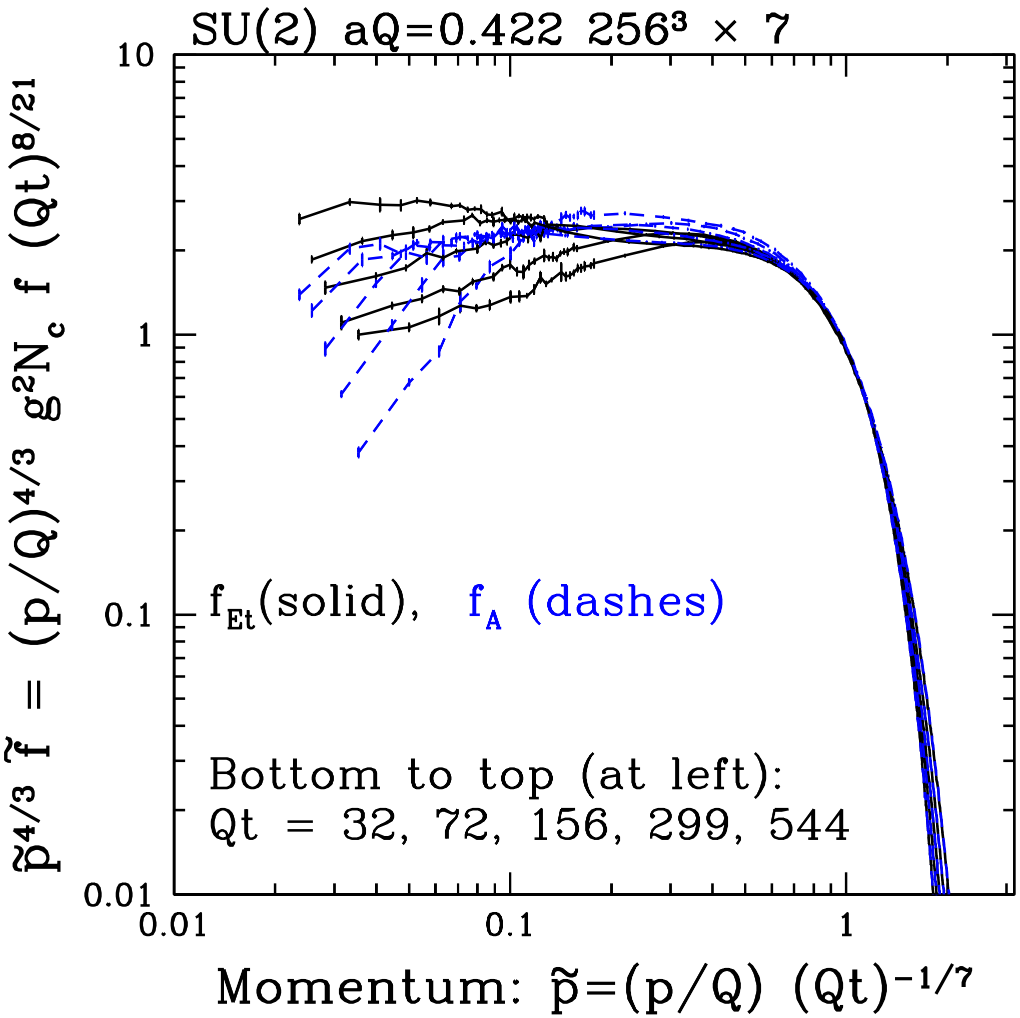


Rather rapid convergence to scaling solution:



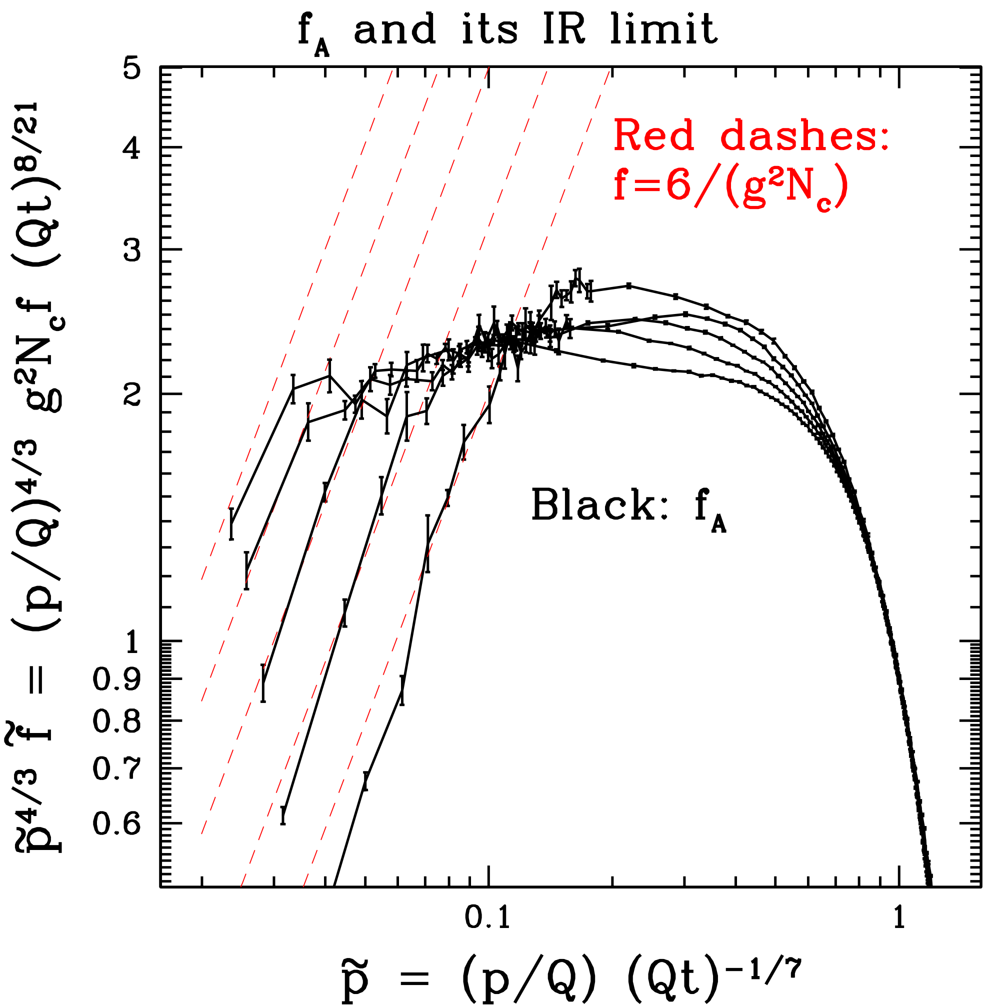
6 distinct initial conditions, but soon they all look same.

"Scaling" solution evolves in IR



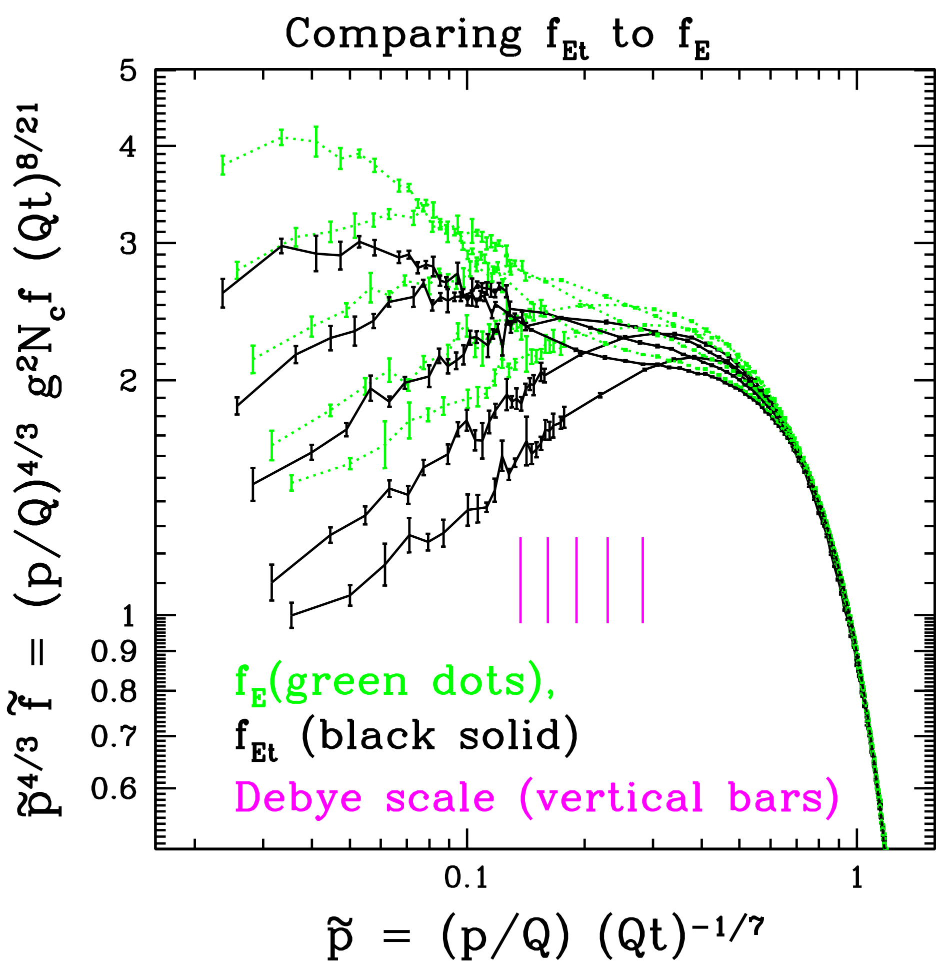
f_A, f_{E_t} tell same story except in IR

