Effective potential for O(N) quantum scalar field in de Sitter geometry

Julien Serreau

APC - University Denis Diderot
(Paris 7)

Motivations

QFT on curved background / geometry

Understanding loop effects

- Radiative corrections to inflationary dynamics (power spectra, nongaussianities, decoherence, reheating ...)
- (Analog) Black Hole radiation
- Curvature-induced phase transitions

...

- Mathematical consistency of QFT on non-trivial geometries

Motivations

A test scalar field in de Sitter

(Inflation)

- Expanding geometry ≡ nonequilibrium

⇒ perturbation theory is <u>secular</u>

[Tsamis, Woodard ('95); Weinberg ('05); ...]

- Massless, minimally couped field

⇒ perturbation theory is <u>IR divergent</u>

[e.g. Van der Meulen, Smit ('07) ...]

⇒ Alternative schemes (re-summations) : e.g. 2PI

[Berges, J.S., SEWM 04]

<u>See also</u> : [Ramsey, Hu ('97); Riotto, Sloth ('08); Tranberg ('08); Garbrecht, Rigopoulos ('11)]

DRG → [Burgess, Leblond, Holman, Shandera ('10)]

Model and approximation

$\boxed{D = d+1}$

$$S = -\int d^D x \sqrt{-g} \left\{ \varphi_a (\Box + \mu^2) \varphi_a + \frac{\lambda}{4! N} (\varphi_a \varphi_a)^2 \right\}$$

$ds^2 = dt^2 - e^{2Ht} dx^2$ (flat spatial sections)

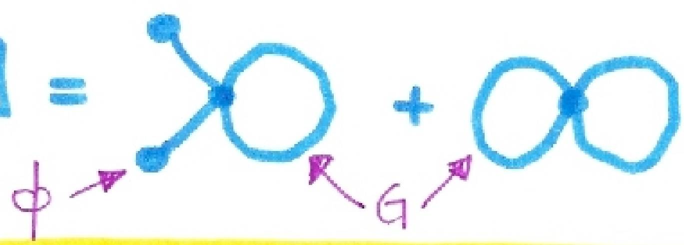
$\Box = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \right)$

$$\mu^2 = m^2 + \xi R \quad \text{and} \quad R = d(d+1) H^2$$

+

(2PI) $1/N$ - expansion at L.O.

$\Gamma_2[\phi, G] =$



=

$$[\Box + M^2(x)]\, \phi(x) = 0$$

$$[\Box + M^2(x)]\, G(x,y) = -i\, \delta^{(D)}(x,y)$$

$$M^2(x) = \mu^2 + \frac{\lambda}{6} \phi^2(x) + \frac{\lambda}{6} G(x,x)$$

Reminder : flat space-time (ST)

$(H \to 0)$

① ST translation invariant state (e.g. vacuum)

↳ $\phi(x) = \phi = \text{const.}$ and $G(x,y) \equiv G(x-y)$

↳ $G(x,x) = \text{const.}$ and $M^2(x) = \text{const.}$

$$\Rightarrow \boxed{M^2 = m^2 + \frac{\lambda}{6}\phi^2 + \frac{\lambda}{6}\int \frac{d^Dk}{(2\pi)^D}\frac{i}{k^2 - M^2}}$$

(+ renormalization)

$\Rightarrow \boxed{M^2\phi = 0}$ → $\boxed{\phi = 0}$ Sym. phase

or

→ $\boxed{\begin{matrix} M^2 = 0 \\ \phi \neq 0 \end{matrix}}$ Spont. broken symmetry

