Technicolor and conformal window on the lattice

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Strong and Electroweak Matter, Swansea, 11.7.2012

Introduction:

- **Infrared conformality**: gauge theories with with large enough (but not too large!) number of fermions generically feature an infrared fixed point.
- phenomenology: **Extended technicolor**
- theoretical curiosity: strongly coupled conformal phase, sQGP, "unparticles"

- Lot of recent activity both on and off the lattice
- *Slow running* of the coupling g^2

→ Lattice studies very difficult

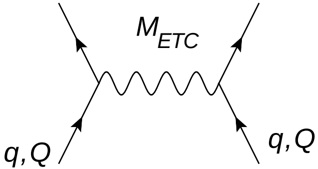
- Here we mostly discuss SU(2) with $N_f = 4, 6$ and 10 fundamental rep. fermions

Technicolor

- Technigauge + massless techniquarks Q
- Techniquarks have both technicolor and EW charge (exactly like quarks in the SM)
- Chiral symmetry breaking in technicolor $\longrightarrow$ Electroweak symmetry breaking
- Scale: $\Lambda_{\rm TC} \sim f_{\rm TC} \sim \Lambda_{\rm EW}$
- After chiral symmetry breaking:
  - $\Rightarrow$ decay constant $f_{\rm TC} \leftrightarrow$ Higgs expectation value v.
  - $\Rightarrow$ scalar $\bar{Q}Q$ -meson $\leftrightarrow$ Higgs
  - $\Rightarrow$ pseudoscalars $\leftrightarrow$ W,Z -longitudinal modes
  - $\Rightarrow$ exotic technihadrons (observable!)
- Describes well the W, Z+Higgs sector (depending on the model, may have too many Goldstone bosons)
- Elegant, "proven" mechanism in the Standard Model
- Does *not* explain fermion masses (Yukawa). For that, we need additional structure $\rightarrow$ *Extended technicolor*

Extended technicolor

- In addition to the "pure" technicolor, introduce a new higher-energy interaction coupling Standard Model fermions q (quarks, leptons) and techniquarks (Q): extended technicolor (ETC)
  Several options, e.g. massive gauge boson, $M_{\rm ETC}$:



[Eichten,Lane,Holdom,Appelquist,Sannino,Luty. . .]

- $\frac{1}{M_{\rm ETC}^2}\bar{Q}Q\bar{q}q \longrightarrow$ SM fermion mass $m_q \propto \frac{1}{M_{\rm ETC}^2}\langle\bar{Q}Q\rangle_{\rm ETC}$
- $\frac{1}{M_{\rm ETC}^2}\bar{q}q\bar{q}q \longrightarrow$ extra FCNC's (harmful!)
- $\frac{1}{M_{\rm ETC}^2}\bar{Q}Q\bar{Q}Q \longrightarrow$ explicit χSB in the techniquark sector

$\langle\bar{Q}Q\rangle_{\rm ETC}$: condensate evaluated at the ETC scale
$\langle\bar{Q}Q\rangle_{\rm EW}$: condensate at TC$\sim$EW) scale

Extended technicolor

I) $\bar{q}q\bar{q}q$ -term leads to unwanted FCNC's. In order to be compatible with precision electroweak tests, we must have $\Lambda_{\rm ETC} \approx M_{\rm ETC} \gtrsim 1000 \times \Lambda_{\rm EW} (\Lambda_{\rm TC} \approx \Lambda_{EW})$

II) For EWSB we must have $\langle \bar{Q}Q \rangle_{\rm EW} \propto \Lambda^3_{\rm EW}$

III) On the other hand, $\langle \bar{Q}Q \rangle_{\rm ETC} \propto m_q M^2_{\rm ETC}$ (top quark!)

