The (better) fate of nonabelian plasma instabilities in 3+1 dimensions with longitudinal expansion

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work in collaboration with:

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History

Abelian:

E. S. Weibel: “Spontaneously growing transverse waves in a plasma due to an anisotropic velocity distribution”, PRL 2 (1959) 83-84

QGP:

- U. W. Heinz: Quark Matter 1984 (NPA418 (1984) 603c)
- St. Mrówczyński; Pokrovsky & Selikhov 1988ff
  Mrówczyński & Thoma 2000: Hard loop approach...
  Randrup & Mrówczyński 2003
- P. Romatschke & M. Strickland: PRD68 (2003) 036004
  P. Arnold, J. Lenaghan, G. D. Moore: JHEP 0308 (2003) 002
  St. Mrówczyński, AR, M. Strickland: PRD70 (2004) 025004

Fate of non-Abelian plasma instabilities

Numerical simulations of anisotropic hard loop effective theory:

- 1D+3V: AR, Romatschke, Strickland 2004: PRL 94 (2005)
- 3D+3V: Arnold, Moore, Yaffe: PRD72 (2005) – The Fate…
  AR, Romatschke, Strickland: JHEP 09 (2005)
  Bödeker, Rummukainen: JHEP 07 (2007)

Complete (parametric) isotropization scenario for $\alpha_s \ll 1$:

- A. Kurkela & G. D. Moore: JHEP 11,12 (2011) cf. talk by Aleksi K.

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Here:

Fate in 3D+3V hard loop theory with longitudinal expansion

(boldly extrapolated to $\alpha_s \sim 0.3$ and $n(\tau_0) \propto Q_s^3 \alpha_s^{-1}$)

Hard anisotropic loop gauge boson self energy

Spectrum of unstable modes from hard loop self energy

$$\Pi^{\mu\nu}(k) = g^2 \int \frac{d^3p}{(2\pi)^3} v^\mu \partial_\beta^{(p)} f(\mathbf{p}) \left(g^{\nu\beta} - \frac{v^\nu k^\beta}{k \cdot v + i\epsilon} \right), \quad v^\mu \equiv \frac{p^\mu}{p^0}, \quad p^0 = |\mathbf{p}|$$

Special case: $f(\mathbf{p}) = f_{\rm iso}\left(\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2\right)$

$\xi = 0$: isotropic; $-1 < \xi < 0$: prolate (cigar-shaped); $0 < \xi < \infty$: oblate (squashed)

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can be evaluated in closed form: [Romatschke & Strickland 2003]

Change variables $\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2 = \bar{\mathbf{p}}^2$

$$\Pi^{ij}(k) = m^2 \int \frac{d\Omega}{4\pi} v^i \frac{v^l + \xi(\mathbf{v}.\mathbf{n})n^l}{(1 + \xi(\mathbf{v}.\mathbf{n})^2)^2} \left(\delta^{jl} + \frac{v^j k^l}{k \cdot v + i\epsilon} \right)$$

$$m^2 \equiv -\frac{g^2}{2\pi^2} \int_0^\infty d\bar{p}\, \bar{p}^2 \frac{df_{\text{iso}}(\bar{p}^2)}{d\bar{p}}$$

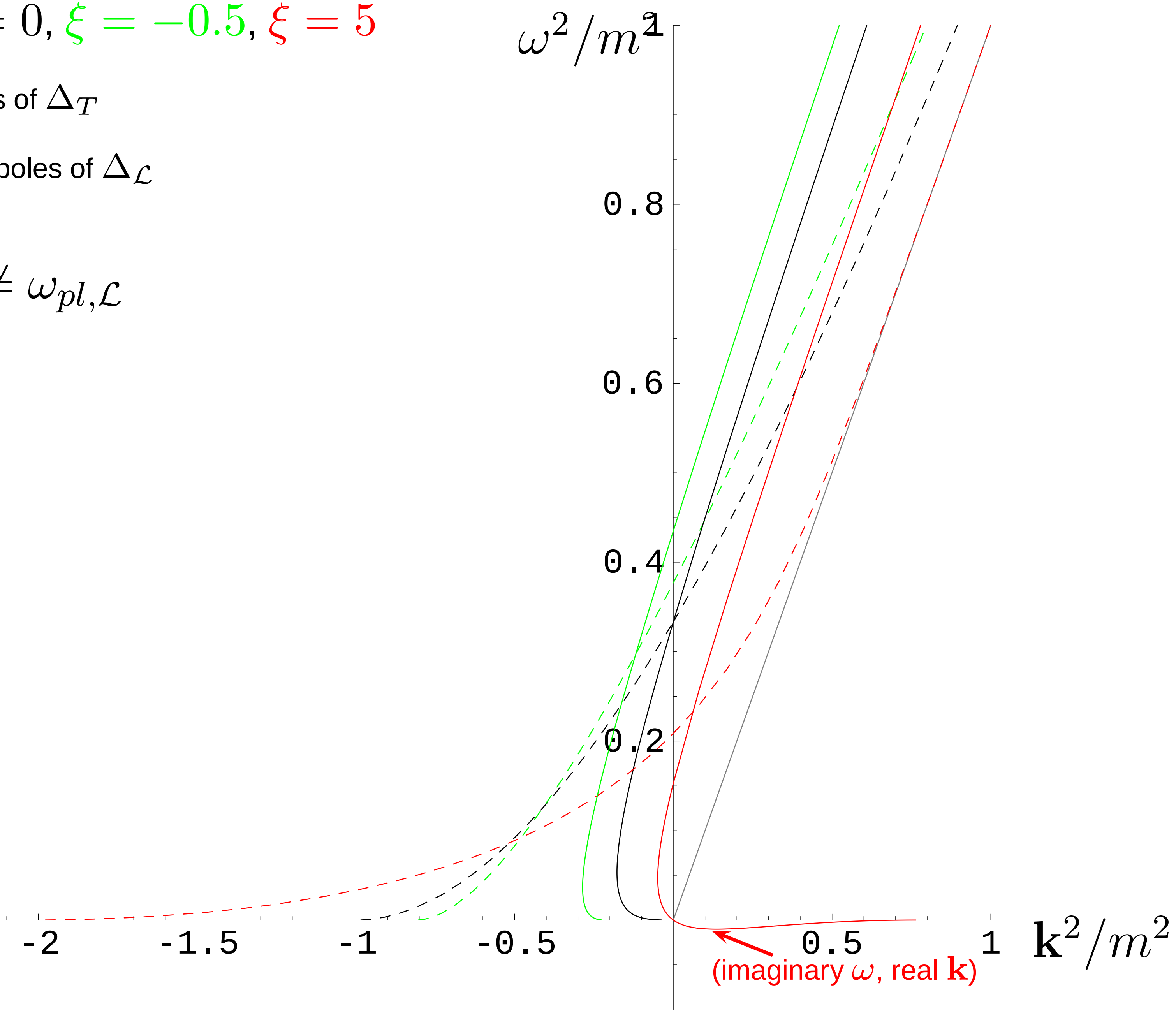
Dispersion laws in anisotropic plasma for $\mathbf{k}||\mathbf{n}$

Comparing $\xi = 0$, $\xi = -0.5$, $\xi = 5$

full lines: poles of Δ_T

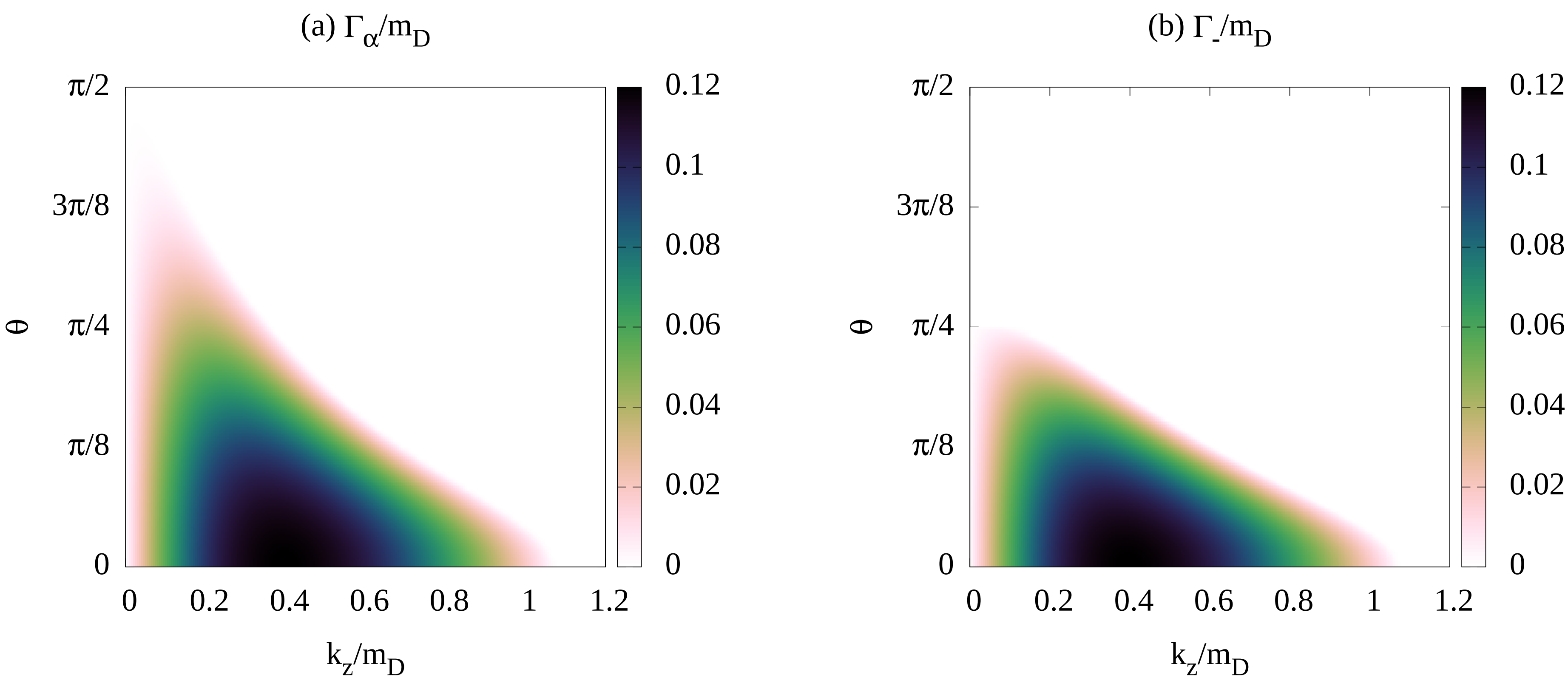
dashed lines: poles of $\Delta_{\mathcal{L}}$