First look at f_A



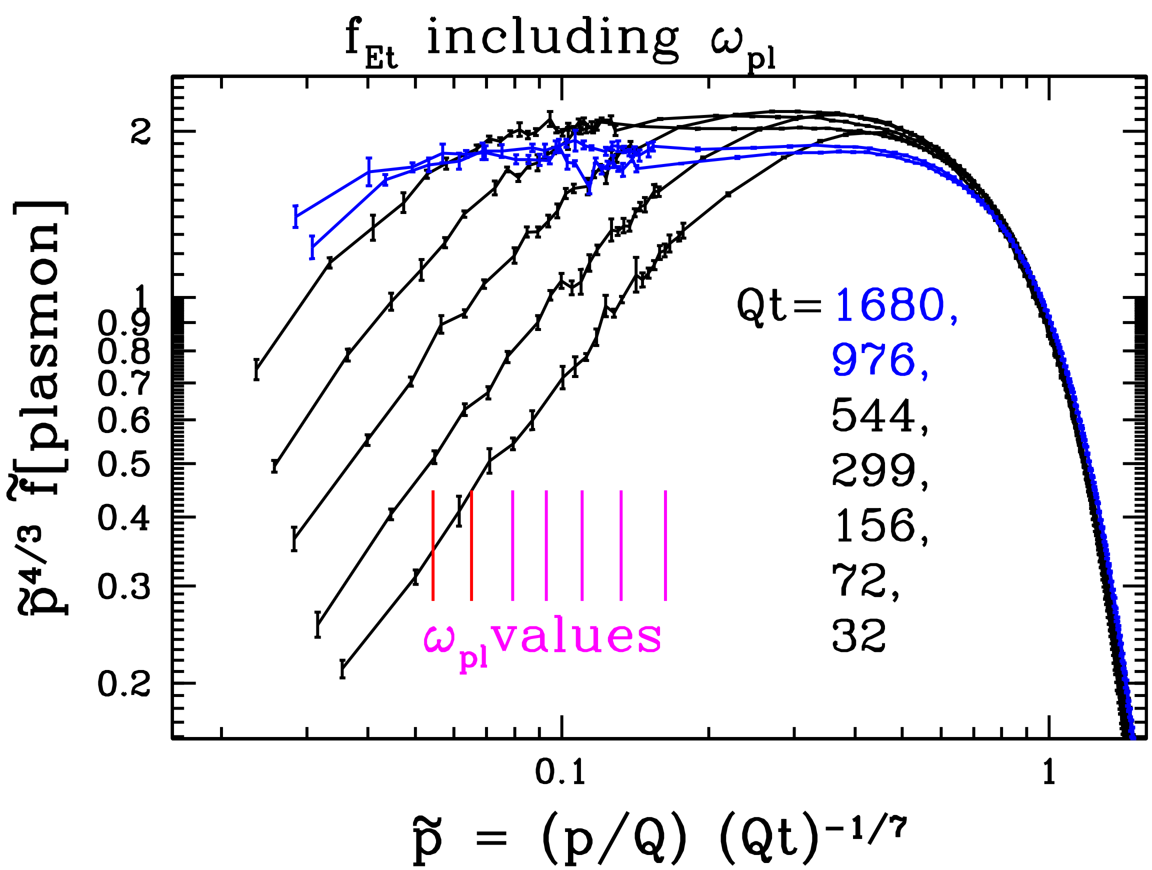
f_A rises, reaches $6/(g^2 N_c)$, saturates.