- Using RG evolution

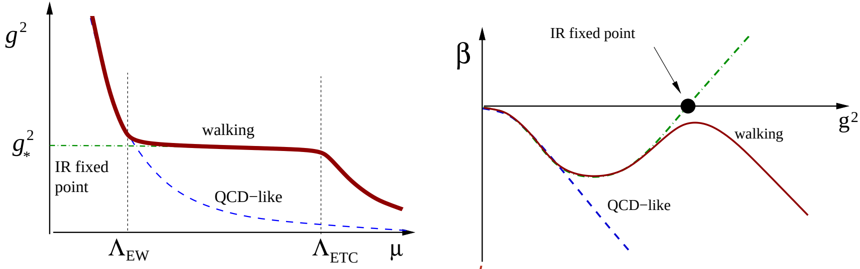
$$\langle \bar{Q}Q \rangle_{\rm ETC} = \langle \bar{Q}Q \rangle_{\rm EW} \exp \left[\int_{\Lambda_{\rm EW}}^{M_{\rm ETC}} \frac{\gamma(g^2)}{\mu} d\mu \right]$$

  where $\gamma(g^2)$ is the mass anomalous dimension.
- In weakly coupled theory γ is small, and $\langle \bar{Q}Q \rangle$ is $\sim$ constant.
- *Thus, it is not possible to satisfy the constraints I), II), II) in a QCD-like theory, where the coupling is large only on a narrow energy range above χSB.*

Walking coupling

- If the coupling *walks*, i.e. if $g^2 \approx g_*^2$ (constant) over the range from TC to ETC, then we can solve $\langle \bar{Q}Q \rangle_{\mathrm{ETC}} \approx \left(\frac{\Lambda_{\mathrm{ETC}}}{\Lambda_{\mathrm{TC}}}\right)^{\gamma(g_*^2)} \langle \bar{Q}Q \rangle_{\mathrm{TC}}$ (condensate enhancement)
- Inserting II) and III) we obtain

$$\gamma(g_*^2) \approx 1 - 2$$



- In a walking theory the β-function $\beta = \mu \dfrac{dg}{d\mu}$ reaches almost zero near g_*^2.
- If the β-function hits zero there is an IR fixed point, where the system becomes *conformal*.

Perturbative β-function

2-loop universal β-function for SU(N_c) gauge theory with N_f fermions:

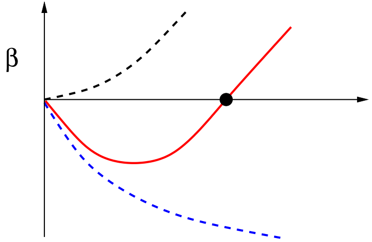
$$\beta(g) = -\mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

where the coefficients are

$$\beta_0 = \frac{11}{3} C_r - \frac{4}{3} T_r N_f, \qquad \beta_1 = \frac{34}{3} C_r^2 - \frac{20}{3} C_r T_r N_f - 4 C_r T_r N_f$$

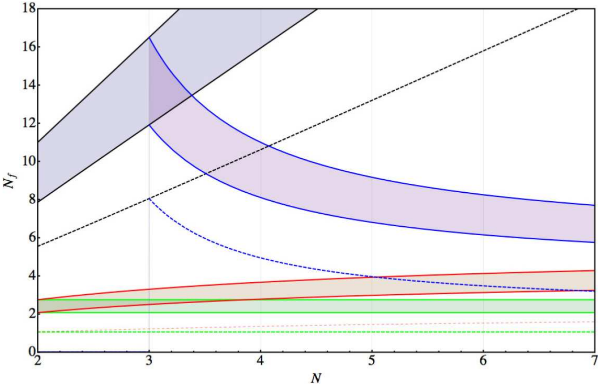
When N_f is varied, generically 3 different behaviours seen:

- confinement and χSB at small N_f
- IR fixed point (conformal window) at medium N_f [Banks,Zaks]
- Asymptotic freedom lost at large N_f



Conformal window in SU(N) gauge

fundamental



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

2-index antisymmetric

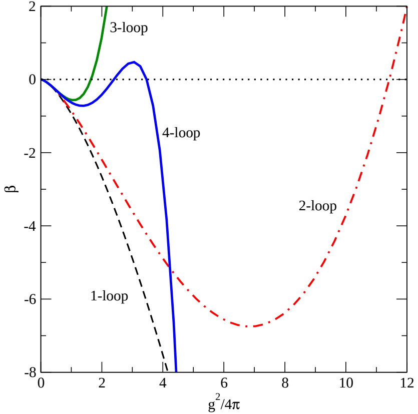
2-index symmetric

adjoint

- Upper edge of band: asymptotic freedom lost
- Lower edge of band: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- In higher reps it is easier to satisfy EW constraints [Sannino,Tuominen,Dietrich] → lot of recent activity!