$\xi \neq 0$: $\omega_{pl,T} \neq \omega_{pl,\mathcal{L}}$



Growth rates of unstable modes – general $\mathbf{k}$

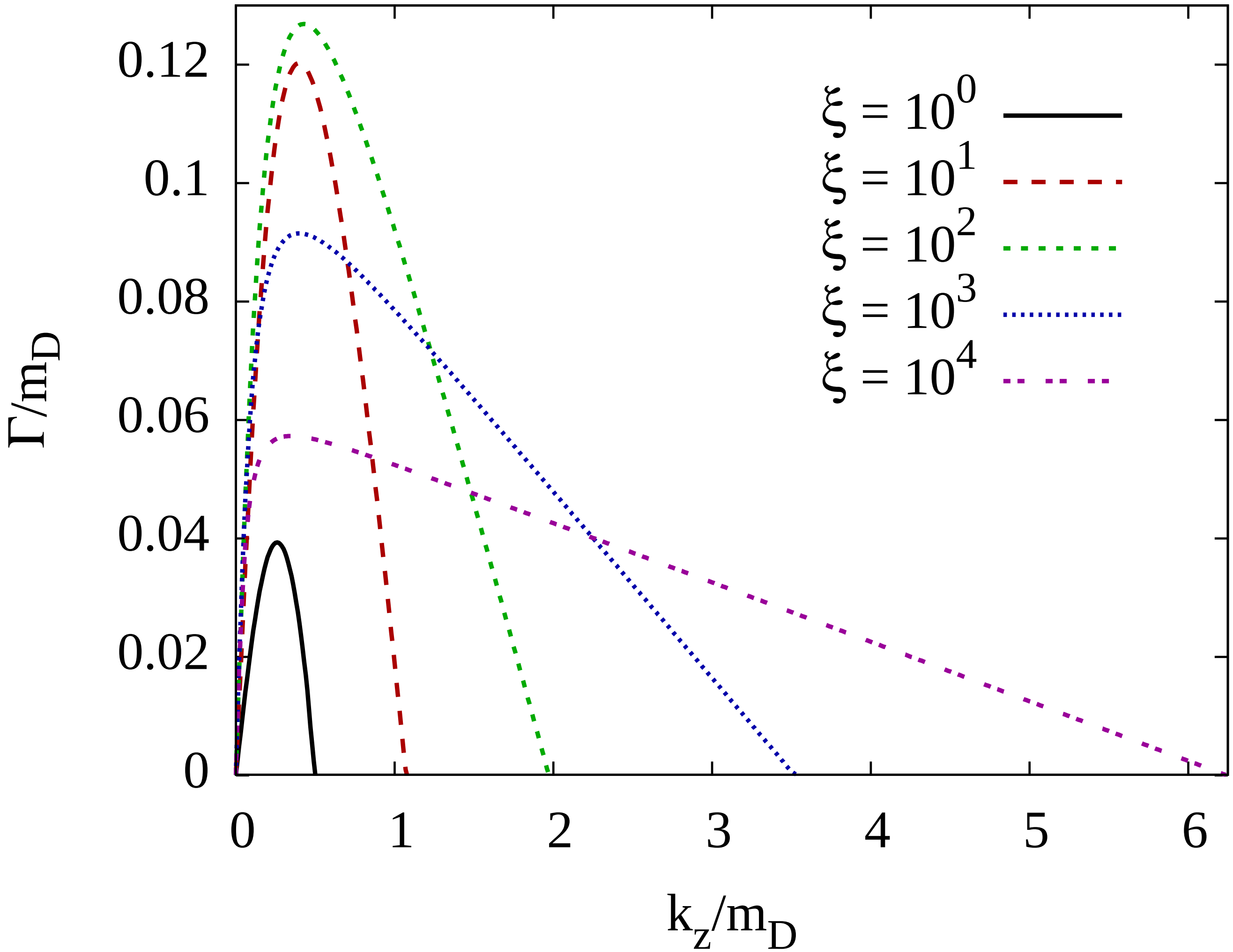
Unstable growth rates for $\xi = 10$



$\theta = \arctan(k_T/k_z)$

$m_D \ldots$ Debye mass in isotropic plasma obtained by compressing back to $\xi = 0$

Growth rates of most unstable modes ($\mathbf{k}||\mathbf{n}$)



(m_D: isotropic Debye mass before longitudinal expansion)

Hard Anisotropic Loop Effective Action

Exponentially growing <u>non-Abelian</u> gluon fields
$\Rightarrow$ linear response regime quickly left
$\Rightarrow$ need the (infinitely many) vertex functions required by gauge invariance

Hard (Thermal) Loop Effective Action formally still given by

[Pisarski 1993, Mrówczyński, Strickland, AR 2004]

$$S_{\mathrm{aniso}} = -\frac{g^2}{2} \int_x \int_{\mathbf{p}} \left\{ f(\mathbf{p}) F^a_{\mu\nu}(x) \left(\frac{p^\nu p^\rho}{(p \cdot D[A])^2} \right)_{ab} F_\rho{}^{b\,\mu}(x) \right\}$$

nonlinear and nonlocal!

— only useful as generating functional of hard anisotropic loops

Discretized Hard Loop Effective Theory

Useful:

auxiliary field formulation: [Nair; Blaizot & Iancu 1994; Mrówczyński, AR & Strickland 2004]

$$\delta f^a(x;p) = -gW^a_\mu(t,\mathbf{x};\mathbf{v})\partial^\mu_{(p)} f_0(\mathbf{p})$$

$$\boxed{[v\cdot D(A)]W_\mu(x;\mathbf{v}) = F_{\mu\gamma}(A)v^\gamma}$$

$$v^\mu \equiv p^\mu/|\mathbf{p}| = (1,\mathbf{v})$$

$$\boxed{D_\rho(A)F^{\rho\mu} = j^\mu(x) = -g^2\int \frac{d^3p}{(2\pi)^3}\frac{1}{2|\mathbf{p}|}\,p^\mu\,\frac{\partial f(\mathbf{p})}{\partial p^\nu}W^\nu(x;\mathbf{v})}$$

Hard Loop effective theory: (hard) scale $|\mathbf{p}|$ can be integrated out

Auxiliary field version: local in terms of field living also on velocity space S_2

Nonlinear response $\longrightarrow$ real-time lattice simulation

$\longrightarrow$ discretize also velocity space

$$D_\rho(A)F^{\rho\mu} = j^\mu(x) = \frac{1}{\mathcal{N}}\sum_{\mathbf{v}} v^\mu \mathcal{W}_{\mathbf{v}}(x)$$

Transversely constant modes: 1D+3V

Most unstable modes in linear response: $\mathbf{k} \parallel \mathbf{n}$

$\Longrightarrow$ no dependence on transverse coordinates;
dimensional reduction to 1 spatial dimension