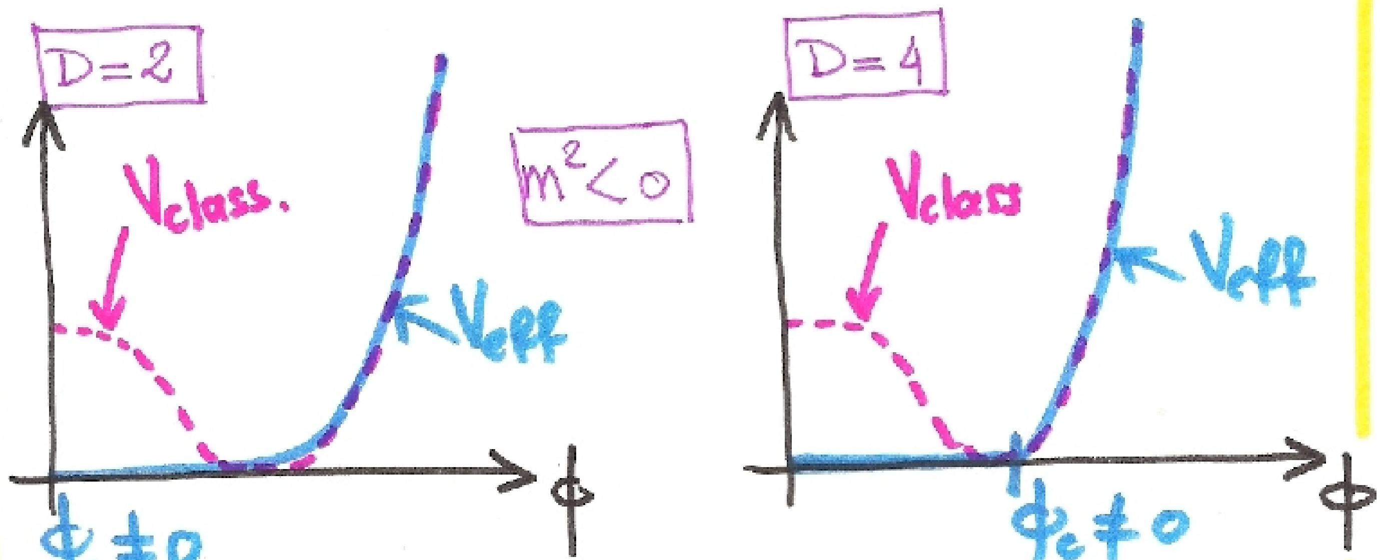
Only for $D > 2$

Infra-Red (IR) effects prevent SSB in $D = 2$

Reminder : flat ST

Effective potential

$$V_{eff}(\phi) = \frac{1}{2}\int_0^{\phi^2} d\varphi^2\, M^2(\varphi^2) + \text{const.}$$



Finite temperature

- Phase transition at $\underline{T_c \neq 0}$ for $D>2$
- Thermal mass generation

  e.g. high T : $\boxed{M^2 \sim \lambda T^2}$

de Sitter space-time

$(H \neq 0)$

dS-invariant state

$\hookrightarrow \phi(x) = \phi = \text{const.}\ ;\ G(x,x') \equiv G(\underline{\underline{z}})$

$$z(x,x') = \operatorname{ch} H(t-t') - \frac{|\vec{x}-\vec{x}'|^2}{2} e^{H(t+t')}$$

$\hookrightarrow G(x,x) = \text{const.}$ and $M^2(x) = \text{const.}$

$$\boxed{M^2 = \mu^2 + \frac{\lambda}{6}\phi^2 + \frac{\lambda}{6}\kappa \int \frac{d^d z}{(2\pi)^d} |H_{\underline{\underline{\nu}}}(z)|^2}$$

(+renormalization)

Hankel function

$$\boxed{\nu = \sqrt{\frac{d^2}{4} - \frac{M^2}{H^2}}}$$

IR issues in dS.

$$|H_\nu(z)|^2 \underset{z\to 0}{\sim} z^{-2\nu} \underset{(M^2\to 0)}{\sim} z^{-d}$$

Strongly enhanced IR fluctuations

(Compare to z^{-1} in Minkowsky ST)

Perturbative expansion is ill-defined <u>in any dimension</u> (for $\mu^2=0$)

Large-N resums IR div. in a well-defined expression

$$\boxed{\text{Requires } \nu < \frac{d}{2} \Leftrightarrow M^2 > 0}$$

$\boxed{\text{IR effects prevent the existence of a dS-inv. broken phase in any } d \text{ !!}}$

