Gauging Away the Big-Bang:
Toy Models from Higher Spins and Strings

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Background and Philosophy

General Relativity requires short distance modifications.
One reason: the existence of an infinite number of independent UV divergences and the accompanying loss of predictivity.

String theory solves this problem because it has an enormous gauge symmetry, called conformal invariance. Even though the theory contains gravity and new UV particles, this gauge symmetry relates all the new couplings arising in perturbation theory.

Analogue: Fermi theory vs massive vector boson theory vs broken $SU(2) \times U(1)$ gauge theory.

Apart from the quantum problem of divergences, there is also a purely classical reason why we expect that gravity might require modifications at short distances.

This is because in Einstein gravity, spacetime singularities are generic. Problematic, if we believe in causality even in the most primitive sense.

This is again (possibly) a short-distance problem. Since strings are finite in the UV, they may resolve singularities.

Some mechanisms for handling singularities are known in string theory, eg: honest-to-god resolutions of various kinds [Strominger, APS, ...] or cover-ups behind horizon [Dabholkar-Kallosh-Maloney, ...],...

Cosmological singularities are even harder. Why?

Because Cosmology $\Longrightarrow$ time dependence, and we don't know how to quantize string theory in time-dependent backgrounds.

Typically we only understand how to quantize string theory in supersymmetric backgrounds, and supersymmetry makes states automatically time independent.

Punchline: understanding singularities in cosmology from the context of string theory is doubly hard.

One way forward is to consider cosmological quotients of flat space as simple examples of time dependent singular backgrounds. The idea is that since the covering space is flat, we should be able to use the tools from flat space string theory to explore these singular geometries [numerous papers].

Two popular examples: Null orbifold [Horowitz-Steif, Simon,...] & Milne orbifold [pre-historic], time dependent orbifolds of flat space.

However, it is known that 4-point tree level string scattering amplitudes on the null orbifold [Liu-Moore-Seiberg] and Milne orbifold [Berkooz-Craps-Kutasov-Rajesh-Pioline-Nekrasov-...] has divergences.

So things dont look good. State of the art in [Horowitz-Polchinski]: their philosophy is to take the amplitude at face value and interpret the divergence as due to black hole formation when the string is inside its own Schwarzschild radius, a.k.a large backreaction when it is pointlike ($\alpha' \to 0$).

What can be learnt from long and floppy strings?

Enter Higher Spin Theories: Vasiliev has constructed interacting theories with gravity and higher spin fields.

There is evidence that higher spin theories capture the $\alpha' \to \infty$ (tensionless) limit of string theory [Sundborg, Witten, Minwalla-Yin-..., Gaberdiel-Gopakumar]

So one can roughly think of higher spin theory as a theory in spacetime for the worldsheet spectrum of states of the string, in which all the massive modes have become massless (masses are inversely related to α').

We will adopt this philosophy.

In the tensionless limit, higher spin theories capture some(?) of the stringy gauge invariances as bigger spacetime gauge symmetries than diffeomorphisms. Diffeomorphisms $\subset$ Higher spin symmetries.

Diffeomorphisms (freedom to change coordinates) cannot remove singularities, but these bigger gauge invariances might change that.

So: Maybe **some** of the singularities are just artifacts of a choice of gauge in string theory? Can a higher spin gauge transformation put Milne/Null orbifold in a non-singular gauge?

We will embed Milne in three dimensions and work with flat space higher spin theory, because the theory has a Chern-Simons formulation, which allows a finite spin truncation.

[Afshar-Bagchi-Fareghbal-Grumiller-Rosseel,Gonzalez-Matulich-Pino-Troncoso]

This is good because higher dimensional Vasiliev theories are far more complicated, and do not allow a simple truncation of this kind.

Because the orbifolds we will consider are quotients of 2D flat space, we can always embed them in 3D.