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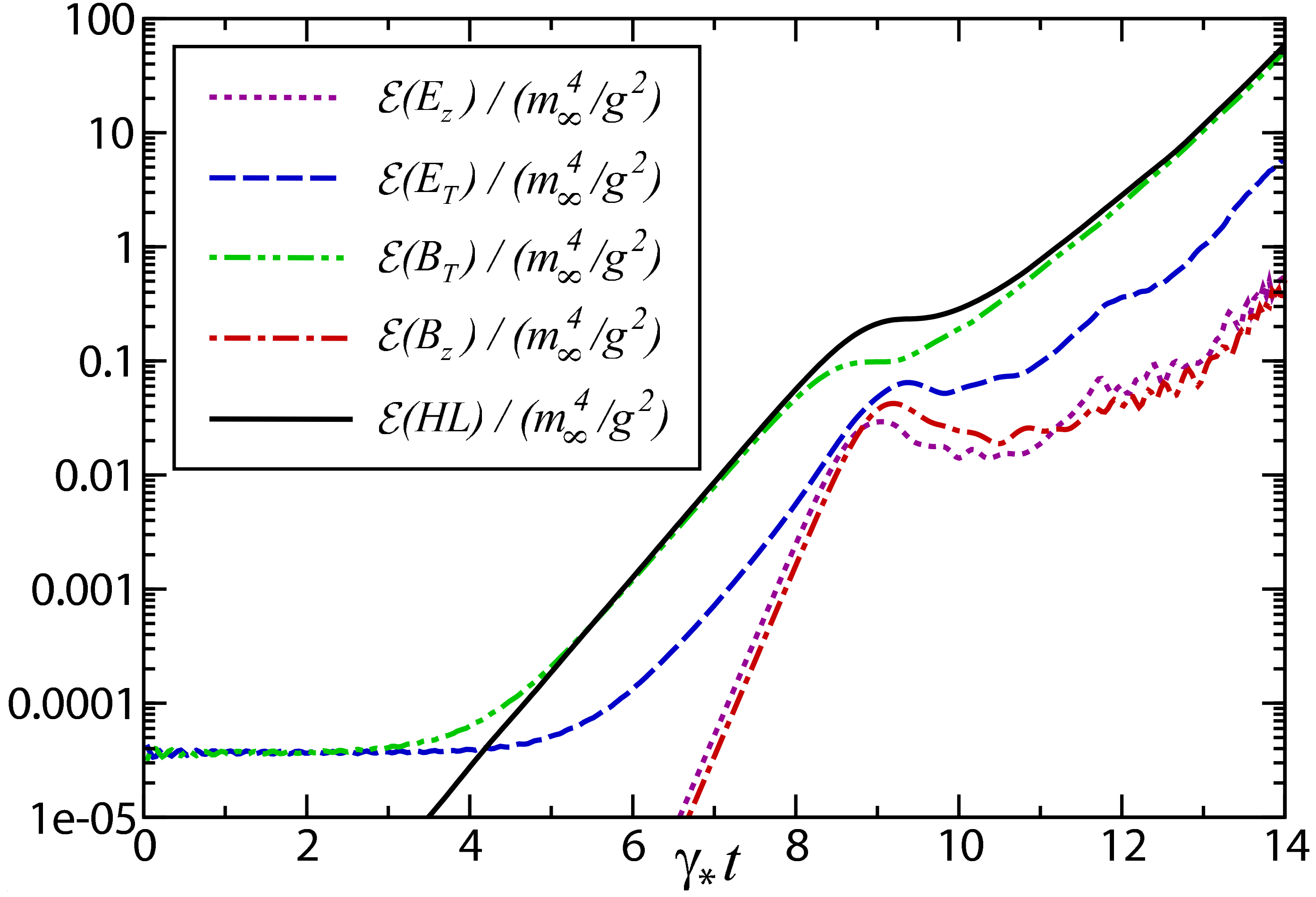
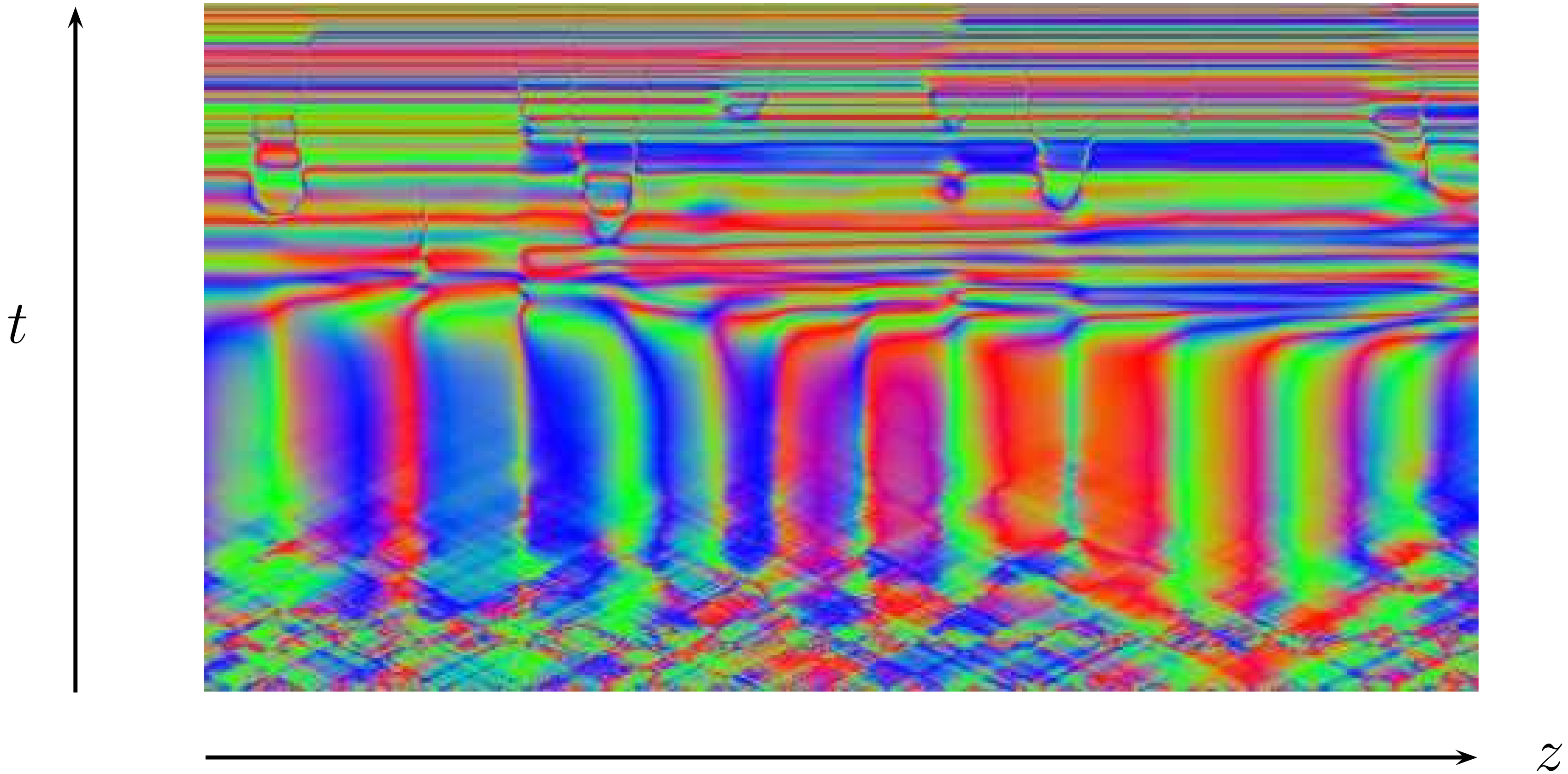
First simulation of nonabelian instabilities: [AR, Romatschke & Strickland, PRL 94 ('05) 102303]



Transversely constant modes: 1D+3V

Evolution of color degrees of freedom:
(parallel-transported color from fixed spatial point)



Late-time (non-linear) regime: Abelianization over extended spatial domains
– responsible for continued Abelian-like growth in non-linear regime

3D+3V

However: local Abelianization can be destroyed by interactions with not perfectly transversely constant modes

3D+3V

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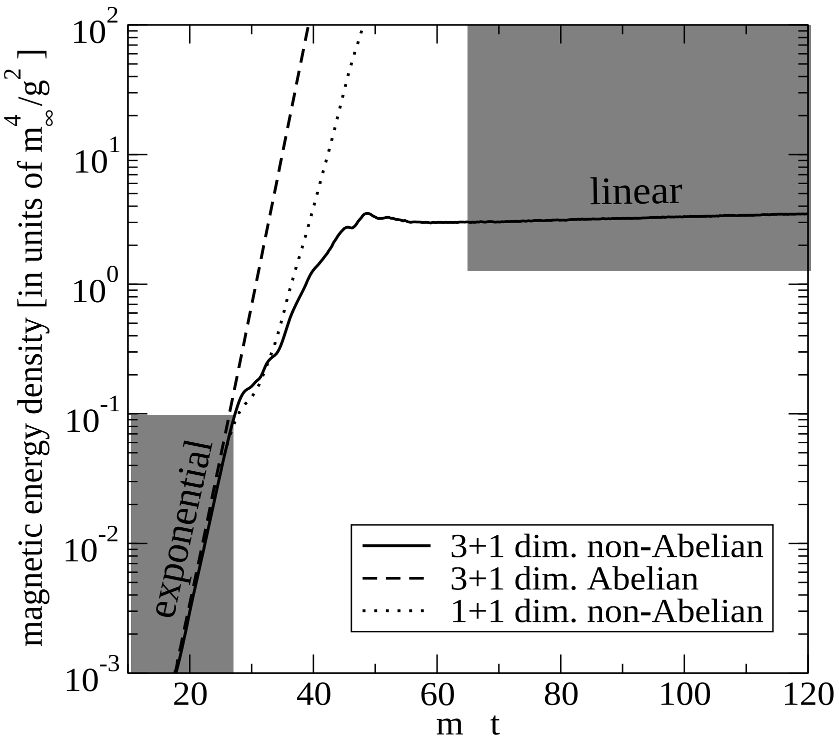
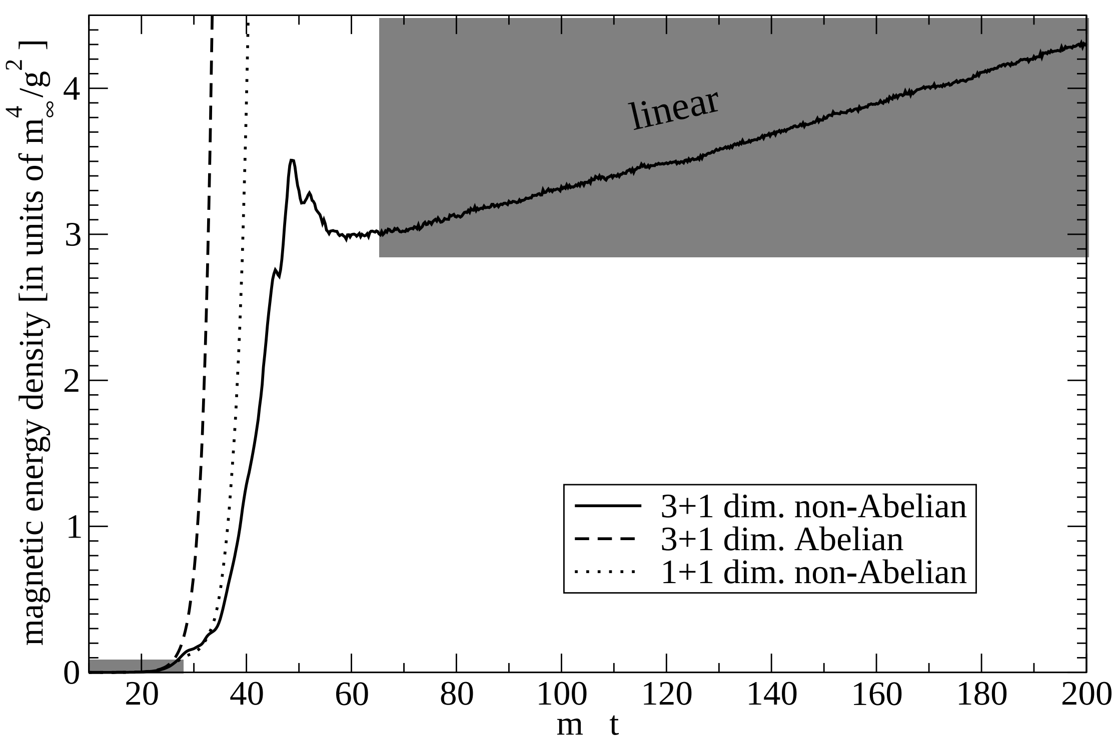
$\longrightarrow$ attenuation of exponential growth to only linear in regime where backreaction still parametrically small! [Arnold, Moore & Yaffe, PRD72 (2005) 054003]

[AR, Romatschke & Strickland, JHEP 09 (2005) 041]