Solving the gap-equation

Case $\frac{M^2}{H^2} \gg 1$ $\longrightarrow$ $\nu \approx i\frac{M}{H}$

$$M^2(\phi^2) = \mu^2(\phi^2) + \frac{\lambda}{6}\int\frac{d^d z}{(2\pi)^d}\frac{1}{2\sqrt{z^2+M^2}} + \mathcal{O}\left(e^{-M/H}\right)$$

$$\mu^2 + \frac{\lambda}{6}\phi^2$$

Minkowsky limit ✓

Case $\frac{M^2}{H^2} \ll 1$ $\longrightarrow$ $\nu = \frac{3}{2} - \frac{M^2}{3H^2}$ $\quad d=3$

$$M^2(\phi^2) = \mu^2(\phi^2) + \frac{\lambda H^2}{48\pi^2}\left(\frac{3H^2}{M^2(\phi^2)} + c\right)$$

$$\boxed{M^2(\phi^2) = a(\phi^2) + \sqrt{a^2(\phi^2) + b}}$$

$$a(\phi^2) = \frac{1}{2}\mu^2(\phi^2) + \frac{c\lambda H^2}{96\pi^2} \qquad b = \frac{\lambda H^4}{16\pi^2}$$

see also [Garbrecht, Rigopoulos ('14)]

Mass generation

- Minimally coupled massless field

$$\boxed{\mu^2 = m^2 + \xi R = 0}$$

- Loop diagrams are IR divergent
- Self-interactions generate a non-zero mass (at $\phi = 0$)

$$\boxed{\frac{M^2}{H^2} = \frac{c\lambda}{96\pi^2} + \sqrt{\frac{\lambda}{16\pi^2} + \left(\frac{c\lambda}{96\pi^2}\right)^2}}$$

$$\boxed{M^2 \sim \sqrt{\lambda}\, H^2}$$

[Starobinsky, Yokoyama ('94)]

[Compare to $M^2 \sim \lambda T^2$ at finite T]

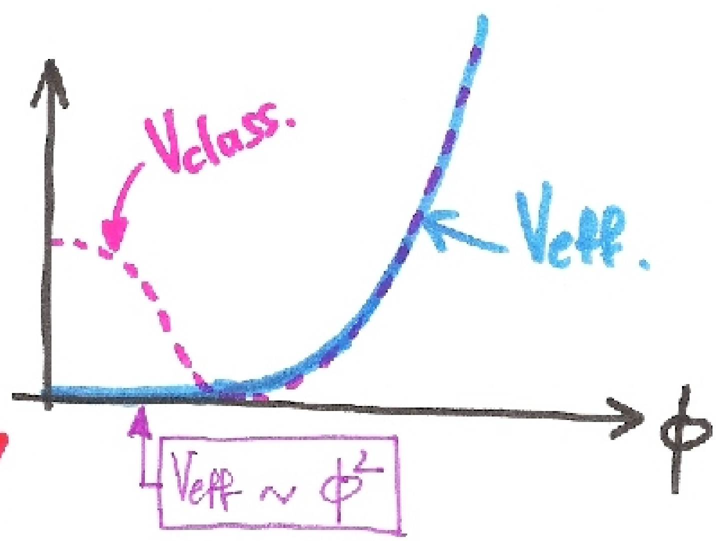
Enhancement of IR-sensitive obs. (e.g. non-gaussianities) [Riotto, Sloth ('08)]

Curvature-induced symmetry restoration

- SSB at classical level : $\boxed{\mu^2 < 0}$

- The effective potential gets lifted

$$\boxed{V_{eff} \simeq \frac{3}{2\lambda} M^4(\phi^2) + \frac{3H^4}{16\pi^2} \ln \frac{M^2(\phi^2)}{M^2(0)} + \text{const.}}$$



H arbitrary !

$$\boxed{M^2(0) \simeq \frac{\lambda H^4}{16\pi |\mu^2|} \sim \frac{H^4}{\phi_c^2}}$$

Stronger enhancement of IR sensitive observables ?

Conclusions

- A new, exact result of QFT on dS.
  (Large-N limit : exact solution)
- $\exists$ dS-invariant state for a massless minimally coupled interacting scalar field
- dS IR effects prevent the existence of (dS-inv.) spontaneously broken phase in any d and for arbitrary weak curvature
  (No curvature-induced phase transition)
- Gravitational lifting of Minkowsky results (modes $P/H \sim e^{-3H^2/2M^2}$)

PROSPECTS

- Non Gaussian correlators in the large-N limit.

IR enhancement at <u>tree level</u>

[Riotto, Sloth ('08)]

Large-N :

[Serreau, in progress]

- $1/N$ corrections

NLO : non vanishing self energy

$$\Sigma_F, \Sigma_\rho$$

Crucial to study the damping of unequal time correlators.

[Parentani, Serreau, in progress]