Now f_E versus f_{E_t}



f_E rises more than $4/3$ power, but ..

$$f_E = \frac{\mathcal{P}_{\mathrm{T}}^{ij}\delta_{ab}}{2(N_{\mathrm{c}}^2 - 1)\sqrt{p^2+\omega_{\mathrm{pl}}^2}} \int d^3x \, e^{i\mathbf{p}\cdot\mathbf{x}} \langle E_i^a(x) E_j^b(0) \rangle_{\mathrm{coul}}$$



So far, IR occupancy

$$f(p,t) \sim \frac{1}{g^2 N_{\rm c}} (Qt)^{\frac{-4}{7}} (p_{\rm max}/p)^{\frac{4}{3}}$$

For f_A, saturates at $f = 6/(g^2 N_{\rm c})$. f_E a bit lower.

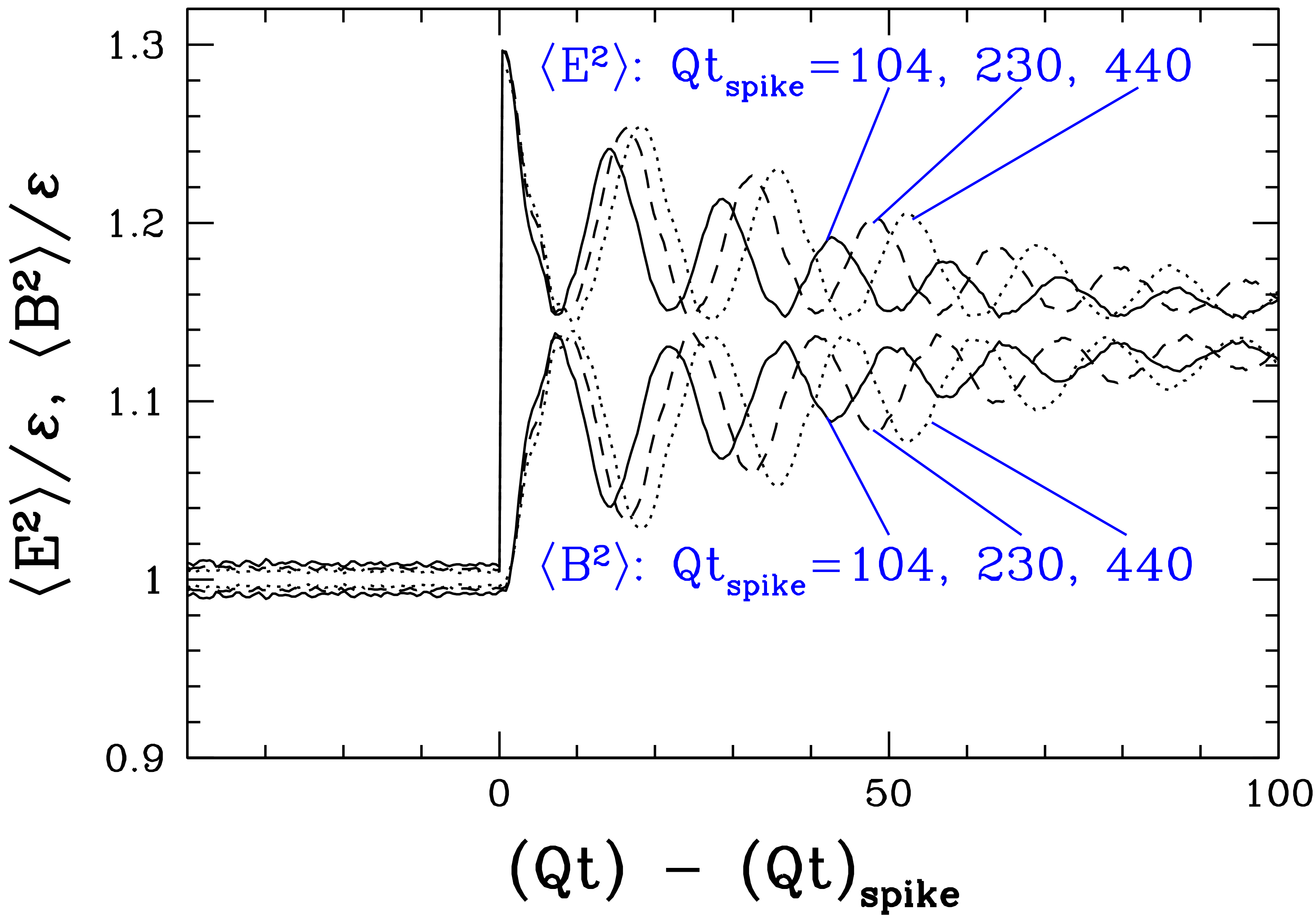
Part. number with $f \geq \frac{1}{g^2 N_{\rm c}}$ falls with time as $n_{\rm cond.}/n_{\rm tot} \sim (Qt)^{\frac{-5}{7}}$. Fairly small coefficient.