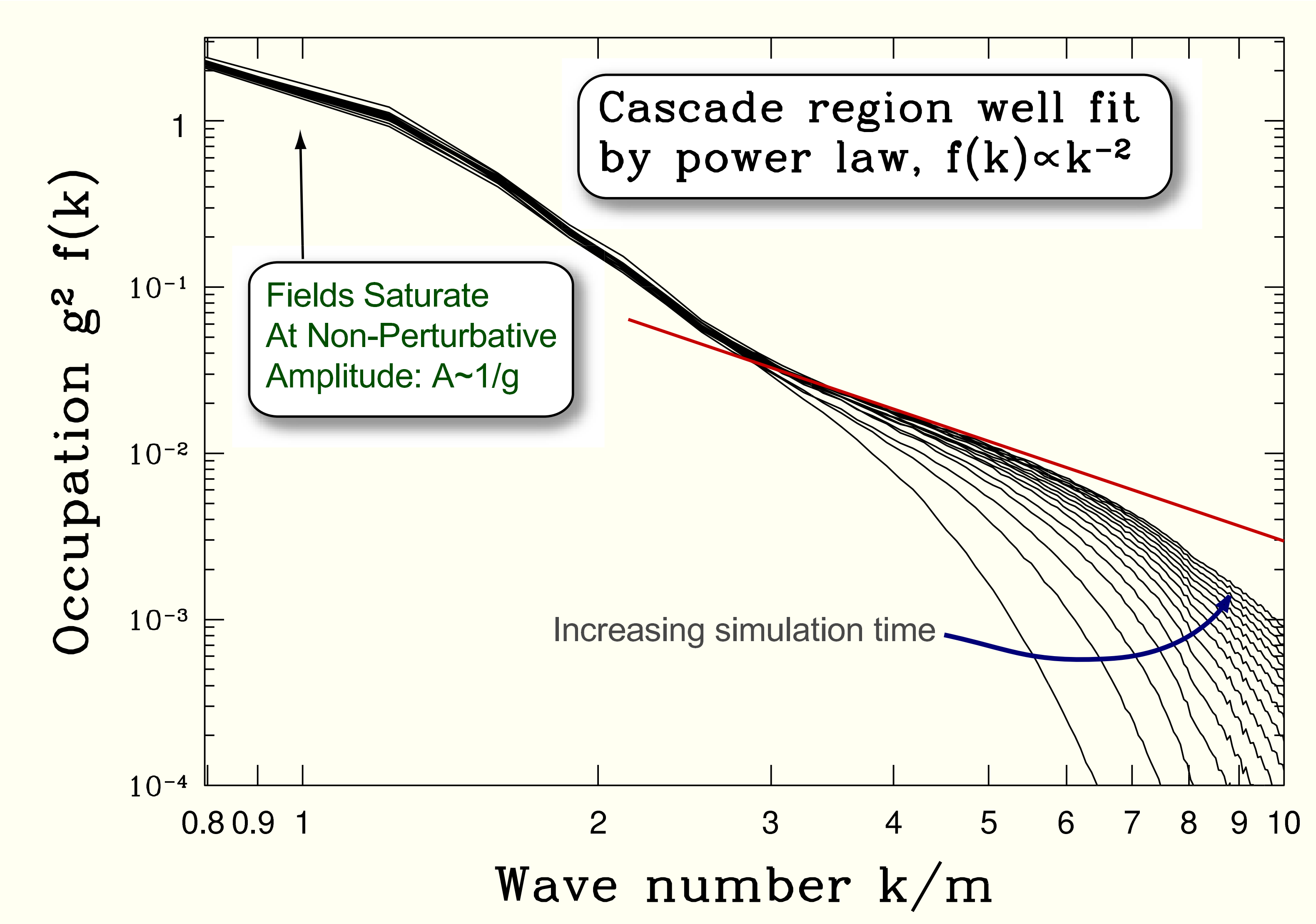




stronger anisotropies: no saturation? (when initialized with tiny seed fields)

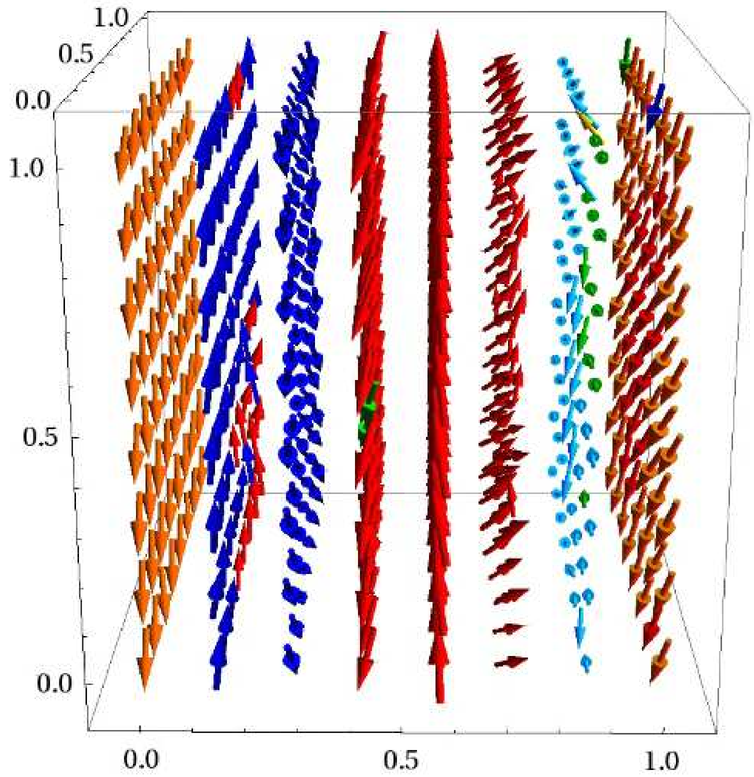
Bödeker & Rummukainen, JHEP 07 (2007) 02

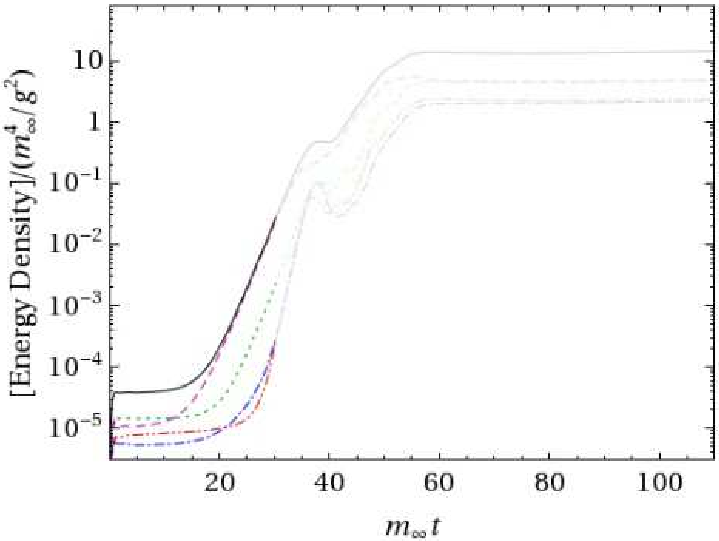
Turbulent energy cascade

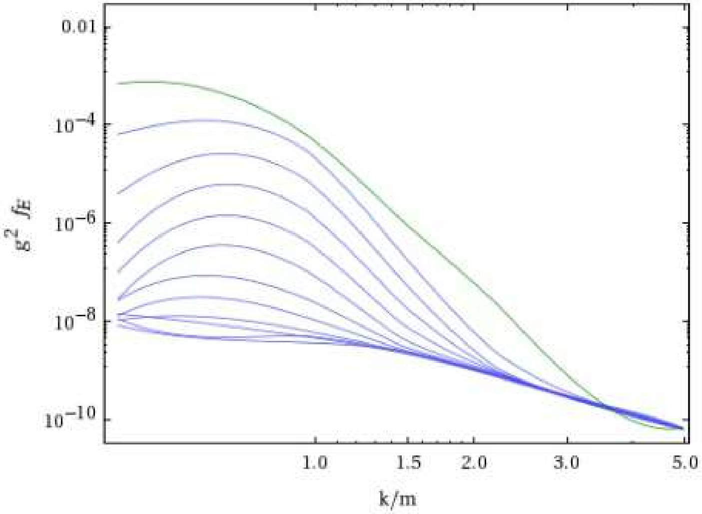


3D+3V and SU(3)

A. Ipp, AR, M. Strickland, Phys.Rev. D84 (2011) 056003







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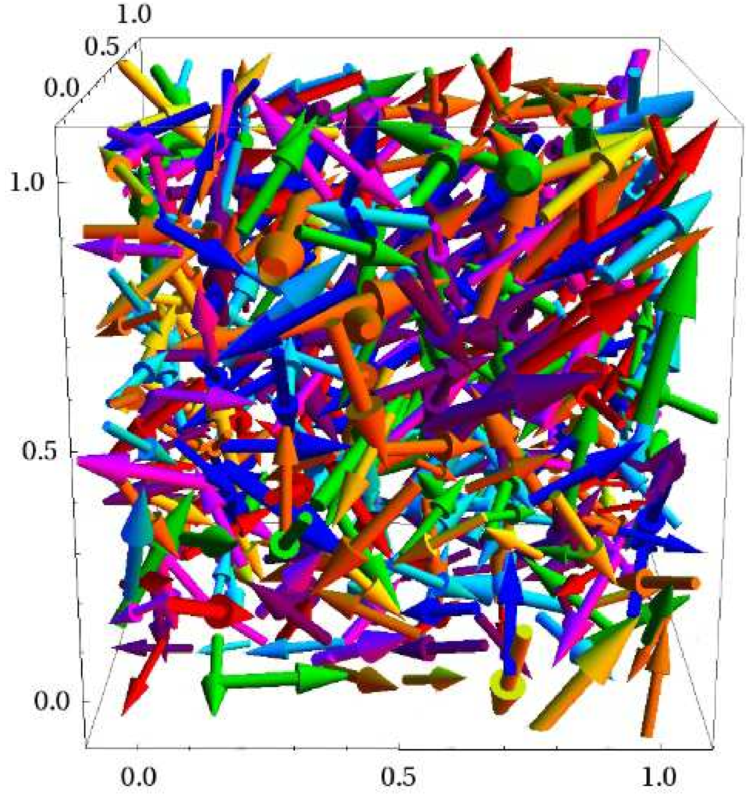