Existence of the IRFP essentially non-perturbative

Example: Perturbative β-function of SU(2) gauge with $N_f = 6$ fundamental rep fermions

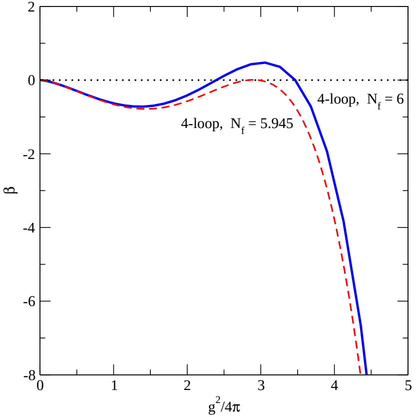


[4-loop MS: Ritbergen, Vermaseren, Larin]

Results from lattice: existence of IRFP inconclusive, maybe at $g_*^2/4\pi \sim 1.2$ ($g^2 \sim 15$) [Karavirta et al]

“Walking” at $N_f \lesssim 6$

Interestingly, the fixed point vanishes from 4-loop MS beta function if N_f is slightly lowered from 6:



Goals:

Take SU(N) gauge theory with N_f fermions in some representation.

- Locate the lower edge of the conformal window
- Measure $\beta(g^2)$-function
- Measure $\gamma(g^2)$
- We want to find a theory which
  - is walking or
  - is just within conformal window (easy to deform into walking)
  - has large anomalous exponent γ near FP
    - AdS-QFT: Indications that $\gamma = 1$ at the lower edge of the conformal window [Järvinen et al.]
  - Compatible with EW precision measurements (S,T,U -parameters) $\rightarrow$ small N_f preferred!
- Note: walking is automatic just below the conformal window!
- Technicolor phenomenology: SU(2) or SU(3) gauge theory with $N_f = 2$ adjoint or 2-index symmetric representation fermions.
- "Hadron" spectrum, chiral symmetry breaking pattern

Models studied

Red: conformal Blue: χSB Black: unclear

- SU(3) + $N_f = 8$–16 fundamental rep:
  - $N_f = 8$: Appelquist et al; Deuzeman et al; Fodor et al; Jin et al
  - $N_f = 9$: Fodor et al
  - $N_f = 10$: Hayakawa et al; Appelquist et al
  - $N_f = 12$: Hasenfratz; Appelquist et al; Deuzeman et al; Xin and Mawhinney; Fodor et al
  - $N_f = 16$: Damgaard et al; Heller; Hasenfratz; Fodor et al
- SU(2) + fundamental rep fermions:
  - $N_f = 4$: Karavirta et al
  - $N_f = 6$: Del Debbio et al; Karavirta et al; Appelquist et al (unclear)
  - $N_f = 8$: Iwasaki et al
  - $N_f = 10$: Karavirta et al
- SU(2) + $N_f = 2$ adjoint rep: Catterall et al; Bursa et al; Hietanen et al; De Grand et al
- SU(3) + $N_f = 2$ 2-index symmetric rep: DeGrand et al; Sinclair and Kogut; Fodor et al
- SU(4) + $N_f = 2$ 2-index symmetric rep: DeGrand et al

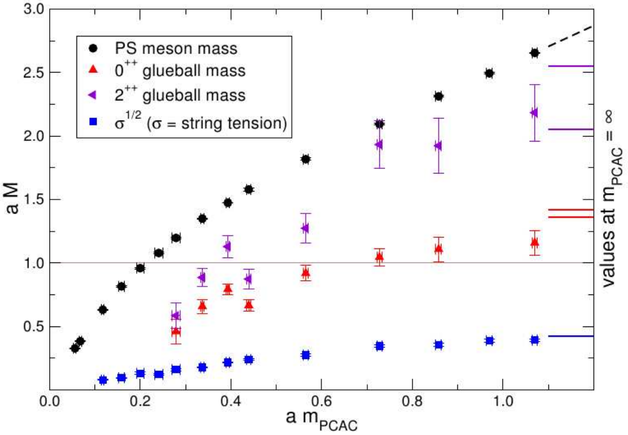
at SEWM '12:

- Rago: SU(2) + $N_f = 2$ adjoint rep.
- de Forcrand: SU(3), many N_f fundamental rep.
- Bennett: SU(2) + $N_f = 1$ adjoint rep.
- Bursa: SU(2) + $N_f = 2$ adjoint rep.
- Lucini: SU(2) gauge w. fundamental/adjoint action
- Miura: SU(3) + fundamental rep.
- da Silva: SU(3) + fundamental rep.