In 3D, the higher spin situation is formally very similar to pure gravity (spin-2). For spin-2, the idea is that Einstein-Hilbert action can be written in 3 dimensions as

$$I_{EH} \sim I_{CS}[A] - I_{CS}[\tilde{A}] \tag{1}$$

where

$$I_{CS}[A] = \frac{k}{4\pi} \int_M \mathrm{Tr}\Big[A \wedge dA + \frac{2}{3} A \wedge A \wedge A\Big] \tag{2}$$

with

$$A = (\omega + \epsilon e), \;\; \tilde{A} = (\omega - \epsilon e) \tag{3}$$

where

$$A = A^a_\mu T_a dx^\mu = (\omega^a_\mu + \epsilon e^a_\mu) T_a dx^\mu \equiv (\omega + \epsilon e), \tag{4}$$

and similarly for $\tilde{A}$. Here T_a are the generators of $SL(2)$:

$$[T_a, T_b] = \epsilon_{abc} T^c \tag{5}$$

Even thought the generators satisfy SL(2) algebra, the theory is flat space gravity if we take ϵ to be a Grassmann parameter [CK-S.Roy-A.Raju].
Aside: The asymptotic symmetries also go over from Brown-Henneaux to BMS with correct central charge as expected from earlier work [Compere-Barnich-Bagchi-...].

If we increase the rank of the gauge group from SL(2) to SL(N) the theory becomes a higher spin theory [Campoleoni-et-al.]. A very simple set up.

Milne Geometry a quotient of flat space:

$$ds^2 = -dT^2 + r_C^2 dX^2 + \alpha^2 T^2 d\varphi^2, \tag{6}$$

X is noncompact and $\phi \sim \phi + 2\pi$. The parameters can be set to 1.

Space time behaves like a double cone and there is a causal singularity at $T = 0$ where ϕ-circle crunches to a point before expanding in a big-bang. From the triads and spin connection

$$e^0 = dT, \;\; e^1 = r_C dX, \;\; e^2 = \alpha T d\varphi, \tag{7}$$

$$\omega^0 = 0, \;\; \omega^1 = \alpha d\varphi, \;\; \omega^2 = 0. \tag{8}$$

the Chern-Simons connection for Milne is then

$$A^{\pm} = \pm\left(\epsilon\, dT\right) T_0 + \left(\alpha d\varphi \pm \epsilon\, r_C dX\right) T_1 \pm \left(\epsilon\, \alpha T d\varphi\right) T_2. \tag{9}$$

We want to do a gauge transformation that preserves the holonomy of the solution, since Einstein solutions correspond to flat connections.

The ϕ-circle holonomy matrix can be directly computed to be $\omega_\phi^\pm = 2\pi\alpha\,(T_1 \pm \epsilon T\,T_2)$ it has the eigen values $(0, 2\pi\alpha)$. The characteristic polynomials coefficients of these holonomy matrices are

$$\Theta^0_\varphi \equiv \det(w_\varphi) = 0, \quad \Theta^1_\varphi = \mathrm{tr}\left(w^2_\varphi\right) = 8\pi^2\alpha^2, . \tag{10}$$

the $\pm$ superscript is dropped as the polynomials are identical for both.

The higher spin gauge transformed solution that we consider should also have same characteristic polynomial for it to describe the same physical configuration.

Adding the higher spin components, we get

$$A' = A + \sum_{n=-2}^{2} (C^n + \epsilon\, D^n)\, W_n. \tag{11}$$

where C^n and D^n are frame fields and connection associated with new higher spin generators W_n. If work with a spin-3 theory $n \in -2, -1, 0, 1, 2$.

Our goal is to find the simplest resolution, so we try the coefficients to be constants, turns out that this works.

We make two demands-
1. The holonomy is preserved for the new connection.
2. It is still a flat connection.

Turns out we can satisfy both these conditions if we set $D_\phi^{\pm} = 0, D_\phi^{-2} = 0$ together with $D_\phi^0 = 3D_\phi^2$ and all the C's set to zero. The resultant metric is identical to Milne except now,

$$g'_{\phi\phi} = g_{\phi\phi} + 12(D_\phi^2)^2 \tag{12}$$

Since $g_{\phi\phi} \sim r^2$, this means that the singularity is now resolved to a circle of minimum radius, there is a bounce.

The curvature scalars are everywhere finite.

The solution now contains non-trivial (but regular, upto a subtlety) higher spin fields which can be thought of as the matter supporting the throat.