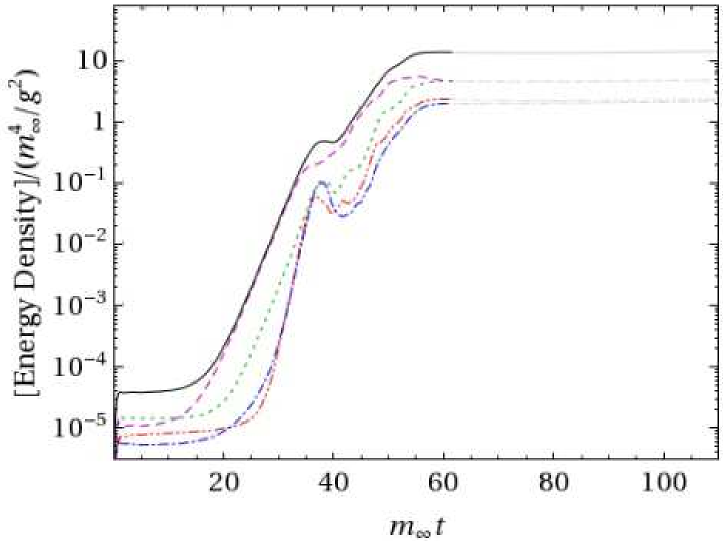

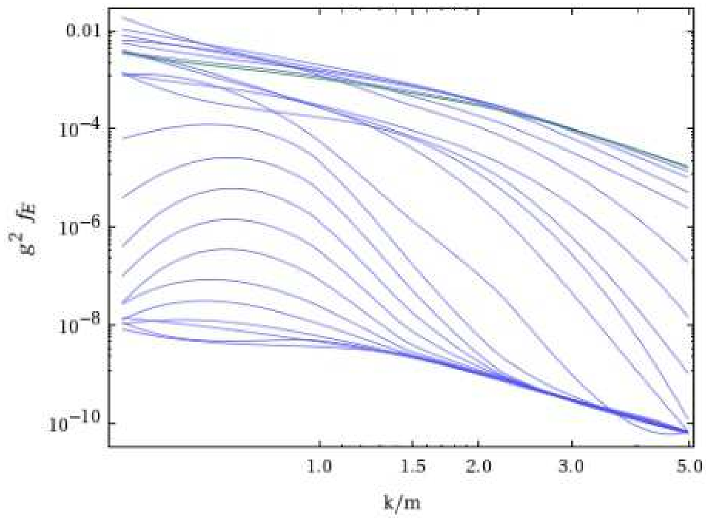

formation of turbulent cascade, again with $f \sim k^{-2}$

Plasma instabilities in Bjorken expansion

Longitudinal (Bjorken) expansion: Competition between

- increasing anisotropy (more and more modes become unstable)
- and decreasing density ($\longleftrightarrow$ growth rate)

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Notation: proper time $\tau = \sqrt{t^2 - z^2}$ and space-time rapidity $\eta = \mathrm{atanh}\frac{z}{t}$

$$x^\mu \to x^\alpha = (\tau, x^i, \eta) \text{ with } g_{\alpha\beta} = (1, -1, -1, -\tau^2)$$

momentum rapidity $y = \mathrm{atanh}\frac{p^0}{p^z}$:

$$p^\mu \to p^\alpha = |\mathbf{p}_\perp|(\cosh(y-\eta), \cos\phi, \sin\phi, \tau^{-1} \underbrace{\sinh(y-\eta)}_{p'^z/|\mathbf{p}_\perp|})$$

Boost-invariant free-streaming background e.g.

$$f_0(\mathbf{p}, x) = f_{\mathrm{iso}}\left(\sqrt{p_\perp^2 + p_\eta^2/\tau_{\mathrm{iso}}^2}\right) = f_{iso}\left(\sqrt{p_\perp^2 + (p'^z\tau/\tau_{iso})^2}\right)$$

with space-time dependent anisotropy parameter $\xi(\tau) = (\tau/\tau_{iso})^2 - 1$

Hard-Expanding-Loop formalism

[Romatschke & AR, PRL 97 (2006)]

$$\boxed{v \cdot D\, W_\alpha(\tau, x^i, \eta; \phi, y)|_{\phi,y} = v^\beta F_{\alpha\beta},}$$

$$\boxed{\frac{1}{\tau} D_\alpha \left[\tau g^{\alpha\gamma}(\tau) g^{\beta\delta}(\tau) F_{\gamma\delta}\right] = j^\beta,}$$

where for $f_0(\mathbf{p}, x) = f_{\rm iso}\left(\sqrt{p_\perp^2 + p_\eta^2/\tau_{\rm iso}^2}\right)$

$$j^\alpha(\tau, x^i, \eta) \quad = \quad -\frac{m_D^2(\tau = \tau_{iso})}{2} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{-\infty}^{\infty} dy\, v^\alpha \left(1 + \frac{\tau^2}{\tau_{iso}^2} \sinh^2(y-\eta)\right)^{-2}$$

$$\times \left\{ \cos\phi\, W_1 + \sin\phi\, W_2 - \frac{\tau}{\tau_{iso}^2} \sinh(y-\eta)\, W_\eta \right\}$$

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discretized HEL: [AR, Strickland, Attems, PRD 78 (2008): 1D+3V]

$\mathcal{W}(\tau, x^i, \eta; \phi, y)$ with discretized ϕ_n, y_m

3D+3V: (massively parallelized) on new Vienna supercomputer VSC

3D+3V HEL

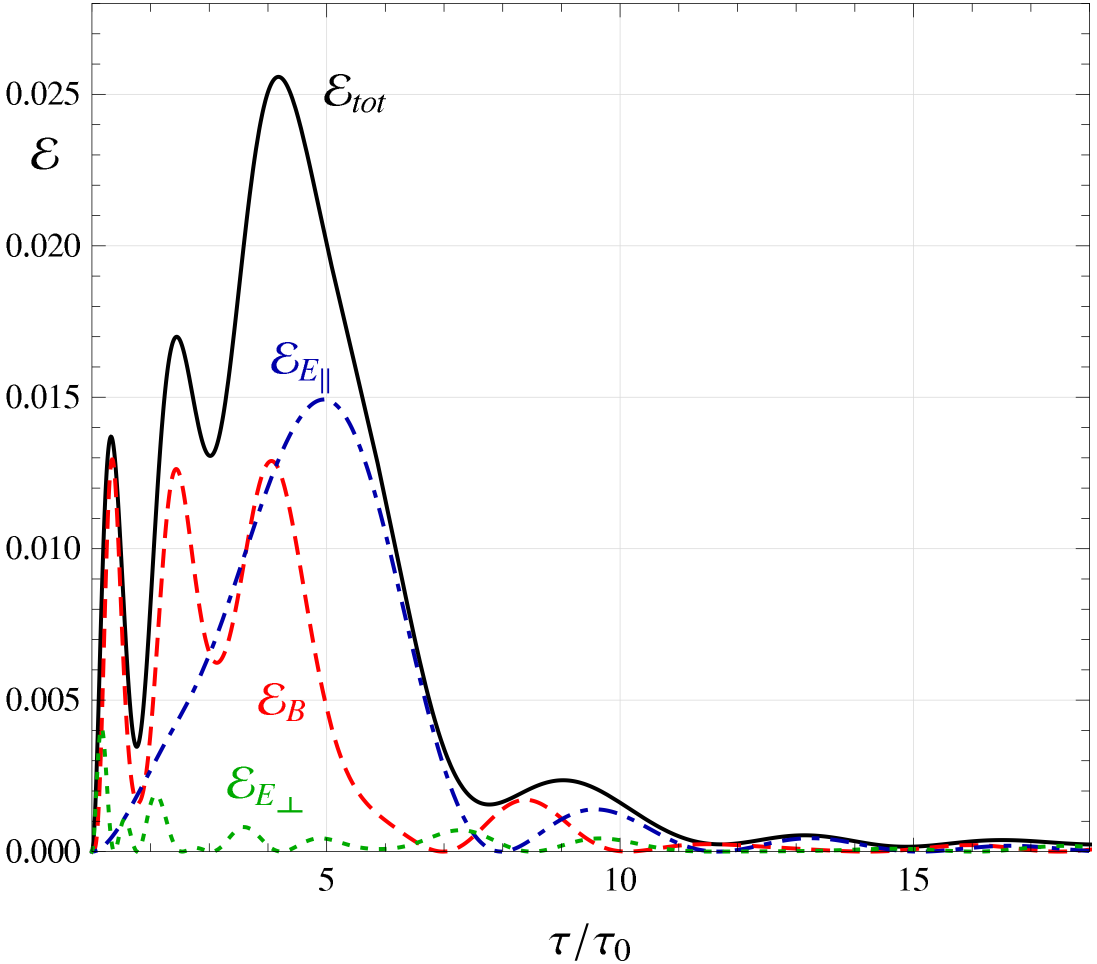
Abelian checks:

Full 3+1-dimensional semi-analytic study of linear (effectively abelian) regime

[AR & D. Steineder, Phys.Rev. D81 (2010) 085044]

E.g.: electric (Buneman) instability for wave vector not parallel to z-axis but initially within 45^o cone to the z-axis:

(turned off by expansion because wave vector rotates away from z axis)



Parameters for 3D+3V HEL

starting at earliest time where particle picture may become relevant:

$\tau_0 \sim Q_s^{-1} \sim 0.1$ fm/c (LHC?)