Could there *be* a condensate? If so, how would it evolve?

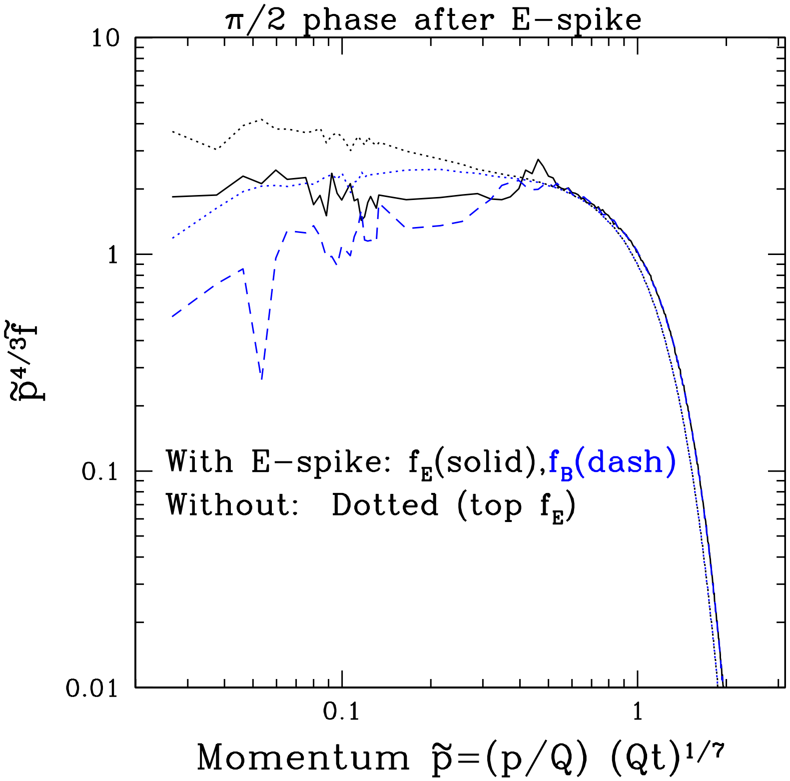
We can put one in by hand!

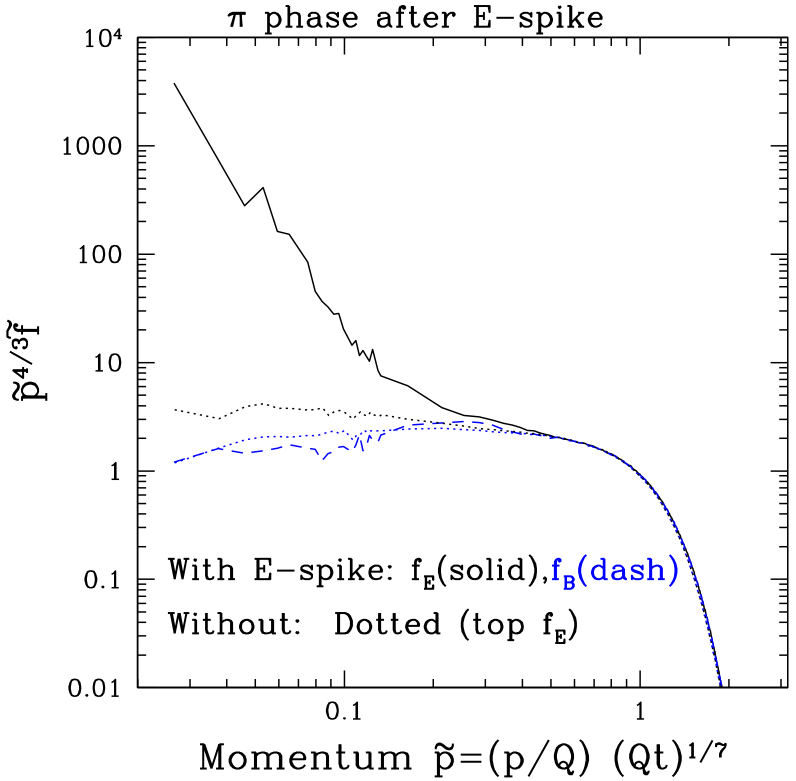
Evolve for a while, fix Coulomb gauge, insert uniform E field:

Energies with E-spike



Occupancies with E-spike





at $\mathbf{E}$-minimum

at $\mathbf{E}$-maximum

Conclusions

- Classical Yang-Mills dynamics: cascade to UV
- Scaling with time, $p_{\rm max} \sim Q(Qt)^{\frac{1}{7}}$ and $f \sim \frac{1}{g^2 N_{\rm c}}(Qt)^{\frac{-4}{7}}$
- $f(p,t)$ approaches scaling solution, with $f(p > p_{\rm max})$ exponential and $f(p < p_{\rm max}) \propto p^{\frac{-4}{3}}$
- IR corrections to scaling – mostly saturation of $\mathbf{A}$-fields at $f \sim 6/g^2 N_{\rm c}$ and screening effects on $\mathbf{E}$-fields
- No evidence of occupancies larger than above. Plasmon condensate possible, not realized and would decay fast

Extra slide: $\langle \mathbf{E}\mathbf{E} \rangle$ correlators

Consider classical partition function:

$$\int \mathcal{D}(\mathbf{A},\mathbf{E}) \exp\left(-\frac{\mathbf{E}^2+\mathbf{B}^2}{2T}\right)\delta(\mathbf{D}\cdot\mathbf{E})$$

Int. out UV $\longrightarrow$ white-noise charge fluct. in Gauss' Law:

$$\int \mathcal{D}(\mathbf{A},\mathbf{E},\rho) \exp\left(-\frac{\mathbf{E}^2+\mathbf{B}^2}{2T}+\frac{\rho^2}{2m^2T}\right)\delta(\mathbf{D}\cdot E-\rho)$$

Now do ρ integral using delta function:

$$\int \mathcal{D}(\mathbf{A},\mathbf{E}) \exp\left(\frac{\mathbf{B}^2+\mathbf{E}^2+(\mathbf{D}\cdot\mathbf{E})^2/m^2}{2T}\right)$$

which gives claimed correlators.

Identification of m^2 with $m_{\rm D}^2$:

Apply δ function via Lagrange multiplier

$$\int \mathcal{D}(\mathbf{A},\mathbf{E},\rho)\exp\left(-\frac{\mathbf{E}^2+\mathbf{B}^2}{2T}+\frac{\rho^2}{2m^2T}\right)\delta(\mathbf{D}\cdot E-\rho)$$

$$= \int \mathcal{D}(\mathbf{A},\mathbf{E},\rho,A_0)\exp\left(-\frac{\mathbf{E}^2+\mathbf{B}^2+\frac{\rho^2}{m^2}+iA_0(\mathbf{D}\cdot\mathbf{E}-\rho)}{2T}\right)$$

Perform gaussian $\mathbf{E}$ and ρ integrals:

$$\int \mathcal{D}(\mathbf{A},A_0)\exp\left(-\frac{\mathbf{B}^2+(\mathbf{D}A_0)^2+m^2A_0^2}{2T}\right)$$

We know the m^2 in front of A_0 is $m_{\rm D}^2$.

Nielsen-Olesen Instability

Hard to make coherent $\mathbf{B}$ field with $f \gg 1/g^2$. If you do, unstable!

In uniform $\mathbf{B}$, charged particle $p_\perp^2$ is quantized (Landau levels):

$$p_\perp^2 = gB(1+2n)\,, \quad n = (0,1,2,\ldots)$$

Energy-squared is $\quad E^2 = p_z^2 + p_\perp^2 - 2g\mathbf{S}\cdot\mathbf{B}$

For spin-$\frac{1}{2}$, $2g\mathbf{S}\cdot\mathbf{B} = \pm gB$ and the lowest level has $E^2 = p_z^2$.

But for spin-1, $2g\mathbf{S}\cdot\mathbf{B} = \pm 2gB$:

$$E^2_{\rm lowest} = p_z^2 + gB - 2gB = -gB + p_z^2$$

Negative, corresponding to exponential growth

$$\exp(\pm\gamma t)\,, \quad \gamma = \sqrt{gB - p_z^2}$$

We observe N-O instability