Classifying conformal / χ SB ?

- Measure β-function directly
  - Schrödinger functional
  - MCRG
- Measure technihadron masses as functions of the techniquark mass m_Q:
  - Conformal: $M \propto m_Q^{1/(1+\gamma)}$
  - χSB: $M_\pi \propto m_Q^{1/2}$, others remain massive

Mass spectrum of SU(2)+$N_f = 2$ adjoint fermions



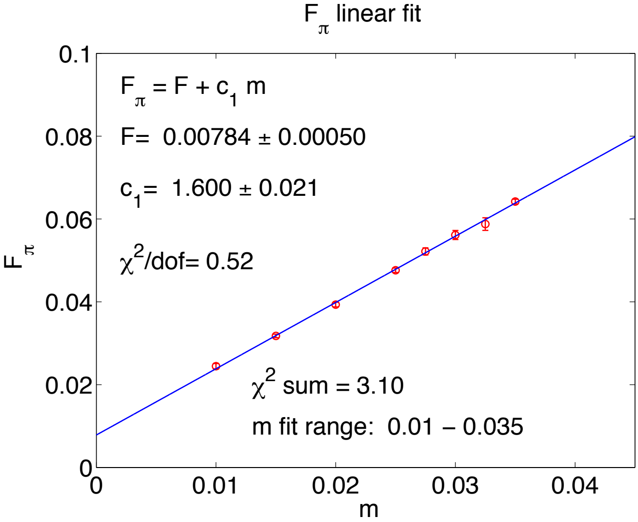
[Del Debbio et al]

Spectrum becomes massless, inverted hierarchy when compared with QCD [Miransky]

Talk by Rago

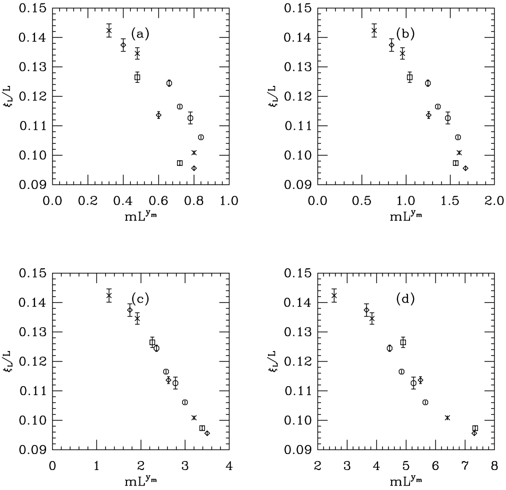
SU(3) $N_f = 12$ spectrum

F_π: non-zero intercept as $m_Q \to 0$? Looks QCD-like (χSB)



[Fodor, Holland, Kuti, Nogradi, Schroeder, 2011]

However: using critical scaling ansatz $ML = f(L^{y_m} m_Q), y_m = 1 + \gamma$ the same data appears to fall on a scaling curve:

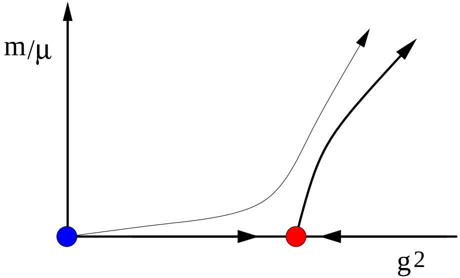


Result: No χSB, $\gamma \approx 0.4$

[DeGrand 2011]

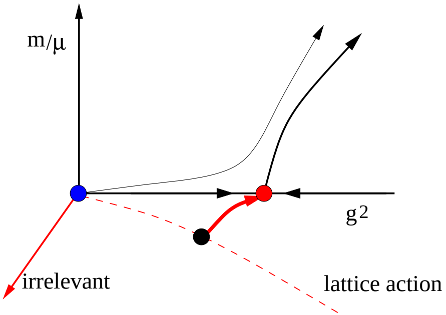
RG flow in the conformal case

- Relevant parameters at UV: g^2 and m_Q



- Only m_Q is relevant at the IRFP.
- Scaling near IRFP: masses of physical particles $M \propto (m_Q)^{1/(1+\gamma)}$