CGC: $n(\tau_0) \sim Q_s^3/\alpha_s$ with gluon liberation factor as estimated by Kovchegov/Lappi

$\rightarrow \boxed{m_D^2(\tau_{\rm iso}) \approx 1.285 Q_s^2 \frac{\tau_0}{\tau_{\rm iso}}}$

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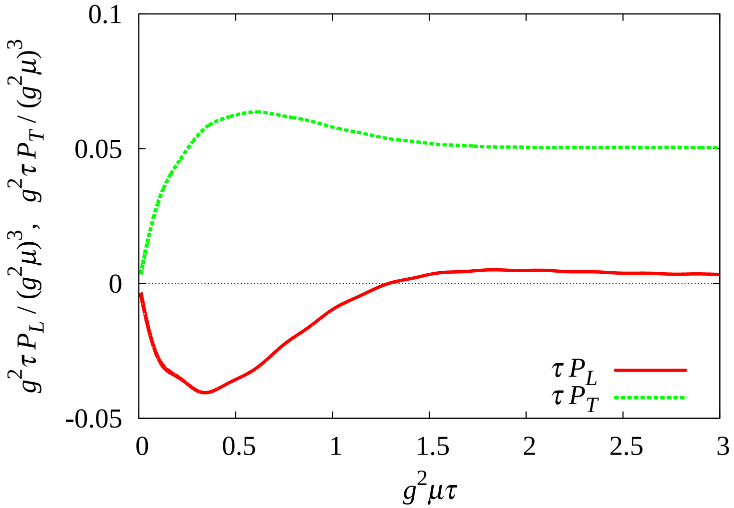
CGC/glasma simulations: [K. Fukushima & F. Gelis, NPA874 (2012) 108]



strong (oblate) anisotropy around $\tau = \tau_0 \sim Q_s^{-1} \sim (g^2\mu)^{-1}$

$\rightarrow \boxed{\text{start with } \frac{\tau_{\text{iso}}}{\tau_0} = 0.1}$

Lattice parameters for 3D+3V HEL

⋆ Spatial lattice $N_T^2 \times N_\eta = 40^2 \times 128$, $a_T = 0.1$fm, $a_\eta = 0.025$

⋆ Auxiliary fields: $N_W = N_y \times N_\phi = 128 \times 32 = 4096$

⋆ Initial conditions in terms of W-fields (fluctuations in hard particle distribution)

— white noise in transverse plane

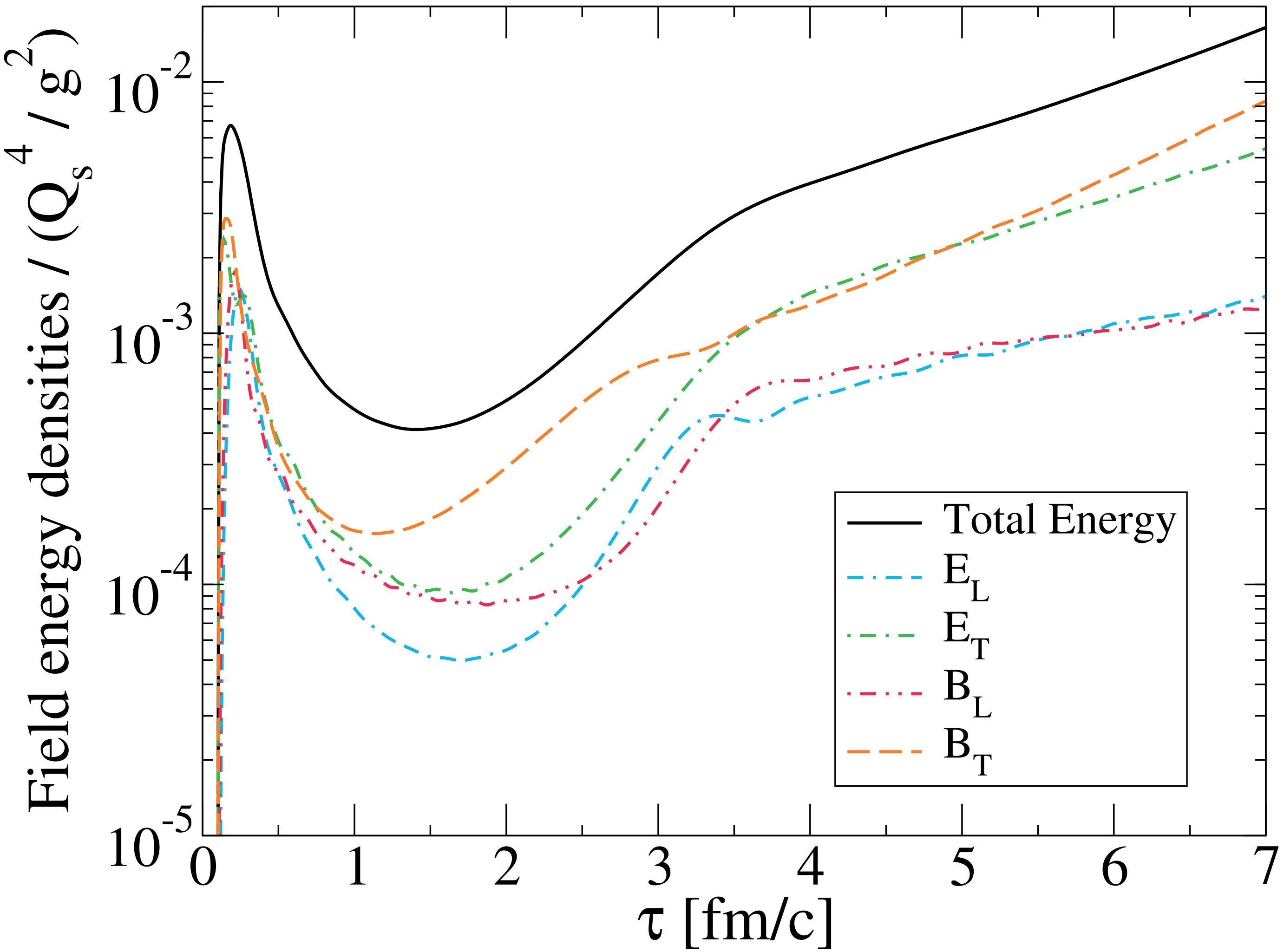
— spectral cutoff in longitudinal wave number ν

— initial current fluctuations such that initial occupation numbers $\lesssim \frac{1}{2}$
(sizeable initial fields)

3D+3V HEL - Field energy densities

Nonabelian lattice simulations: **No saturation!**

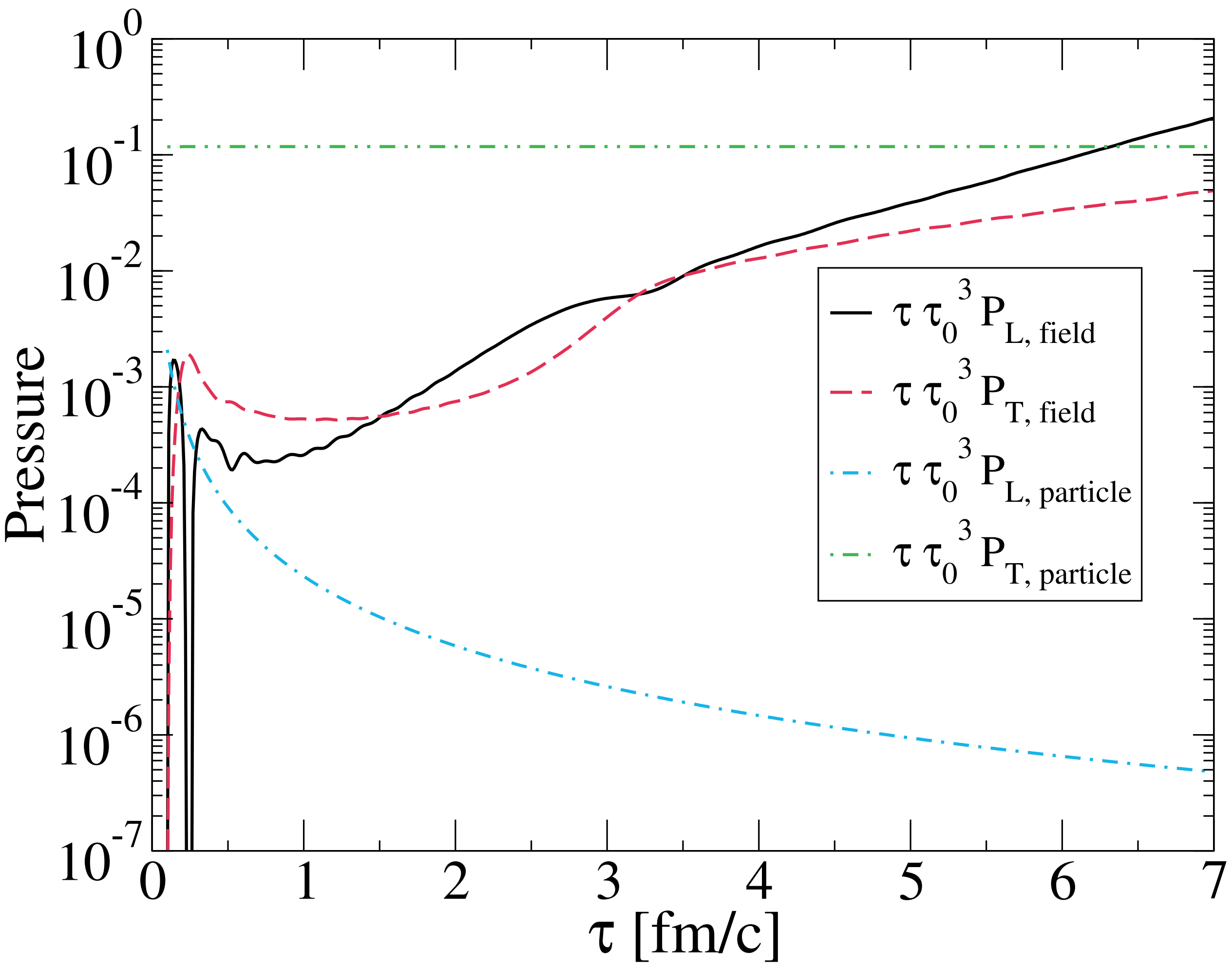
[M. Attems, AR & M. Strickland: arXiv:120?.S00N]



scales by matching to CGC at LHC energies ($Q_s \simeq 2$ GeV)