Connection to string theory

We expect morally that the tensionless limit of string theory should capture aspects of higher spin theories, even though precise proposals exist only in specific cases [Minwalla-et-al(AdS4), Gaberdiel-Gopakumar(AdS3)].

If this is true, the divergences in the amplitudes that people have considered previously should not arise in the tensionless limit, if our resolutions are capturing something physical.

(The scattering amplitude captures gauge-invariant information).

When we say tensionless (ie. $\alpha' \to \infty$) what we mean is the dimensionless α'.

Since we are working with quotients of flat space, the only available dimensionless α' has to be constructed from the the momenta of the scattering states - there is no background scale.

(In AdS the AdS radius works as a one-paramter family along which we can send the dimensionless α' to ∞).

The string scattering amplitudes are complicated beasts with integrals over numerous Gamma functions and such. The two basic results that hold for Milne are [Craps-CK-Saurabh] (very similar statements hold also for Null orbifold [Kiran-CK-Saurabh-Simon]):

- All UV divergences that were previously identified as arising from the pathological singularity arise when $\alpha'(\mathbf{p}_1 - \mathbf{p}_3)^2 \leq 2$ or similar conditions hold. Roughly speaking the α' is being measured in units of momentum transfer and when it is large enough, there are no UV divergences.
- We exhaustively scan for all divergences, and all the other divergences are sensible IR divergences that are unrelated to the singularity. Eg: a whole sequence of poles giving rise to logarithmic divergences which have interpretation as the tower of intermediate string states going on-shell.

Comments, Problems, What Next

- Quotients of de Sitter space, with a cosmological interpretation, have been resolved before [CK-Roy]. Also, even before, in AdS_3 it has been noticed that horizons etc are gauge-dependent [Kraus-Gutperle, Maloney et al.].

  But Milne and null orbifold are interesting because they are flat space quotients, and allow connections with string theory. We needed flat space higher spin theories in three dimensions.
- Even though we focused on Milne, a roughly similar story holds for the null orbifold cosmology of Liu-Moore-Seiberg: Higher spins can resolve it, string scattering amplitudes are well behaved at large α' etc [Kiran-CK-Saurabh-Simon].

- One interesting aspect of the LMS orbifold is that its **C-S holonomy has trivial eigenvalues**, even though the singularity is known to be pathological.

  There are implied claims/hopes in the literature that demanding trivial CS holonomy is an indicator of regularity of the geometry: this is a counter-example.
- The simplest resolution of the LMS orbifold has the interesting peoperty that all its curvature scalars vanish. The geometry is in fact a pp-wave/Kundt geometry and in 2+1 dimensions these geometries have the VSI property.

- When we construct resolutions, we need to turn on higher spin fields. How do we know we haven't created some new kind of singularity in the higher spin field? They typically vanish somewhere in the bulk. Is this bad? Part of the reason to look at the string scattering amplitude was this question. The well-definedness of the string scattering amplitude at large α' we'll take as an indication that the resolution is legitimate. To answer this question purely from a higher spin point of view, without resorting to string theory, we will need a higher spin generalization of Riemannian geometry.

- The resolutions we constructed do not fall into the flat space boundary conditions considered in [ABFGR, GMPT]. Our hope was that there will be a class of boundary conditions which contain the resolution. Indeed such a set of boundary conditions was constructed recently by [Gary-Grumiller-Riegler-Rosseel] starting with the Grassmann approach of [CK-S.Roy-A.Raju] and non-trivial amounts of stamina and ingenuity.
- They also checked that the gauge transformations which take the singular geometry to our resolved geometry have zero canonical charge - ie., they are the same state, as one would want. This is heartening: one would expect this, since the string amplitude captures gauge invariant information.

- The $\alpha' \to \infty$ limit is precisely the opposite limit of the GR limit in string theory: strings are long and floppy, not pointlike.

  But the message that singularities might be gauge artefacts and might be resolved via gauge transformations is perhaps a useful paradigm to keep in mind.
- Can a more physical big-bang be understood as a similar gauge artefact in a symmetry-broken phase of higher spin theory?....

Thank You!