RG flow on the lattice

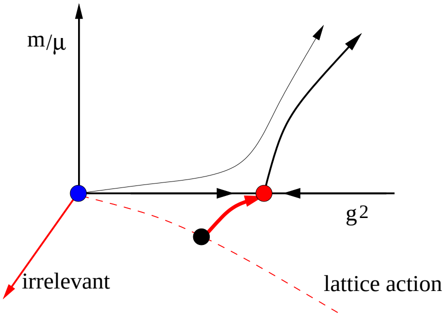


- Irrelevant operators (cutooff effects) die out as a/L, $(a/L)^2 \ldots$ (L: IR scale)
- Evolution of g^2 along the physical axis very slow

$\Rightarrow$ irrelevant operators can (and do!) mask the physical evolution

- Need either:
  - Very large lattices (large L/a) – impractical
  - Very high quality lattice action – small cutoff effects

RG flow on the lattice



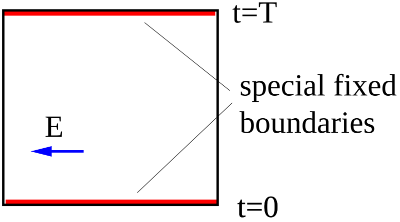
- Near the IRFP, the continuum limit cannot be taken at weak bare coupling:
- Even when the lattice spacing $a \to a/100$, the bare coupling barely decreases
- Present experience: we really need highly improved lattice actions which work at strong coupling
  - Non-perturbative (thin-link) clover – not sufficient?
  - nHYP smeared clover
  - perfect gauge? perfect fermions?
  - do staggered quarks work?

Measuring the coupling

Schrödinger functional: Generate a *background* chromoelectric field using non-trivial fixed boundary conditions, parametrised with a **twist angle** η

At the classical level, we have

$$\frac{dS_{\text{class.}}}{d\eta} = \frac{A}{g^2}$$



where $A(\eta)$ is a known constant.
At the quantum level, we define the coupling through

$$\frac{1}{g^2} = \left\langle \frac{1}{A} \frac{dS}{d\eta} \right\rangle$$

- Evaluates g^2 directly at scale $\mu = 1/L$, the lattice size
- Can use $m_Q = 0$
- Has been used very succesfully in QCD by the Alpha collaboration

Step scaling function

- Step scaling: coupling when the lattice size is (e.g.) doubled

$$\Sigma(u, L/a) = g^2(g_0^2, 2L/a)_{u=g^2(g_0^2, L/a)}$$

- Continuum limit:

$$\sigma(u) = \lim_{a/L \to 0} \Sigma(u, L/a)$$

- Step scaling is related to β-function:

$$-2 \ln 2 = \int_u^{\sigma(u)} \frac{dx}{\sqrt{x}\beta(\sqrt{x})}$$

- Close to the fixed point:

$$\beta(g) \approx \frac{g}{2 \ln 2} \left(1 - \frac{\sigma(g^2)}{g^2}\right)$$

- 1-loop analysis indicates that finite lattice spacing effects large ($\sim 50\%$ at $L/a = 10$) $\Rightarrow$ improvement! [Alpha; Karavirta et al.]

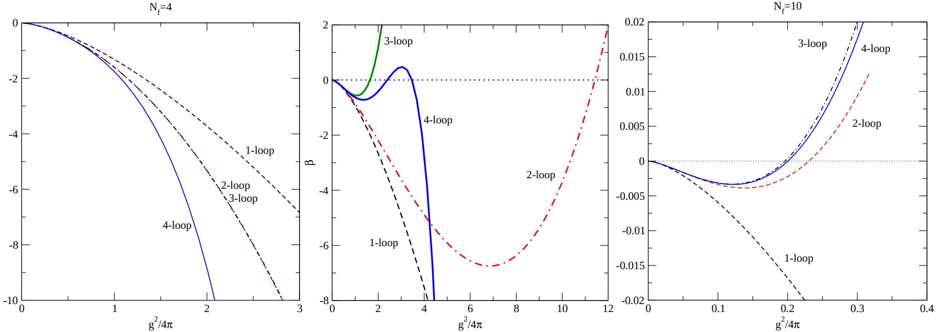
SU(2) fundamental representation at $N_f = 4, 6, 10$