3D+3V HEL - Pressures

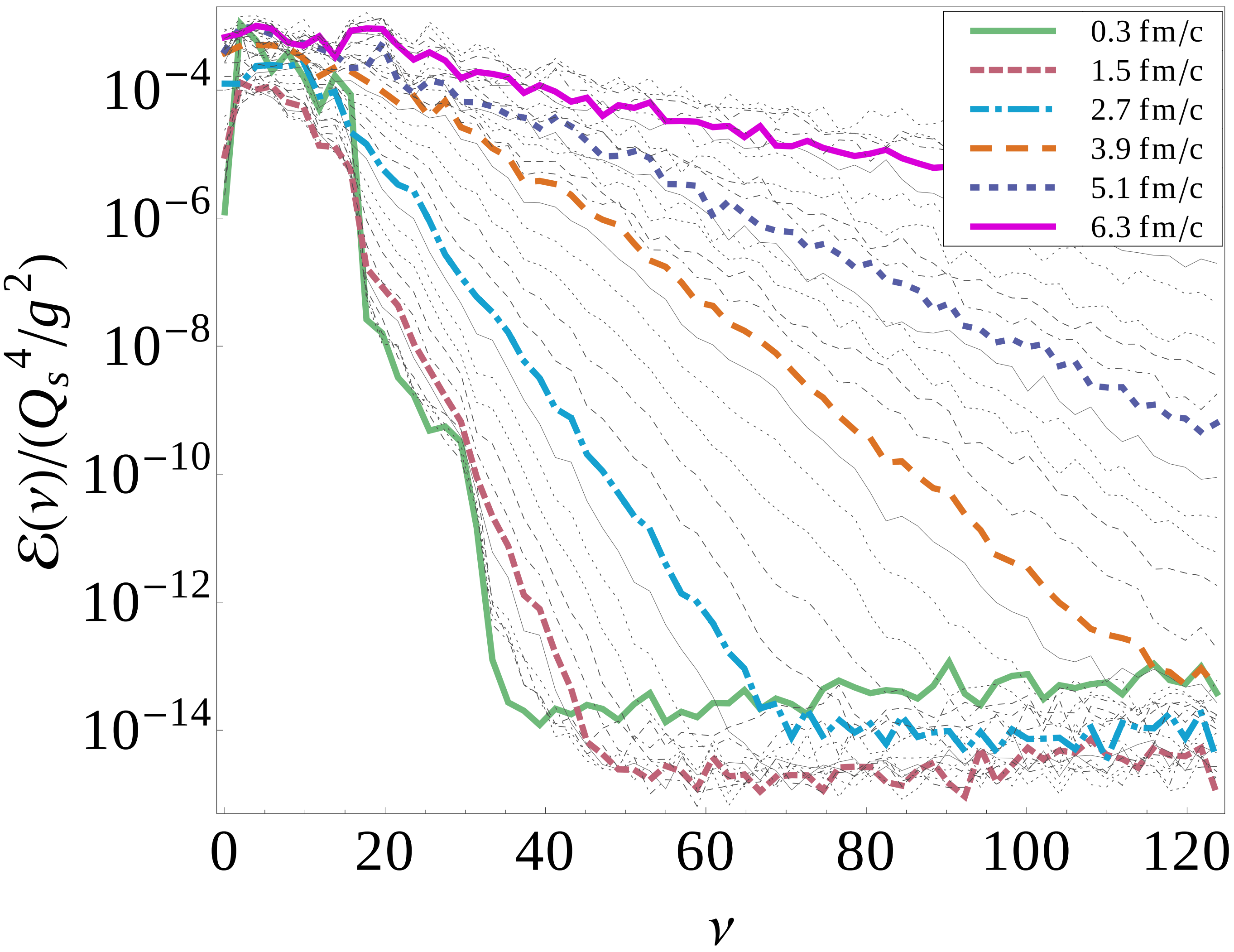
Comparison of hard (particle) and soft (field) contributions at semi-realistic coupling ($\alpha_s = 0.3$):



Isotropization after ∼5 fm/c (hard-loop approximation breaks down then)

3D+3V HEL - Longitudinal spectra

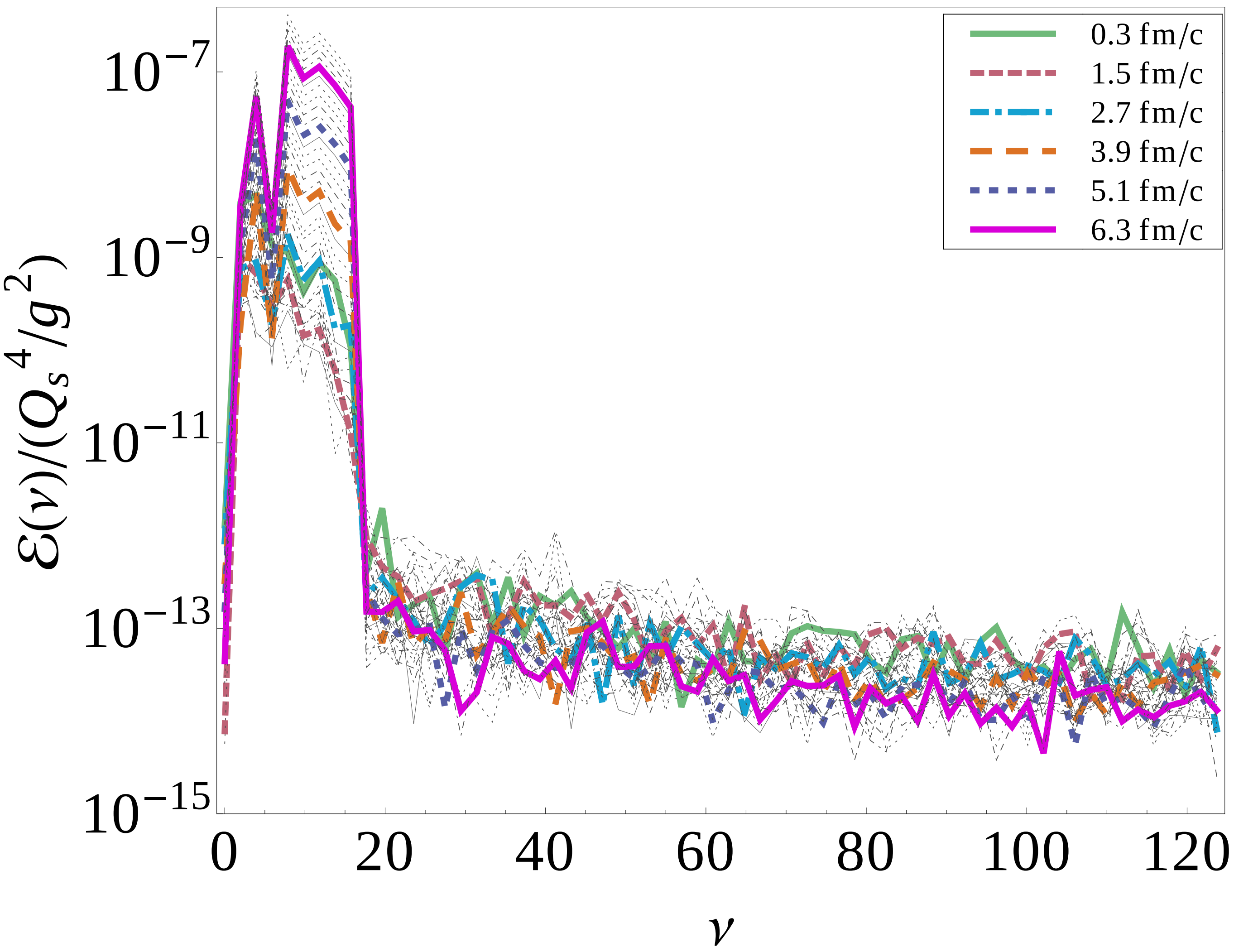
no saturation — instead of power-law: exponential!