Fundamental rep SU(2) with $N_f = 4, 6$ and 10

- Measure coupling using SF
- Measure γ also using SF (different)
- Choose:
  - $N_f = 4$: QCD-like, chiral symmetry breaking
  - $N_f = 6$: $\sim$ lower edge of conformal window
  - $N_f = 10$: upper edge of conformal window
- We use 1-loop perturbative c_{SW}, with perturbative boundary improvement coefficients

Fundamental rep: perturbation theory

Perturbative β-function w. $N_f = 10$ and $N_f = 6$ [3,4-loop MS: Ritbergen, Vermaseren, Larin]

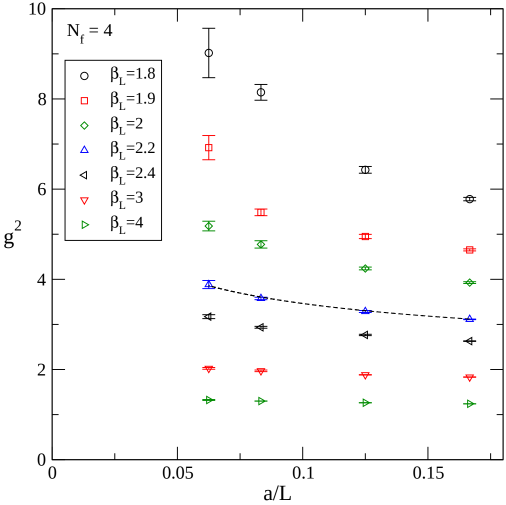


$N_f = 4$ QCD-like, confining

$N_f = 6$ completely non-perturbative

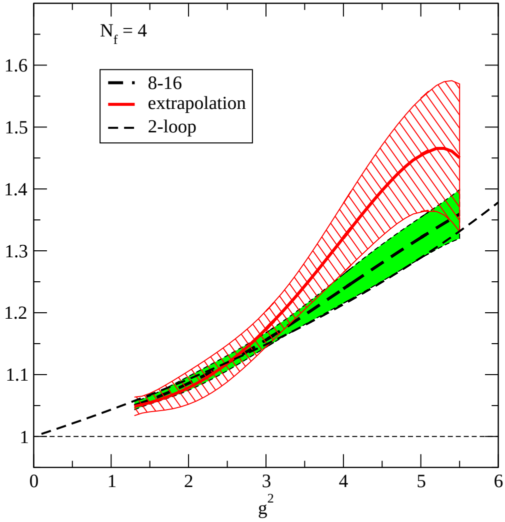
$N_f = 10$ perturbative Banks-Zaks FP, "test case".

$N_f = 4$:



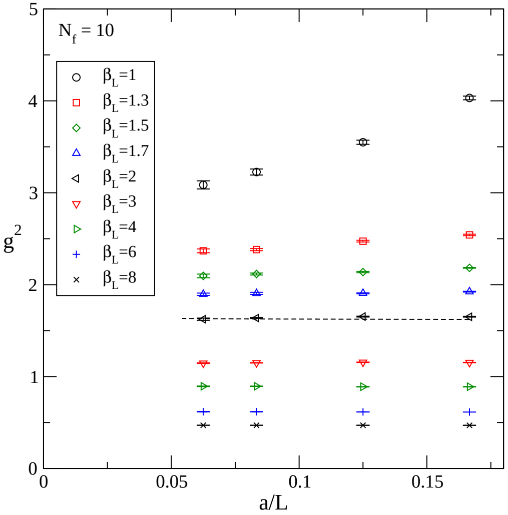
g^2 grows as L grows – QCD-like

Step scaling function: $N_f = 4$



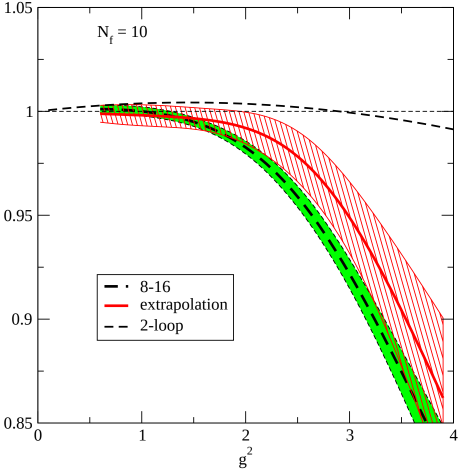
QCD-like behaviour

$N_f = 10$



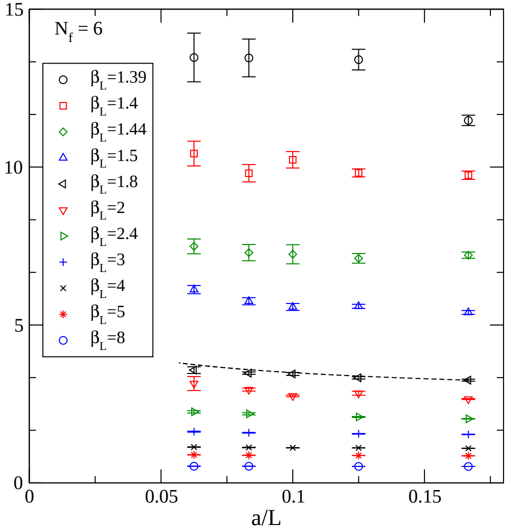
N_f = 10
β_L=1
β_L=1.3
β_L=1.5
β_L=1.7
β_L=2
β_L=3
β_L=4
β_L=6
β_L=8
g²
a/L
0
0.05
0.1
0.15
0
1
2
3
4
5

Step scaling function: $N_f = 10$



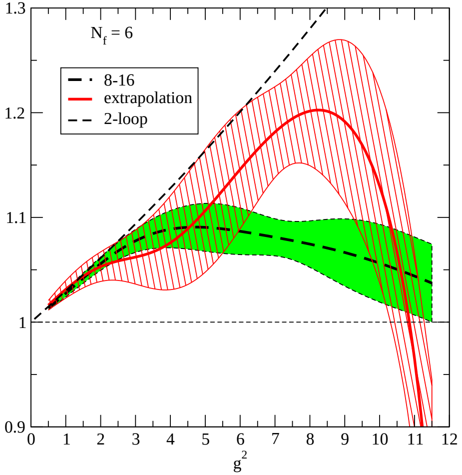
- We see $\sim$ zero evolution below $g^2 \sim 2.5$
- Above this step scaling diverges from perturbative curve.
- It is caused by our $\beta_L = 4/g_0^2 = 1$ data set – strong coupling, lattice artefact?

$N_f = 6$



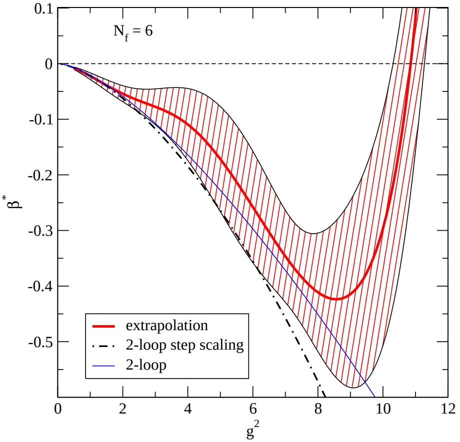
- Does it grow any more? Borderline
- Very strong coupling

Step scaling function: $N_f = 6$



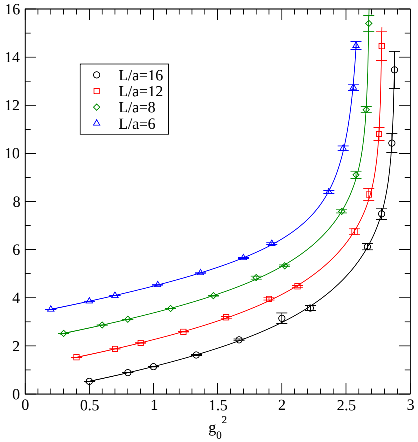
- Perhaps IRFP at $g^2 \gtrsim 12$ ($\alpha \gtrsim 1$)?
- Lose control at $g^2 \sim 12 - 14$ ($\beta_L \approx 1.39$)
- Need to have actions which work there