ν: co-moving wave number in longitudinal direction

3D+3V HEL - Longitudinal spectra

by comparison: same with Abelian gauge group



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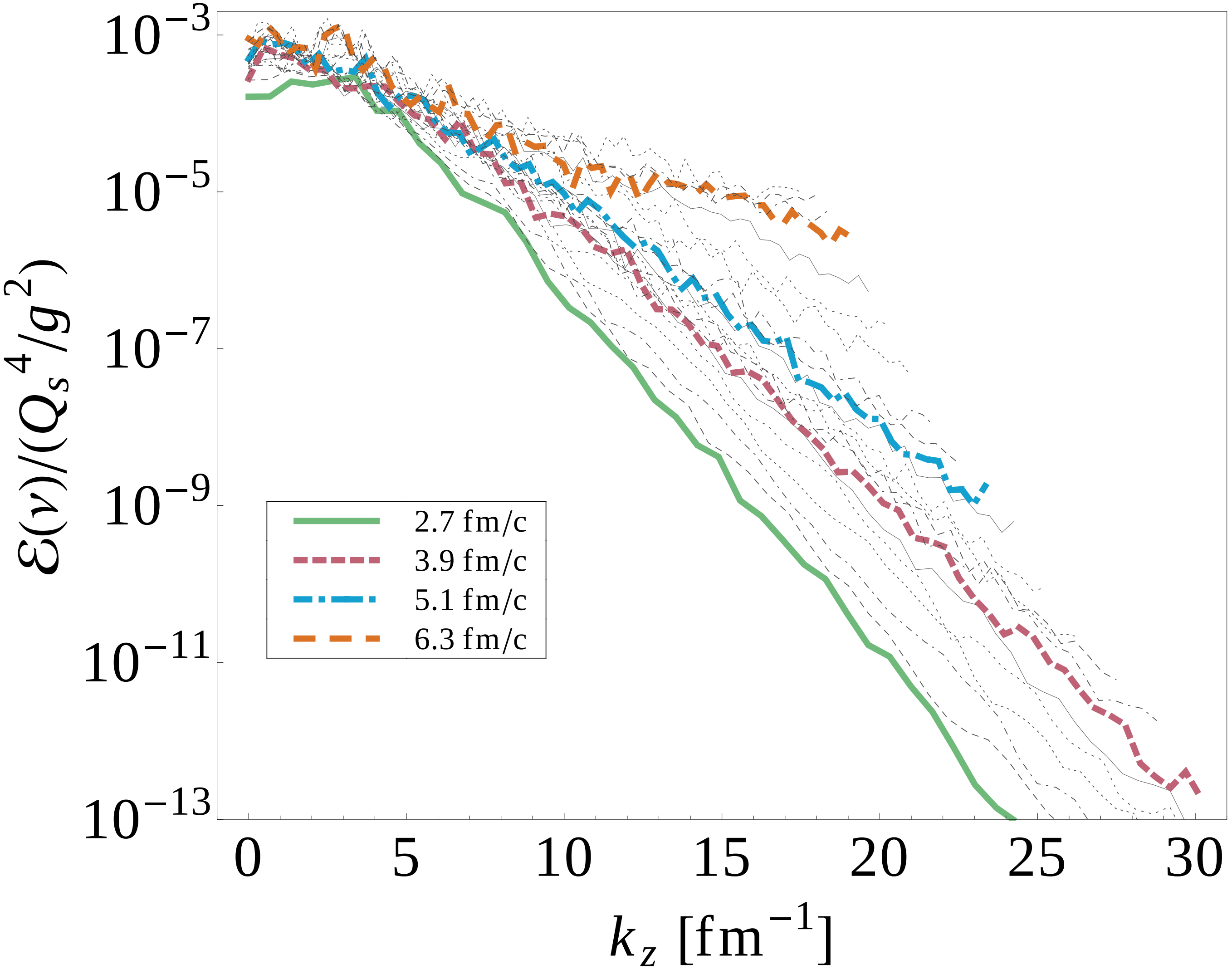
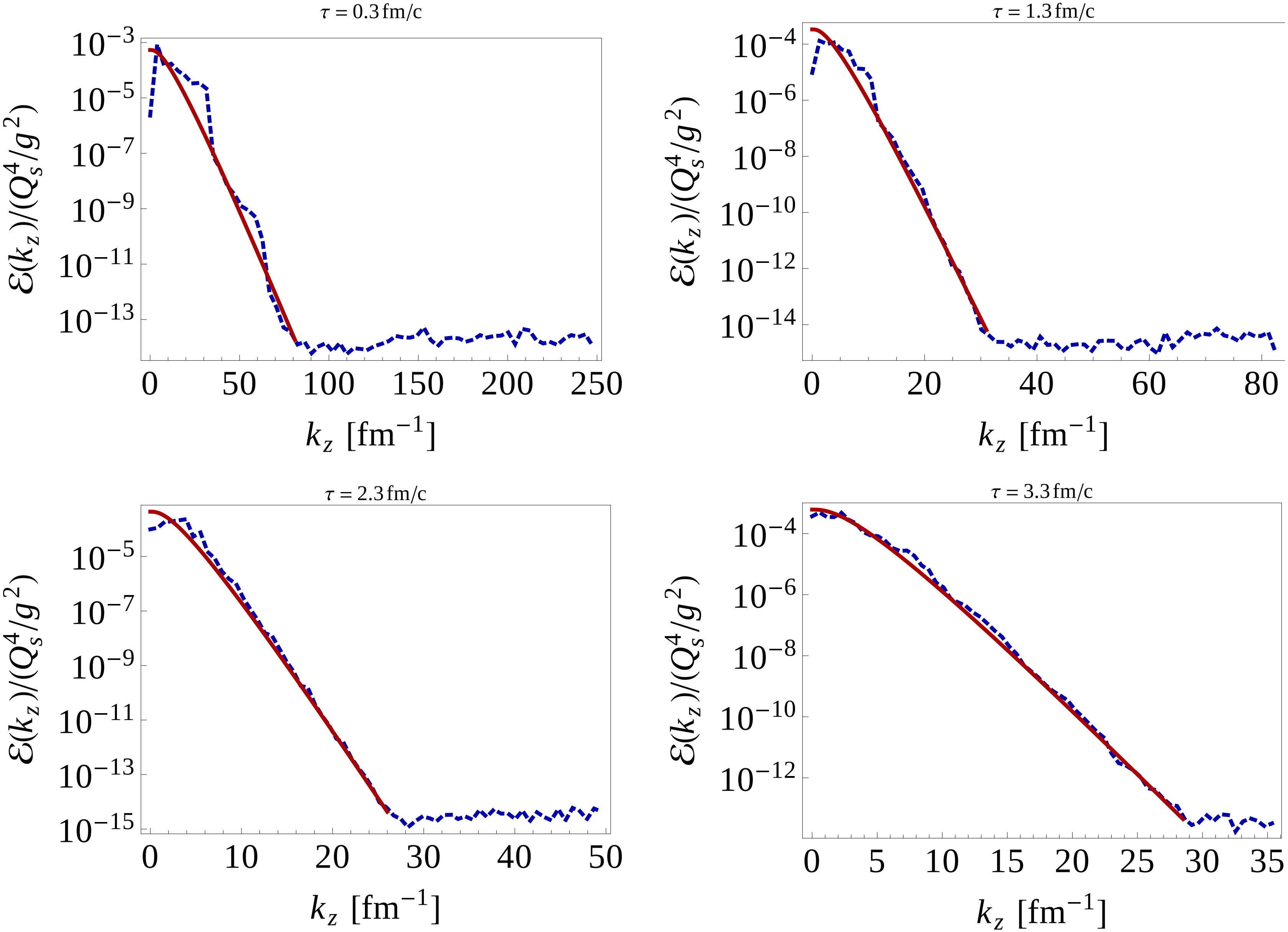


k_z: redshifted actual longitudinal wave numbers

3D+3V HEL - Longitudinal thermalization

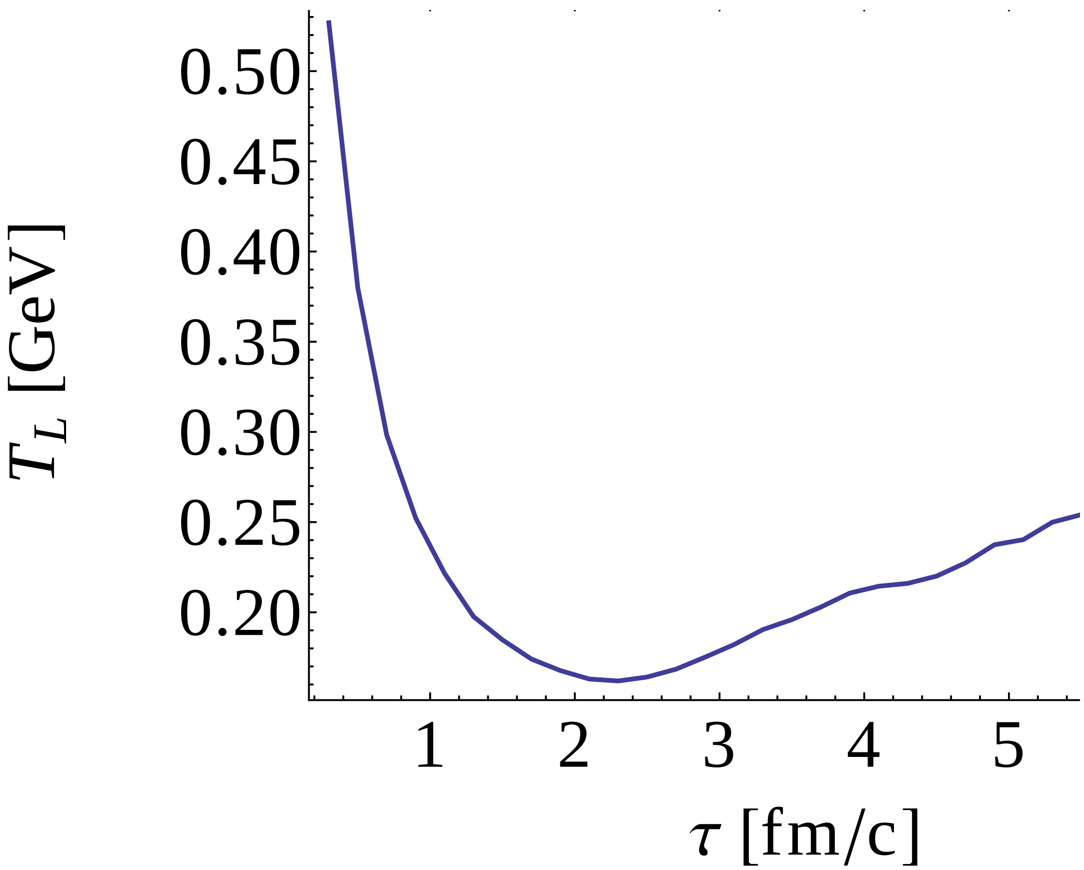
$$\mathcal{E}_{\rm fit}(k_z) = A\left(k_z^2 + 2|k_z|T + 2T^2\right)\exp\left(-|k_z|/T\right)$$



3D+3V HEL - Longitudinal thermalization

Energy distribution $\mathcal{E}(\nu)$ well fitted by Boltzmann distribution

with increasing temperature for $\tau \gtrsim 2$ fm/c despite free-streaming expansion:



consistent with glasma evolution: K. Fukushima & F. Gelis, NPA874 (2012) 108

(power-law Kolmogorov behavior there only at $\tau \gtrsim 100$ fm/c !)

Conclusion

- Plasma instabilities are parametrically dominant phenomenon in anisotropic wQGP with interesting characteristic time scales

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- Plasma instabilities are parametrically dominant phenomenon in anisotropic wQGP with interesting characteristic time scales
- Full 3+1-dimensional evolution of nonabelian plasma instabilities in longitudinally expanding plasma:
  - No saturation of growth in regime where backreaction on hard particles still small
  - turbulent cascade with $f \sim k^{-2}$ (only) in stationary anisotropic situation
  - HEL: instead quick *longitudinal thermalization and heating* while total pressure anisotropy remains large over several fm/c

Non-Abelian Discretized HEL

Hard gluon number density and initial fluctuation spectrum from **CGC** $\longrightarrow$

Parameters from saturation scenario $\tau_0 \simeq Q_s^{-1}$:

$n(\tau_0) = c\frac{(N_c^2-1)Q_s^3}{4\pi^2 N_c \alpha_s(Q_s\tau_0)}$

with gluon liberation factor $c = \begin{cases} 0.5 & \text{Krasnitz '99 et al.} \longrightarrow \text{1.1 Lappi '07 (numerical)} \\ \underline{2\ln 2} \approx 1.39 & \text{Kovchegov (analytical estimate)} \end{cases}$

$f_{\rm iso} = \mathcal{N} f_{\rm thermal}$ with (transverse) temperature $T = 0.47 Q_s$ [Krasnitz et al.]

$$\text{pure glue} \qquad \longrightarrow \qquad \mathcal{N} = \frac{1}{\alpha_s}\frac{c}{8N_c(0.47)^3\zeta(3)}\frac{\tau_0}{\tau_{iso}}\frac{1}{Q_s\tau_0}$$

$$\longrightarrow \qquad \frac{\mu}{Q_s} = \frac{1}{8}m_D^2\pi\tau_{\rm iso}Q_s^{-1} = \frac{\pi^2}{48\cdot 0.47\cdot\zeta(3)}\,c \approx \begin{cases} 0.182 & (c=0.5) \\ \underline{0.505} & (c = 2\ln 2) \end{cases}$$

$Q_s \simeq 1$ GeV (RHIC) . . . 2 GeV (LHC) ?