β-function: $N_f = 6$



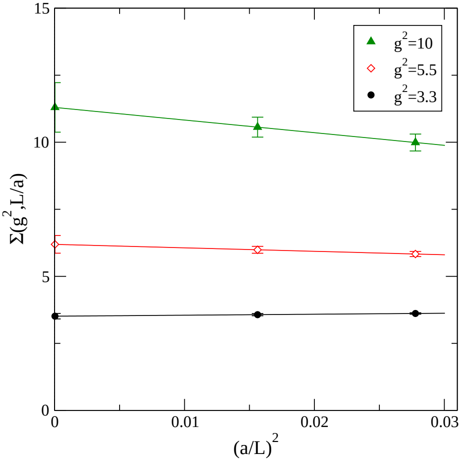
- Interpolate data with the rational function

$$\frac{1}{g^2(\beta_L, L/a)} = \frac{1}{g_0^2}[1 + \sum_{i=1}^{n} a_i g_0^{2i}]/[1 + \sum_{i=1}^{m} b_i g_0^{2i}].$$

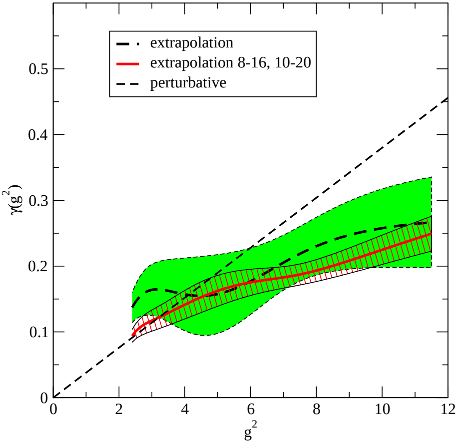


- Construct step scaling using pairs L/a=(6,12) and (8,16)
- $\Sigma(u, L/a) = g^2(2L/a)_{g^2(L/a)=u}$

- We use 2nd order in (a/L) extrapolation to continuum
- Or, use only 8-16 (largest volume step) without interpolation
- $N_f = 6$, 3 arbitrarily chosen $u = g^2$-values:



Result: $N_f = 6$ mass anomalous exponent



- More robust than coupling
- Smaller than perturbative at strong coupling – generic feature?

What do the results imply?

- Measurement of the coupling constant evolution is significantly more difficult than in QCD-like theories:
  - Slow evolution $\rightarrow$ small signal
  - Slow evolution $\rightarrow$ strong bare coupling
- We need to develop actions which
  a) can be used at strong lattice scale coupling
  b) have as small as possible cutoff effects there
- Action engineering:
  - Improvement (non-perturbative?)
  - Smearing
  - More "perfect" actions?
- With Schrödinger functional, incorrect boundary improvement or too large background field (higher reps) can be expected, a priori, to give $\sim$ 50% finite size effects at $L/a \sim 10$.

Conclusions

- Status of the field: early days still. No full consensus yet of the "best practices".
- Small signal, sizable cutoff effects: difficult to get reliable results.

$\rightarrow$ large statistics, improvement

- Lower edge of the conformal window:
  - SU(2) + fundamental rep: $N_f \sim 6$ or slightly above
  - SU(3) + fundamental rep: $N_f \sim 12$(?)
  - $N_f = 2$ adjoint rep SU(2), SU(3) appears to be conformal
  - SU(3) + 2-index symmetric rep: probably conformal
- Walking not observed (except in toy models)

Walking in 2d O(3)

2-d O(3) model with topological charge [de Forcrand, Pepe, Wiese]

$$S = \frac{1}{2g^2} \int d^2x \, \partial_i u_a \partial_i u_a + i\theta Q$$

with $|u| = 1$

$$Q = \int d^2x \epsilon_{ij} \epsilon_{abc} U_a \partial_i U_b \partial_j U_c$$

- asymptotically free
- mass gap
- has a IR fixed point at $\theta = \pi$! (integrable model)
- Adjusting θ the degree of "walking" can be changed

Walking in 2-d O(3)